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Exact travelling solutions of two coupled (2 + 1)-Dimensional Equations

U.M. Abdelsalam

Department of Mathematics, Faculty of Science, Fayoum University, Egypt

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ABSTRACT

Extended tanh method is examined to solve the coupled Burgers system and the modified KdV–Boiti–Leon–Manna–Pempinelli (mKdV–BLMP) system. The exact travelling wave solutions are obtained. The obtained solutions include rational, periodical, singular and solitary wave solutions.

1. Introduction

Nonlinear evolution equations (NLEEs) play a crucial role in modelling nonlinear phenomena in various branches of scientific disciplines, such as optics, plasma physics, fluid mechanics, condensed matter in physics, etc., [1–12]. Studying the nonlinear waves such as soliton, breather, compacton, etc., is one of the most important problems in mathematical physics and engineering. Recently, many powerful mathematical approaches for finding exact solutions of NLEEs have been proposed, such as tanh method [12], extended tanh method [13], sine-cosine method [14], homogeneous balance [15], F-expansion method [16], generalized expansion method [17] and (G'/G) method [18,19]. Extended tanh method [20] is one of the most effective and direct methods to construct solitary wave solutions of NLEEs. Recently, Wang et al. [21] showed that the coupled Burgers system and the modified KdV–Boiti–Leon–Manna–Pempinelli (mKdV-BLMP) system are integrable by using the generalized symmetries and the Painlevé analysis. Such nonlinear systems of equations play an important role in fluid mechanics [22,23]. In this paper, we will use the extended tanh method for examining the coupled Burgers and mKdV–BMLP systems to obtain exact travelling solutions. Hence, the following manuscript is devoted to investigate the applicability of the extended tanh method to coupled Burgers and mKdV-BMLP systems. The manuscript is organised in the following fashion; In Section 2, The extended tanh method is described. In section 3 we apply extended tanh method to solve the coupled Burgers system and the mKdV–BLMP systems. conclusions are given in section V.

2. Description of the method

For the general NLEE

$$P(u, u_x, u_t, u_y, u_{xx}, \ldots) = 0 \quad (1)$$

where $u = u(x, y, t)$ and P is a polynomial in u and its derivatives. We seek its solutions in the form

$$u = \sum_{i=0}^{n} a_i \phi^i(\zeta) \quad (2)$$

with

$$\phi' = \delta + \phi^2 \quad (3)$$

where δ is a nonzero constant and a_i are arbitrary constants to be determined later, and

$$\zeta = x + y - ct \quad (4)$$

where c is the speed of the travelling wave.

step 1. Using transformation (4) we obtain an ordinary differential equation (ODE) for $u = u(\zeta)$:

$$E(u, u', u'', u''', \ldots) = 0 \quad (5)$$

E-mail addresses: usama.ahmad@rub.de, maths_us@hotmail.com

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step 2. By balancing the highest nonlinear terms and the highest-order partial differential terms in the given NLEE we can determine n.

step 3. Substituting Eqs. (2) and (3) into Eq. (5) and collecting coefficients of polynomial of ϕ, then setting each coefficient to zero yields a set of algebraic equations for a_i (i=0,1,2,...,n), δ and c.

step 4. Solving the system of algebraic equations in step 2 for a_i, δ and c using Maple or Mathematica.

step 5. As Eq. (2) possesses the general solutions:

Case 1. If $\delta < 0$, then $\phi = -\sqrt{-\delta} tanh(\sqrt{-\delta}\zeta)$

Case 2. If $\delta < 0$, then $\phi = -\sqrt{-\delta} coth(\sqrt{-\delta}\zeta)$

Case 3. If $\delta > 0$, then $\phi = \sqrt{\delta} tan(\sqrt{\delta}\zeta)$

Case 4. If $\delta > 0$, then $\phi = -\sqrt{\delta} cot(\sqrt{\delta}\zeta)$

Case 5. If $\delta < 0$, then $\phi = -1/\zeta$

3. Application of the method

3.1. The coupled Burgers system

Let us consider the (2+1)-dimensional coupled Burgers system,

$$u_t - 2uu_x - v_{xx} = 0,$$
$$v_{yt} - u_{xxy} - 2uv_{xy} - 2u_x v_y = 0 \tag{6}$$

Balancing the highest derivative term with non-linear terms leads to $n = 1$, hence we may assume that

$$u(x, y, t) = k_0 + k_1\phi(\zeta)$$
$$v(x, y, t) = \mu_0 + \mu_1\phi(\zeta) \tag{7}$$

where $\zeta = x + y - ct$. Substituting Eqs. (7) and (3) into Eq. (6) and collecting coefficients of polynomial of ϕ^i and equating them to zero, we get a system of algebraic equations for k_0, k_1, μ_0 and μ_1

$$-c\delta k_1 - 2\delta k_0 k_1 = 0,$$

$$-2\delta k_1^2 - 2\delta\mu_1 = 0,$$

$$-ck_1 - 2k_0 k_1 = 0,$$

$$-2k_1^2 - 2\mu_1 = 0,$$

$$-2\delta^2 k_1 - 2\delta^2 k_1\mu_1 = 0,$$

$$-2c\delta\mu_1 - 4\delta k_0\mu_1 = 0,$$

$$-8\delta k_1 - 8\delta k_1\mu_1 = 0,$$

$$-2c\mu_1 - 4k_0\mu_1 = 0,$$

$$-6k_1 - 6k_1\mu_1 = 0.$$

Solving the last system of equations we get

$$k_0 = -c/2, k_1 = \pm 1, \mu_1 = -1. \tag{8}$$

Thus we obtain the following solutions of Eq. (6):

For $\delta < 0$ we get the kink solutions

$$u_1(\zeta) = -c/2 \pm \sqrt{-\delta} tanh(\sqrt{-\delta}\zeta)$$
$$v_1(\zeta) = \mu_0 - \sqrt{-\delta} tanh(\sqrt{-\delta}\zeta) \tag{9}$$

and travelling wave solutions

$$u_2(\zeta) = -c/2 \pm \sqrt{-\delta} coth(\sqrt{-\delta}\zeta)$$
$$v_2(\zeta) = \mu_0 - \sqrt{-\delta} coth(\sqrt{-\delta}\zeta) \tag{10}$$

However for $\delta > 0$ we obtain periodic solutions

$$u_3(\zeta) = -c/2 \pm \sqrt{\delta} tan(\sqrt{\delta}\zeta)$$
$$v_3(\zeta) = \mu_0 + \sqrt{\delta} tan(\sqrt{\delta}\zeta) \tag{11}$$

$$u_4(\zeta) = -c/2 \pm \sqrt{-\delta} cot(\sqrt{\delta}\zeta)$$
$$v_4(\zeta) = \mu_0 - \sqrt{\delta} cot(\sqrt{\delta}\zeta) \tag{12}$$

For $\delta = 0$ we obtain rational solutions

$$u_5(\zeta) = -c/2 \pm \frac{1}{\zeta}$$
$$v_5(\zeta) = \mu_0 + \frac{1}{\zeta} \tag{13}$$

3.2. The mKdV–BMLP system

We consider the (2+1)-dimensional mKdV–BMLP system

$$u_t + 3u^2 u_x + u_{xxx} + \frac{3}{2}(uv_x)_x + \frac{3}{2}\partial_y^{-1}(uv_y)_{xx} = 0,$$
$$v_{yt} + \frac{3}{2}(v_x v_y)_x + v_{xxxy} + 3(u^2 v_y)_x + \frac{3}{2}(uu_y)_{xx} + \frac{3}{2}(uu_{xy})_x = 0 \tag{14}$$

The balancing procedure leads to $n = 1$, hence we may assume that

$$u(x, y, t) = k_0 + k_1\phi(\zeta)$$
$$v(x, y, t) = \mu_0 + \mu_1\phi(\zeta) \tag{15}$$

where $\zeta = x + y - ct$ Substituting Eqs. (15) and (3) into Eq. (14) and collecting coefficients of polynomial of ϕ^i and equating them to zero, we get a system of algebraic equations for k_0, k_1, μ_0 and μ_1

$$6\delta^2 k_0 k_1 + 6\delta^2 k_0 k_1\mu_1 = 0,$$

$$18\delta^2 k_1^2 - 2c\delta\mu_1 + 16\delta^2\mu_1 + 6\delta k_0^2\mu_1 + 6\delta^2 k_1^2\mu_1 + 6\delta^2\mu_1^2 = 0,$$

$$24\delta k_0 k_1 + 24\delta k_0 k_1\mu_1 = 0,$$

$$48\delta k_1^2 - 2c\mu_1 + 40\delta\mu_1 + 6k_0^2\mu_1 + 18\delta k_1^2\mu_1 + 12\delta\mu_1^2 = 0,$$

$$18k_0 k_1 + 18k_0 k_1\mu_1 = 0,$$

$$30k_1^2 + 24\mu_1 + 12k_1^2\mu_1 + 6\mu_1^2 = 0$$

Solving the last system of equations we get the first case

$$k_0 = \pm\sqrt{c}/\sqrt{3}, k_1 = \pm 1, \mu_1 = -1, \tag{16}$$

and the second case

$$k_0 = 0, c = -\delta k_1 = 1, \mu_1 = -1. \tag{17}$$

Thus for the first and second case we can obtain kink, periodic, rational and travelling wave solutions of Eq. (14) as follow:

For $\delta < 0$

$$u_1(\zeta) = \pm\sqrt{c}/\sqrt{3} \pm \sqrt{-\delta} tanh(\sqrt{-\delta}\zeta)$$
$$v_1(\zeta) = \mu_0 - \sqrt{-\delta} tanh(\sqrt{-\delta}\zeta) \tag{18}$$

$$u_2(\zeta) = \pm\sqrt{c}/\sqrt{3} \pm \sqrt{-\delta} coth(\sqrt{-\delta}\zeta)$$
$$v_2(\zeta) = \mu_0 - \sqrt{-\delta} coth(\sqrt{-\delta}\zeta) \tag{19}$$

For $\delta > 0$

$$u_3(\zeta) = \pm\sqrt{c}/\sqrt{3} \pm \sqrt{\delta}tan(\sqrt{\delta}\zeta)$$
$$v_3(\zeta) = \mu_0 + \sqrt{\delta}tan(\sqrt{\delta}\zeta) \tag{20}$$

$$u_4(\zeta) = \pm\sqrt{c}/\sqrt{3} \pm \sqrt{-\delta}cot(\sqrt{\delta}\zeta)$$
$$v_4(\zeta) = \mu_0 - \sqrt{\delta}cot(\sqrt{\delta}\zeta) \tag{21}$$

For $\delta = 0$

$$u_5(\zeta) = \pm\sqrt{c}/\sqrt{3} \pm \frac{1}{\zeta}$$
$$v_5(\zeta) = \mu_0 + \frac{1}{\zeta} \tag{22}$$

For $\delta < 0$

$$u_6(\zeta) = -\sqrt{-\delta}tanh(\sqrt{-\delta}\zeta)$$
$$v_6(\zeta) = \mu_0 - \sqrt{-\delta}tanh(\sqrt{-\delta}\zeta) \tag{23}$$

$$u_7(\zeta) = -\sqrt{-\delta}coth(\sqrt{-\delta}\zeta)$$
$$v_7(\zeta) = \mu_0 - \sqrt{-\delta}coth(\sqrt{-\delta}\zeta) \tag{24}$$

For $\delta > 0$

$$u_8(\zeta) = \sqrt{\delta}tan(\sqrt{\delta}\zeta)$$
$$v_8(\zeta) = \mu_0 + \sqrt{\delta}tan(\sqrt{\delta}\zeta) \tag{25}$$

$$u_9(\zeta) = -\sqrt{-\delta}cot(\sqrt{\delta}\zeta)$$
$$v_9(\zeta) = \mu_0 - \sqrt{\delta}cot(\sqrt{\delta}\zeta) \tag{26}$$

For $\delta = 0$

$$u_{10}(\zeta) = -\frac{1}{\zeta}$$
$$v_{10}(\zeta) = \mu_0 + \frac{1}{\zeta} \tag{27}$$

4. Conclusion

In this article, the extended tanh method was applied to give the traveling wave solutions of two Coupled (2 + 1)-Dimensional Equations, the coupled Burgers and mKdV–BMLP systems, The extended tanh method gives different classes of solutions. These solutions include many types like rational, periodical, shock solutions, etc. For example, solutions (11) and (20) are examples exhibiting the sinusoidal-type periodical solutions, which develop a singularity at a finite point, i.e., for any fixed $t = t_0$ there exists a value of æ0 at which these solutions blow up (see Fig. 1). Solutions (10) and (19) are in the form of explosive/blow-up solutions as depicted in Fig. 2.

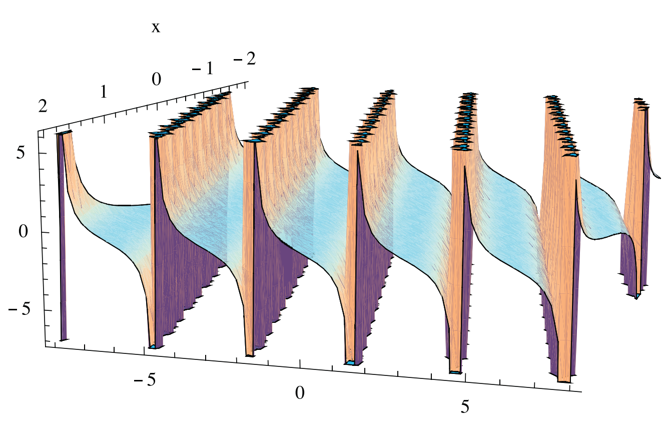

Fig. 1. 3D Plot of the periodic solution (11) with $\delta = 0.9$, $c = 1$ and $y = 1.1$.

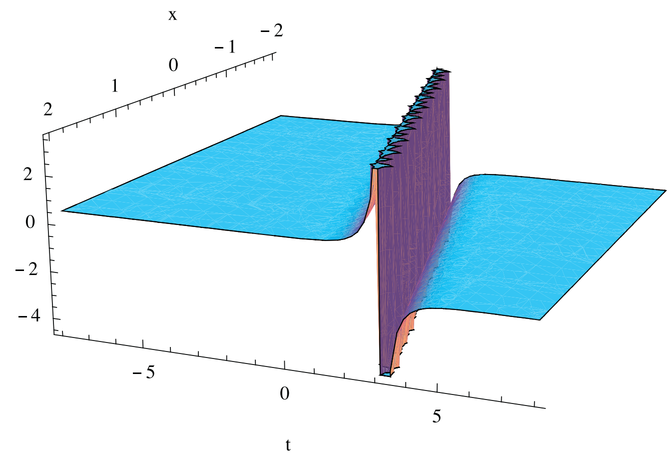

Fig. 2. 3D Plots of the explosive/blow-up solution (10) with $\delta = -0.9$, $c = 1$ and $y = 1.1$.

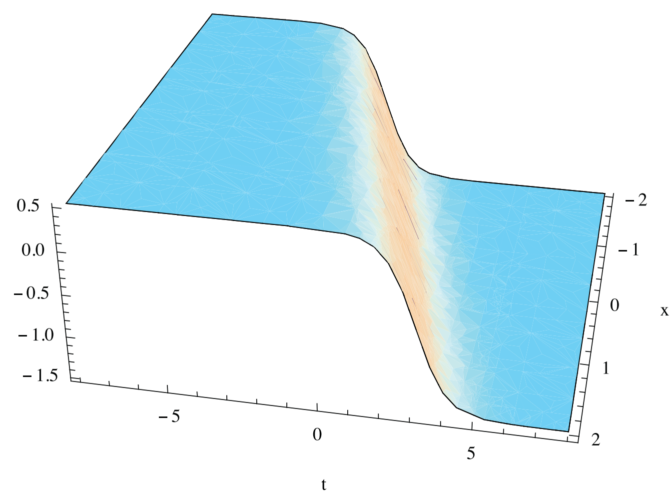

Fig. 3. 3D Plot of the shock wave solution (18) with $\delta = -0.9$, $c = 1$ and $y = 1.1$.

Solution (13) represents the rational-type solutions, the rational solution may be a discrete joint union of manifolds. The solutions (9), (18) and (23) represent shock solutions , these solutions may be of significant importance for the explanation of shock waves in fluid and plasma physics (see Fig. 3).

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