Journal of the Egyptian Mathematical Society 25 (2017) 111–118

Contents lists available at ScienceDirect

Journal of the Egyptian Mathematical Society

journal homepage: www.elsevier.com/locate/joems

Original Article

The corona between cycles and paths

S. Nada[a,*], A. Elrokh[a], E.A. Elsakhawi[b], D.E. Sabra[b]

[a] Dept. of Math., Faculty of Science, Menoyfia University, Shebeen Elkom, Egypt
[b] Dept. of Math., Faculty of Science, Ain Shams University, Cairo, Egypt

ARTICLE INFO

Article history:
Received 3 April 2016
Accepted 23 August 2016
Available online 23 January 2017

MSC:
05C78
05C12
05C15

Keywords:
Graph labeling
Corona

ABSTRACT

A graph is said to be cordial if it has a 0–1 labeling that satisfies certain properties. The corona $G_1 \odot G_2$ of two graphs G_1(with n_1 vertices and m_1 edges) and G_2 (with n_2 vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and then joining the ith vertex of G_1 with an edge to every vertex in the ith copy of G_2. In this paper we investigate the cordiality of the corona between cycles C_n and paths P_n, namely $C_n \odot P_m$.

© 2016 Egyptian Mathematical Society. Production and hosting by Elsevier B.V.
This is an open access article under the CC BY-NC-ND license.
(http://creativecommons.org/licenses/by-nc-nd/4.0/)

1. Introduction

It is known that graph theory and its branches have become interest topics for almost all fields of mathematics and also other area of science such as chemistry, biology, physics, communication, economics, engineering, operations research, and especially computer science.

A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. There are many contributions and different kinds of labeling [1–9].

Two of the most important types of labeling are called graceful and harmonious. Graceful labeling was introduced independently by Rosa [10] in 1966 and Golomb [11] in 1972, while harmonious labeling were first studied by Graham and Sloane [12] in 1980. A third important type of labeling, which contains aspects of both of the other two, is called cordial and was introduced by Cahit [1] in 1990. Whereas the label of an edge vw for graceful and harmonious labeling is given respectively by $|f(v) - f(w)|$ and $(f(v) + f(w))$ (modulo the number of edges), cordial labeling use only labels 0 and 1 and the induced edge label $(f(v) + f(w))$(mod2), which of course equals $|f(v) - f(w)|$. Because arithmetic modulo 2 is an integral part of computer science, cordial labelings have close connections with that field. An excellent reference on this subject is the survey by Gallian [13].

More precisely, cordial graphs are defined as follows: Let $G = (V, E)$ be a graph, let f: $V \to \{0, 1\}$ be a labeling of its vertices, and let f^*: $E \to \{0, 1\}$ is the extension of f to the edges of G by the formula $f^*(vw) = (f(v) + f(w))$ (mod 2). Thus, for any edge $e = uv$, $f^*(e) = 0$ if its two vertices have the same label and $f^*(e) = 1$ if they have different labels. Let v_0 and v_1 be the numbers of vertices labeled 0 and 1 respectively, and let e_0 and e_1 be the corresponding numbers of edges. Such a labeling is called *cordial* if both $|v_0 - v_1| \le 1$ and $|e_0 - e_1| \le 1$ hold. A graph is called *cordial* if it has a cordial labeling.

Suppose that $G = (V, E)$ is a graph, where V is the set of its vertices and E is the set of its edges. Throughout, it is assumed G is connected, finite, simple and undirected.

The *corona* $G_1 \odot G_2$ of two graphs G_1(with n_1 vertices and m_1 edges) and G_2 (with n_2 vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and then joining the ith vertex of G_1 with an edge to every vertex in the ith copy of G_2 [14]. It follows from the definition of the corona that $G_1 \odot G_2$ has $n_1 + n_1 n_2$ vertices and $m_1 + n_1 m_2 + n_1 n_2$ edges. It is easy to see that $G_1 \odot G_2$ is not in general isomorphic to $G_2 \odot G_1$. In [15], the corona of $P_n \odot C_m$ has been studied and proved that $P_n \odot C_m$ is cordial if and only if $(n, m) \ne (1, 3)$(mod4). In this paper we show that the corona $C_n \odot P_m$ is cordial for all $n \ge 3$ and $m \ge 1$.

2. Terminology and notation

A path with n vertices and $n - 1$ edges is denoted by P_n, and a cycle with n vertices and n edges is denoted by C_n. Given a cycle or a path with $4r$ vertices, we let L_{4r} denote the labeling 0011 ... 0011 (repeated $r$times). In most cases, we then modify this by adding symbols at one end or the other (or both); thus $010L_{4r}$ denotes the labeling 010 0011 ... 0011 of the cycle

* Corresponding author.
E-mail addresses: shokrynada@yahoo.com (S. Nada), el-rokh@excite.com (A. Elrokh).

http://dx.doi.org/10.1016/j.joems.2016.08.004
1110-256X/© 2016 Egyptian Mathematical Society. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license. (http://creativecommons.org/licenses/by-nc-nd/4.0/)

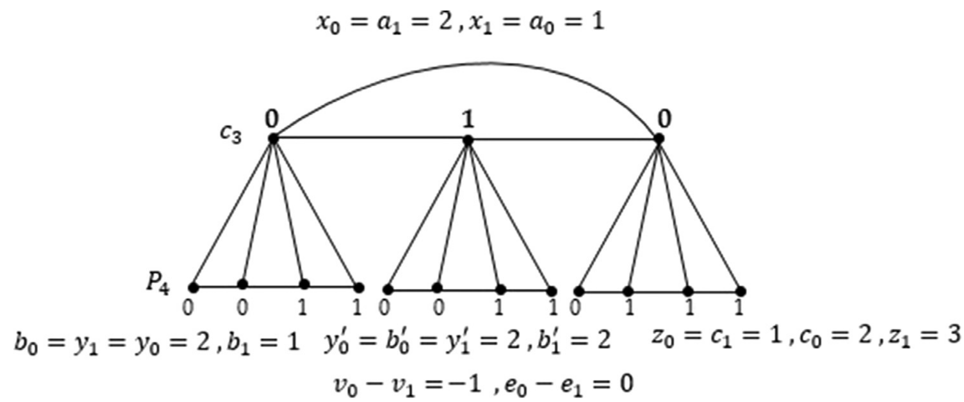

Fig. 1.

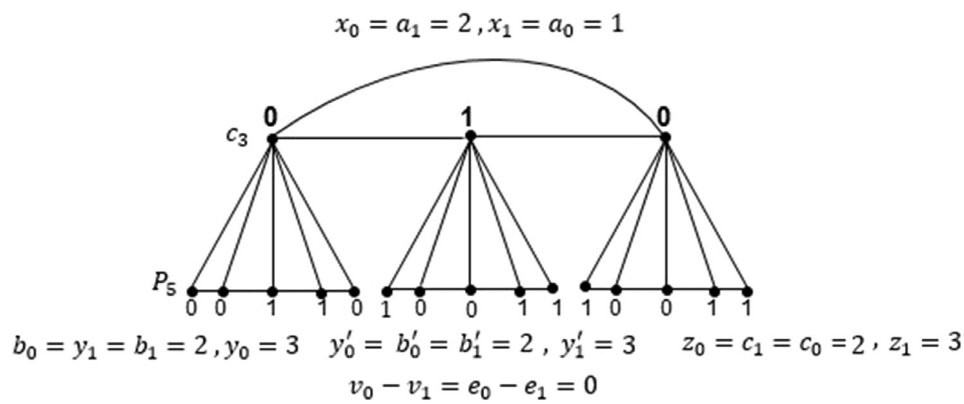

Fig. 2.

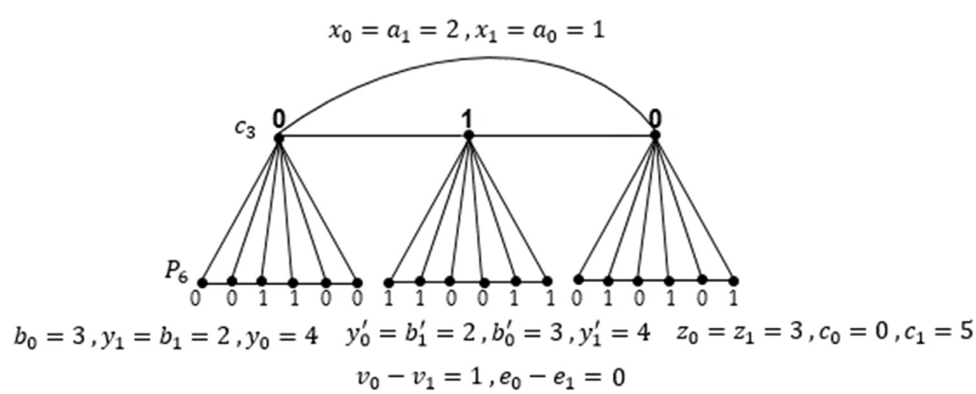

Fig. 3.

C_{4r+3} (or the path P_{4r+3}) when $r \geq 1$ and 010 when $r = 0$. Similarly, $L_{4r}01$ is the labeling 0011 ... 0011 01 of the cycle C_{4r+2} (or the path P_{4r+2}) when $r \geq 1$ and 01 when $r = 0$, and so on. We write M_r for the labeling 01 ... 01 if r is even and 01 ... 010 if r is odd, for example, $M_6 = 010101$ and $M_7 = 0101010$. Also, we write 0_r for the labeling 0 ... 0 ($r$times) and 1_r for the labeling 1 ... 1 ($r$times) [3–8]. If G and H are two graphs, where G has n vertices, the labeling of the corona $G \odot H$ is often denoted by $[A : B_1, B_2, B_3, \ldots, B_n]$, where A is the labeling of the n vertices of G, and B_i, $1 \leq i \leq n$ is the labeling of the vertices of the copy of H that is connected to the ith vertex of G [2]. For a given labeling of the corona $G \odot H$, we denote v_i and e_i $(i = 0, 1)$ to represent the numbers of vertices and edges, respectively, labeled by i. Let us denote x_i and a_i to be the numbers of vertices and edges labeled by i for the graph G. Also, we let y_i and b_i be those for H, which are connected to the vertices labeled 0 of G. Likewise, let y_i' and b_i' be those for H, which are connected to the vertices labeled 1 of G. It is easy to verify that $v_0 = x_0 + x_0 y_0 + x_1 y_0'$, $v_1 = x_1 + x_0 y_1 + x_1 y_1'$, $e_0 = a_0 + x_0 b_0 + x_1 b_0' + x_0 y_0 + x_1 y_1'$ and $e_1 = a_1 + x_0 b_1 + x_1 b_1' + x_0(x_0 y_1) + x_1 y_0'$. Thus $v_0 - v_1 = (x_0 - x_1) + x_0(y_0 - y_1) + x_1(y_0' - y_1')$ and $e_0 - e_1 = (a_0 - a_1) + x_0(b_0 - b_1) + x_1(b_0' - b_1') + x_0(y_0 - y_1) - x_1(y_0' - y_1')$. In particular, if we have only one labeling for all copies of H, i.e., $y_i = y_i'$ and $b_i = b_i'$, then $v_0 = x_0 + n y_0$, $v_1 = x_1 + n y_1$, $e_0 = a_0 + n b_0 + x_0 y_0 + x_1 y_1$ and $e_1 = a_1 + n b_1 + x_0 y_1 + x_1 y_0$. Thus $v_0 - v_1 = (x_0 - x_1) + n(y_0 - y_1)$ and $e_0 - e_1 = (a_0 - a_1) + n(b_0 - b_1) + (x_0 - x_1)(y_0 - y_1)$, where n is the order of G.

3. Corona between cycles and paths

In this section, we show that the corona $C_n \odot P_m$ is cordial for all $n \geq 3$ and $m \geq 1$.

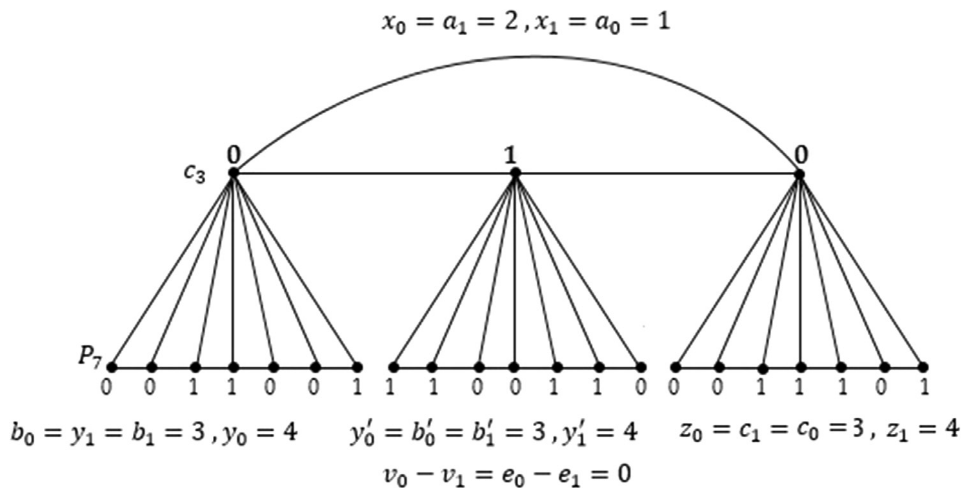

Fig. 4.

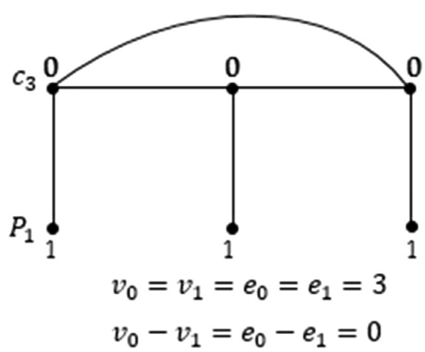

Fig. 5.

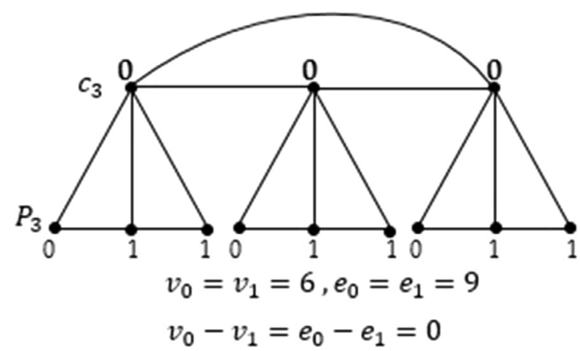

Fig. 7.

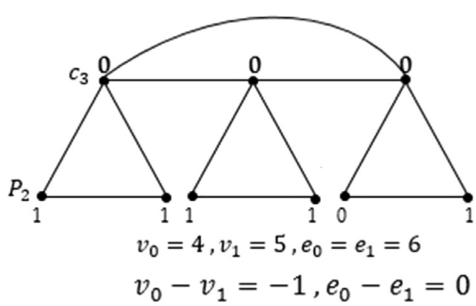

Fig. 6.

This target will be achieved as a consequence of the following series of lemmas.

Lemma 3.1. *The corona* $C_3 \odot P_m$ *is cordial for all* $m \geq 1$.

Proof. Let us first prove that $C_3 \odot P_m$ is cordial for all $m \geq 4$. To do so, we will examine the following four cases:

Case (1). $m \equiv 0(\text{mod}4)$.

Suppose that $m = 4s, s \geq 1$. We choose the labeling $[010 : L_{4s}, L_{4s}, 01_3L_{4s-4}]$ for $C_3 \odot P_{4s}$. Therefore $x_0 = 2,\ x_1 = 1,$ $a_0 = 1,\ a_1 = 2,\ y'_0 = 2s,\ y'_1 = 2s, b'_0 = 2s$ and $b'_1 = 2s - 1$. For the $4s$ vertices of the copy P_{4s} which are connected to the first zero in C_3 we have, $y_0 = 2s,\ y_1 = 2s,\ b_0 = 2s$ and $b_1 = 2s - 1$. For the $4s$

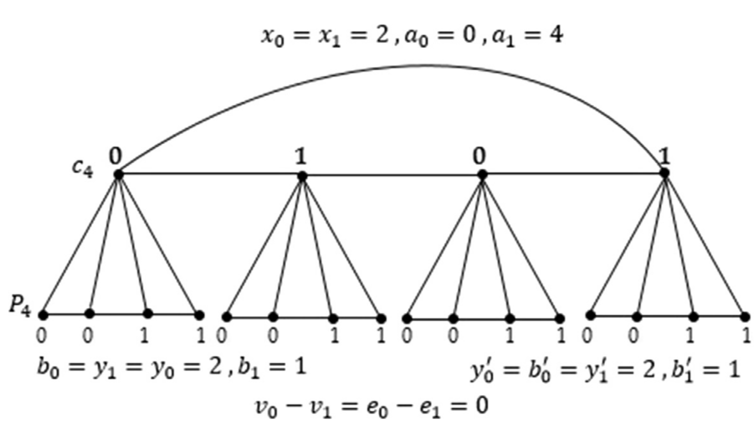

Fig. 8.

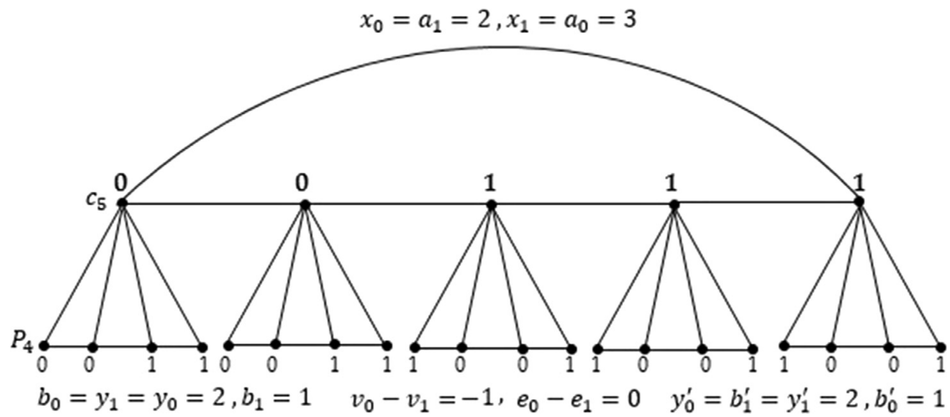

Fig. 9.

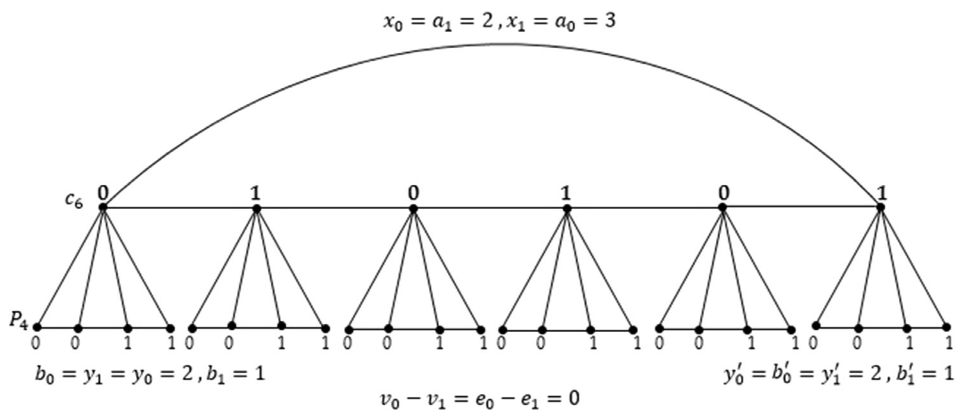

Fig. 10.

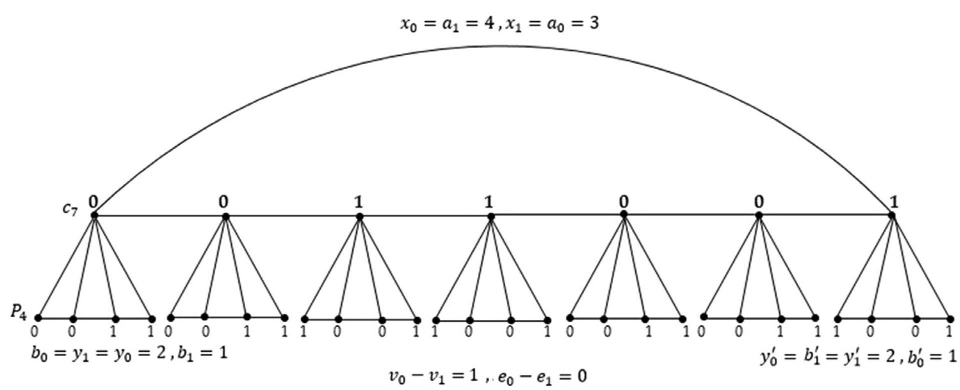

Fig. 11.

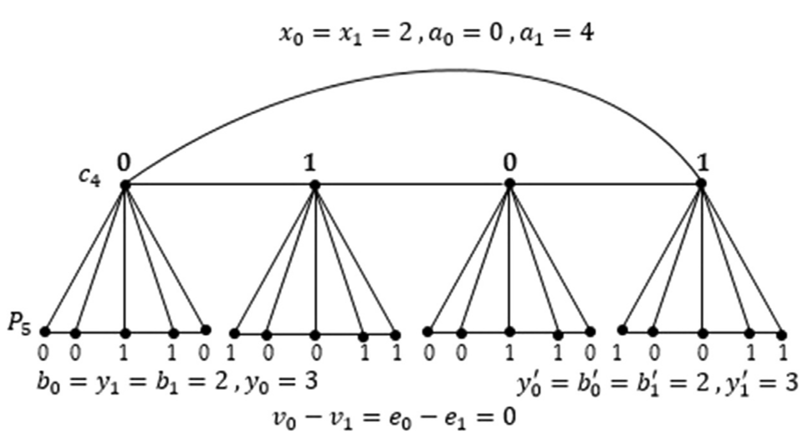

Fig. 12.

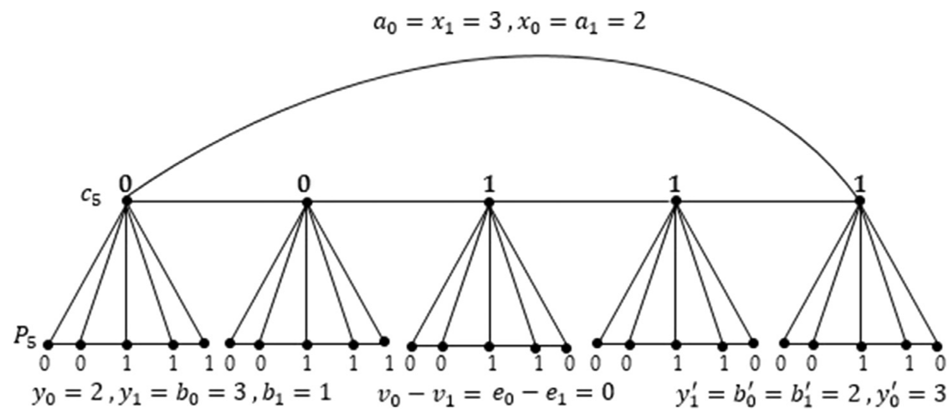

Fig. 13.

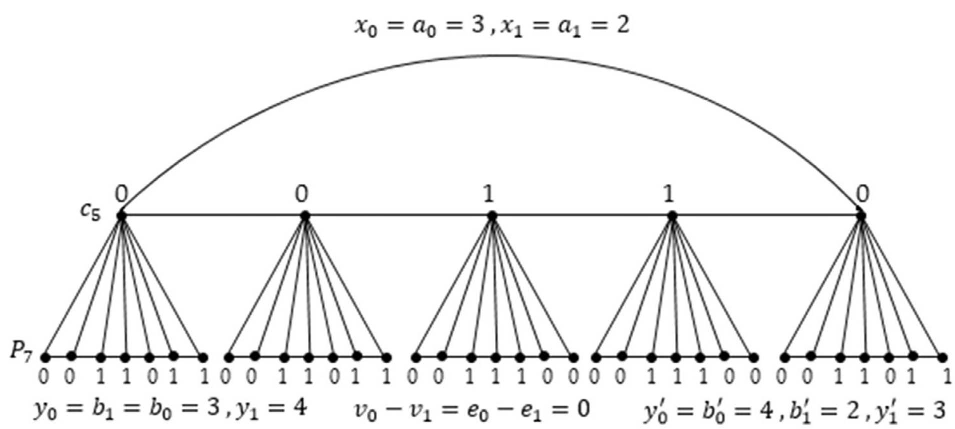

Fig. 14.

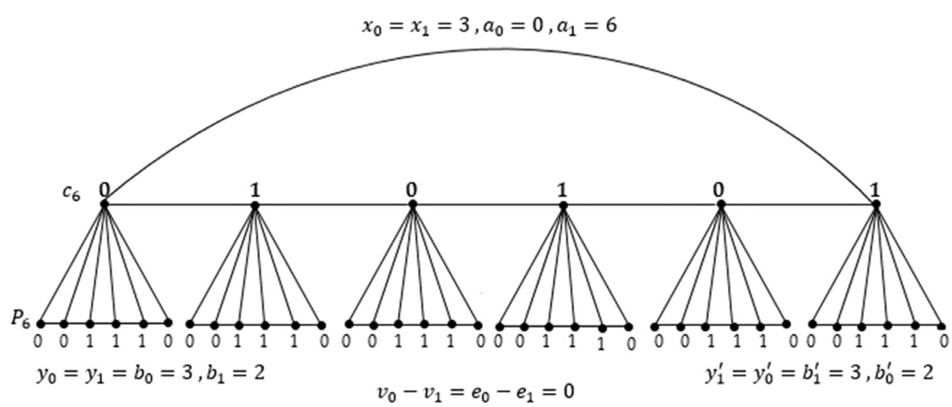

Fig. 15.

vertices of the copy P_{4s} which are connected to the second zero in C_3 we have, $z_0 = 2s - 1,\ z_1 = 2s + 1,\ c_0 = 2s$ and $c_1 = 2s - 1$, where z_i and c_i are the numbers of vertices and edges labeled by i in P_{4s} that are connected to the second zero in C_3. It follows that $v_0 - v_1 = (x_0 - x_1) + (x_0 - 1)(y_0 - y_1) + x_1(y'_0 - y'_1) + (z_0 - z_1) = -1$ and $e_0 - e_1 = (a_0 - a_1) + (x_0 - 1)(b_0 - b_1) + x_1(b'_0 - b'_1) + (x_0 - 1)(y_0 - y_1) - x_1(y'_0 - y'_1) + (c_0 - c_1) + (z_0 - z_1) = 0$. Hence $C_3 \odot P_{4s}$ is cordial. As an example, Fig. (1) illustrates $C_3 \odot P_4$.

Case (2). $m \equiv 1(\text{mod}4)$. Suppose that $m = 4s + 1, s \geq 1$. We choose the labeling [010: $L_{4s}0$, $1L_{4s}$, $1L_{4s}$] for $C_3 \odot P_{4s+1}$. Therefore $x_0 = 2,\ x_1 = 1,\ a_0 = 1,\ a_1 = 2,\ y'_0 = 2s,\ y'_1 = 2s + 1,\ b'_0 = 2s$ and $b'_1 = 2s$. For the $4s + 1$ vertices of the copy P_{4s+1} which are connected to the first zero in C_3 we have, $y_0 = 2s + 1,\ y_1 = 2s$, $b_0 = 2s$ and $b_1 = 2s$. For the $4s + 1$ vertices of the copy P_{4s+1} which are connected to the second zero in C_3 we have, $z_0 = 2s$, $z_1 = 2s + 1,\ c_0 = 2s$ and $c_1 = 2s$, where z_i and c_i are the numbers of vertices and edges labeled by i in P_{4s+1} that are connected to the second zero in C_3. Similar to Case(1), we have that $v_0 - v_1 = (x_0 - x_1) + (x_0 - 1)(y_0 - y_1) + x_1(y'_0 - y'_1) + (z_0 - z_1) = 0$ and $e_0 - e_1 = (a_0 - a_1) + (x_0 - 1)(b_0 - b_1) + x_1(b'_0 - b'_1) + (x_0 - 1)(y_0 - y_1) - x_1(y'_0 - y'_1) + (c_0 - c_1) + (z_0 - z_1) = 0$. Hence $C_3 \odot P_{4s+1}$ is cordial. As an example, Fig. (2) illustrates $C_3 \odot P_5$.

Case (3). $m \equiv 2(\text{mod}4)$. Suppose that $m = 4s + 2, s \geq 1$. We choose the labeling [010: $L_{4s}00$, $11L_{4s}$, $M_6\ L_{4s-4}$] for $C_3 \odot P_{4s+2}$. Therefore $x_0 = 2,\ x_1 = 1,\ a_0 = 1,\ a_1 = 2, y'_0 = 2s,\ y'_1 = 2s + 2$, $b'_0 = 2s + 1$ and $b'_1 = 2s$. For the $4s + 2$ vertices of the copy P_{4s+2} which are connected to the first zero in C_3 we have, $y_0 = 2s + 2$, $y_1 = 2s,\ b_0 = 2s + 1$ and $b_1 = 2s$. For the $4s + 2$ vertices of the

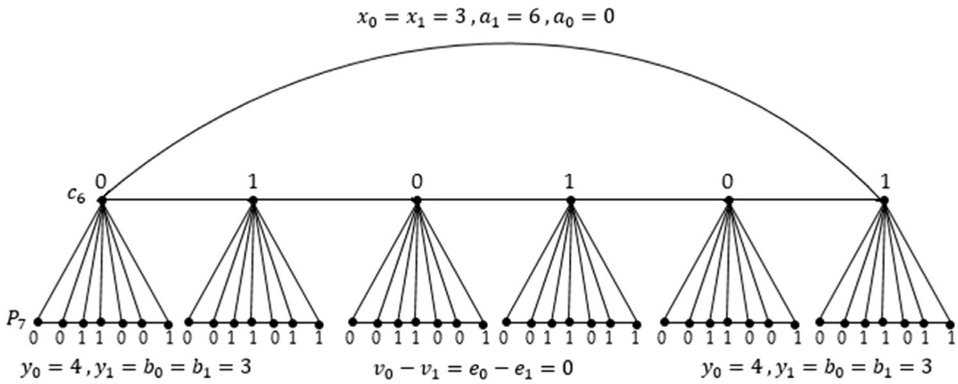

Fig. 16.

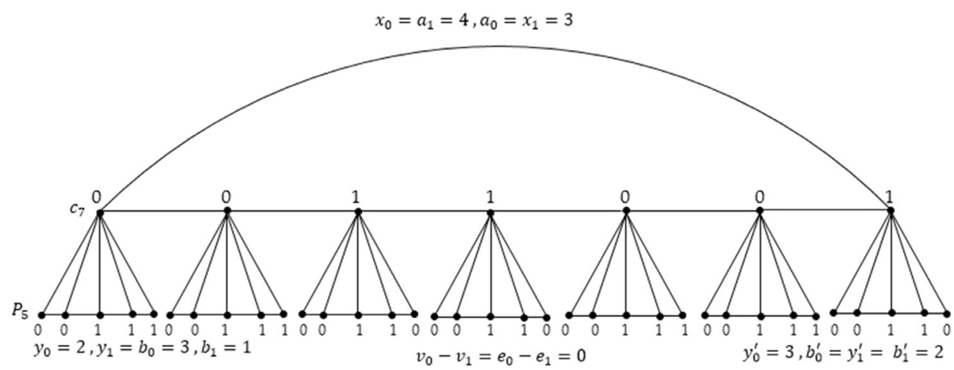

Fig. 17.

copy P_{4s+2} which are connected to the second zero in C_3 we have, $z_0 = 2s,\ z_1 = 2s,\ c_0 = 2s - 2$ and $c_1 = 2s + 3$, where z_i and c_i are the numbers of vertices and edges labeled by i in P_{4s+2} that are connected to the second zero in C_3. As before, we conclude that $v_0 - v_1 = (x_0 - x_1) + (x_0 - 1)(y_0 - y_1) + x_1(y'_0 - y'_1) + (z_0 - z_1) = 1$ and $e_0 - e_1 = (a_0 - a_1) + (x_0 - 1)(b_0 - b_1) + x_1(b'_0 - b'_1) + (x_0 - 1)(y_0 - y_1) - x_1(y'_0 - y'_1) + (c_0 - c_1) + (z_0 - z_1) = 0$. Hence $C_3 \odot P_{4s+2}$ is cordial. As an example, Fig. (3) illustrates $C_3 \odot P_6$.

Case (4). $m \equiv 3(\text{mod}4)$. Suppose that $m = 4s + 3, s \geq 1$. We choose the labeling [010: $L_{4s}001$, $11L_{4s}0$, $L_{4s}101$] for $C_3 \odot P_{4s+3}$. Therefore $x_0 = 2,\ x_1 = 1,\ a_0 = 1,\ a_1 = 2,\ y'_0 = 2s + 1,\ y'_1 = 2s + 2,\ b'_0 = 2s + 1$ and $b'_1 = 2s + 1$. For the $4s + 3$ vertices of the copy P_{4s+3} which are connected to the first zero in C_3 we have, $y_0 = 2s + 2,\ y_1 = 2s + 1,\ b_0 = 2s + 1$ and $b_1 = 2s + 1$. For the $4s + 3$ vertices of the copy P_{4s+3} which are connected to the second zero in C_3 we have, $z_0 = 2s + 1,\ z_1 = 2s + 2,\ c_0 = 2s + 1$ and $c_1 = 2s + 1$, where z_i and c_i in P_{4s+3} that are connected to the second zero in C_3. As before, we conclude that $v_0 - v_1 = (x_0 - x_1) + (x_0 - 1)(y_0 - y_1) + x_1(y'_0 - y'_1) + (z_0 - z_1) = 0$ and $e_0 - e_1 = (a_0 - a_1) + (x_0 - 1)(b_0 - b_1) + x_1(b'_0 - b'_1) + (x_0 - 1)(y_0 - y_1) - x_1(y'_0 - y'_1) + (c_0 - c_1) + (z_0 - z_1) = 0$. Hence $C_3 \odot P_{4s+3},\ s \geq 1$ is cordial as we wanted. As an example, Fig. (4) illustrates $C_3 \odot P_7$. So, $C_3 \odot P_{4s+3}$ is cordial.

It remains to show that $C_3 \odot P_m$ is cordial for all $1 \leq m \leq 3$. The following labeling are sufficient: [000: 1, 1, 1] for the corona $C_3 \odot P_1$, [000: 11, 11, 01] for the corona $C_3 \odot P_2$ and [000: 011, 011, 011] for the corona $C_3 \odot P_3$. It is obvious that $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$ for $C_3 \odot P_1$ and $C_3 \odot P_3$, while $v_0 - v_1 = -1$ and $e_0 - e_1 = 0$ for $C_3 \odot P_2$. See Fig. (5), Fig. (6) and Fig. (7) for these three particular cases. Thus the lemma follows. □

Lemma 3.2. *If $m \equiv 0(mod4)$, then the corona $C_n \odot P_m$ is cordial for all $n \geq 4$.*

Proof. Let $m = 4s, s \geq 1$ and $n = 4r + i, 0 \leq i \leq 3$. Then we have to study the following four cases:

Case(1). $n = 4r, r \geq 1$. We choose the labeling $[M_{4r} : L_{4s}, \ \ldots \ , L_{4s}]$ for $C_{4r} \odot P_{4s}$. Therefore $x_0 = 2r,\ x_1 = 2r,\ a_0 = 0,\ a_1 = 4r,\ y_0 = 2s,\ y_1 = 2s,\ b_0 = 2s,\ b_1 = 2s - 1,\ y'_0 = 2s,\ y'_1 = 2s,\ b'_0 = 2s,\ b'_1 = 2s - 1$. Hence $v_0 - v_1 = (x_0 - x_1) + n(y_0 - y_1) = 0 + 4r(0) = 0$ and $e_0 - e_1 = (a_0 - a_1) + n(b_0 - b_1) + (x_0 - x_1)(y_0 - y_1) = -4r + 4r(1) + 0(0) = 0$. Thus $C_{4r} \odot P_{4s}$ is cordial. As an example, Fig. (8) illustrates $C_4 \odot P_4$.

Case (2). $n = 4r + 1, r \geq 1$. We choose the labeling $[L_{4r}1 : (L_{4s}, L_{4s}, 1L_{4s-4}001, 1L_{4s-4}001, \ \ldots \ (r\text{times})),$ $1L_{4s-4}001]$ for $C_{4r+1} \odot P_{4s}$. Therefore $x_0 = 2r,\ x_1 = 2r + 1,\ a_0 = 2r + 1,\ a_1 = 2r,\ y_0 = 2s,\ y_1 = 2s,\ b_0 = 2s,\ b_1 = 2s - 1,\ y'_0 = 2s,\ y'_1 = 2s,\ b'_0 = 2s - 1,\ b'_1 = 2s$. Hence $v_0 - v_1 = (x_0 - x_1) + x_0(y_0 - y_1) + x_1(y'_0 - y'_1) = -1$ and $e_0 - e_1 = (a_0 - a_1) + x_0(b_0 - b_1) + x_1(b'_0 - b'_1) + x_0(y_0 - y_1) - x_1(y'_0 - y'_1) = 0$. Thus $C_{4r+1} \odot P_{4s}$ is cordial. As an example, Fig. (9) illustrates $C_5 \odot P_4$.

Case (3). $n = 4r + 2, r \geq 1$. We choose the labeling $[M_{4r+2} : L_{4s}, \ \ldots \ , L_{4s}]$ for $C_{4r+2} \odot P_{4s}$. Therefore $x_0 = 2r + 1,\ x_1 = 2r + 1,\ a_0 = 0,\ a_1 = 4r + 2,\ y_0 = 2s,\ y_1 = 2s,\ b_0 = 2s,\ b_1 = 2s - 1,\ y'_0 = 2s,\ y'_1 = 2s,\ b'_0 = 2s,\ b'_1 = 2s - 1$. Hence $v_0 - v_1 = (x_0 - x_1) + n(y_0 - y_1) = 0 + (4r + 2)(0) = 0$ and $e_0 - e_1 = (a_0 - a_1) + n(b_0 - b_1) + (x_0 - x_1)(y_0 - y_1) = -(4r + 2) + (4r + 2)(1) + 0(0) = 0$. Thus $C_{4r+2} \odot P_{4s}$ is cordial. As an example, Fig. (10) illustrates $C_6 \odot P_4$.

Case (4). $n = 4r + 3, r \geq 1$. We choose the labeling $[L_{4r}001 : (L_{4s}, L_{4s}, 1L_{4s-4}001, 1L_{4s-4}001, \ \ldots \ , (r\text{times})),$ $L_{4s}, L_{4s}, 1L_{4s-4}001]$ for $C_{4r+1} \odot P_{4s}$. Therefore $x_0 = 2r + 2,\ x_1 = 2r + 1,\ a_0 = 2r + 1,\ a_1 = 2r + 2,\ y_0 = 2s,\ y_1 = 2s,\ b_0 = 2s,\ b_1 = 2s - 1,\ y'_0 = 2s,\ y'_1 = 2s,\ b'_0 = 2s - 1,\ b'_1 = 2s$. Hence $v_0 - v_1 = (x_0 - x_1) + x_0(y_0 - y_1) + x_1(y'_0 - y'_1) = 1$ and $e_0 - e_1 = (a_0 - a_1) + x_0(b_0 - b_1) + x_1(b'_0 - b'_1) + x_0(y_0 - y_1) - x_1(y'_0 - y'_1) = 0$. Thus $C_{4r+3} \odot P_{4s}$ is cordial. As an example, Fig. (11) illustrates $C_7 \odot P_4$. So, if $m \equiv 0(\text{mod}4)$, then $C_n \odot P_m$ is cordial for all $n \geq 4$ and the lemma follows. □

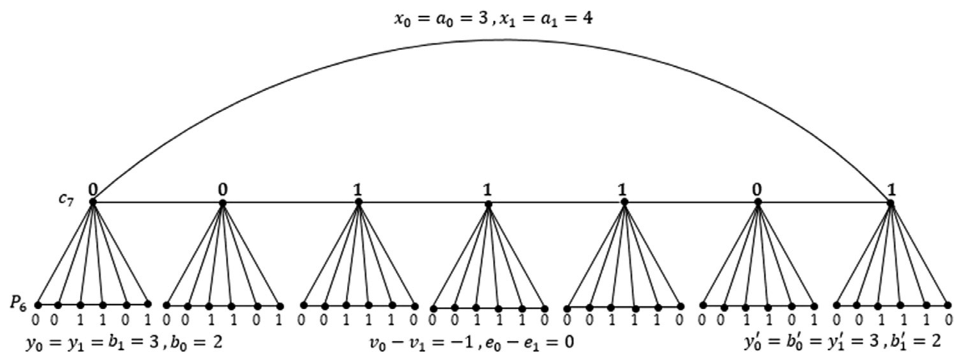

Fig. 18.

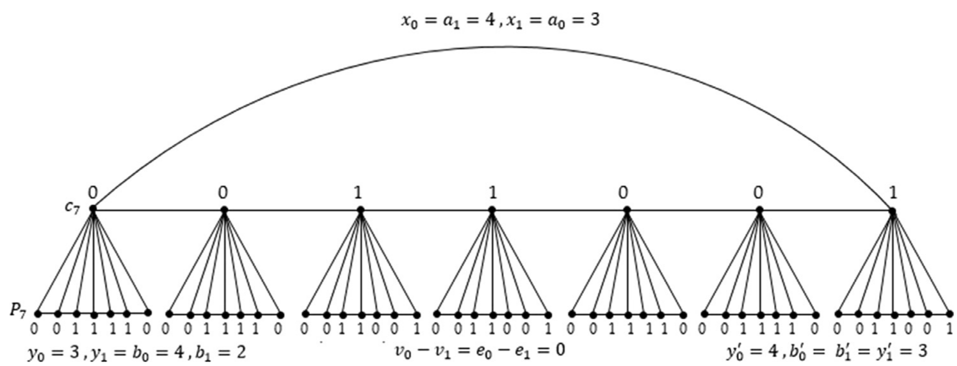

Fig. 19.

Table 3.1
Labeling of C_n and P_m..

$n=4r+i, i=0,1,2,3$	Labeling of C_n				
		x_0	x_1	a_0	a_1
$i=0$	$A_0=M_{4r}$	$2r$	$2r$	0	$4r$
	$A'_0=L_{4r}$	$2r$	$2r$	$2r$	$2r$
$i=1$	$A_1=L_{4r}1$	$2r$	$2r+1$	$2r+1$	$2r$
	$A'_1=L_{4r}0$	$2r+1$	$2r$	$2r+1$	$2r$
$i=2$	$A_2=M_{4r+2}$	$2r+1$	$2r+1$	0	$4r+2$
$i=3$	$A_3=L_{4r}001$	$2r+2$	$2r+1$	$2r+1$	$2r+2$
	$A'_3=L_{4r}101$	$2r+1$	$2r+2$	$2r+1$	$2r+2$
$m=4s+j, j=1,2,3$	Labeling of P_m				
		y_0	y_1	b_0	b_1
$j=1$	$B_1=L_{4s}0$	$2s+1$	$2s$	$2s$	$2s$
	$B^*_1=L_{4s}1$	$2s$	$2s+1$	$2s+1$	$2s-1$
$j=2$	$B_2=L_{4s}10$	$2s+1$	$2s+1$	$2s+1$	$2s$
	$B^*_2=L_{4s}01$	$2s+1$	$2s+1$	$2s$	$2s+1$
$j=3$	$B_3=L_{4s}011$	$2s+1$	$2s+2$	$2s+1$	$2s+1$
	$B^*_3=L_{4s}001$	$2s+2$	$2s+1$	$2s+1$	$2s+1$
	$B^{**}_3=L_{4s}110$	$2s+1$	$2s+2$	$2s+2$	$2s$
$m=4s+j, j=1,2,3$	Labeling of P_m				
		y'_0	y'_1	b'_0	b'_1
$j=1$	$B'_1=1L_{4s}$	$2s$	$2s+1$	$2s$	$2s$
	$B^{*'}_1=L_{4s}0$	$2s+1$	$2s$	$2s$	$2s$
$j=2$	$B'_2=L_{4s}01$	$2s+1$	$2s+1$	$2s$	$2s+1$
	$B^{*'}_2=L_{4s}10$	$2s+1$	$2s+1$	$2s+1$	$2s$
$j=3$	$B'_3=L_{4s}100$	$2s+2$	$2s+1$	$2s+2$	$2s$
	$B^{*'}_3=L_{4s}011$	$2s+1$	$2s+2$	$2s+1$	$2s+1$
	$B^{**'}_3=L_{4s}001$	$2s+2$	$2s+1$	$2s+1$	$2s+1$

Table 3.2
Combinations of labeling.

$n=4r+i$, $i=0,1,2,3$	$m=4s+j$, $j=1,2,3$	P_n	C_m	v_0-v_1	e_0-e_1
0	1	A_0	B_1, B'_1	0	0
0	2	A_0	B_2, $B^{*'}_2$	0	0
0	3	A'_0	B_3, B'_3	0	0
1	1	A_1	B^*_1, $B^{*'}_1$	0	0
1	2	A_1	B_2, B'_2	−1	0
1	3	A'_1	B_3, B'_3	0	0
2	1	A_2	B_1, B'_1	0	0
2	2	A_2	B_2, $B^{*'}_2$	0	0
2	3	A_2	B^*_3, $B^{*'}_3$	0	0
3	1	A_3	B^*_1, $B^{*'}_1$	0	0
3	2	A'_3	B^*_2, $B^{*'}_2$	−1	0
3	3	A_3	B^{**}_3, $B^{**'}_3$	0	0

Lemma 3.3. *If m is not congruent to 0(mod4), then the corona $C_n \odot P_m$ is cordial for all $n \geq 4$ and $m \geq 4$.*

Proof. Let $n = 4r + i$ ($i = 0, 1, 2, 3$ and $r \geq 1$) and $m = 4s + j$ ($j = 1, 2, 3$ and $s \geq 1$), then for a given value of i with $0 \leq i \leq 3$, we may use the labeling A_i or A'_i for C_n as given in Table 3.1. For a given value of j with $1 \leq j \leq 3$, we may use one of the labeling in the set $\{B_j, B^*_j, B^{**}_j, B'_j, B^{*'}_j, B^{**'}_j\}$ for P_m, where B_j, B^*_j and B^{**}_j are the labeling of P_m which are connected to the vertices labeled 0 in C_n, while B'_j, $B^{*'}_j$ and $B^{**'}_j$ are the labeling of P_m which are connected to the vertices labeled 1 in C_n as given in Table 3.1. Using this table and the formulas $v_0 - v_1 = (x_0 - x_1) + x_0(y_0 - y_1) + x_1(y'_0 - y'_1)$ and $e_0 - e_1 = (a_0 - a_1) + x_0(b_0 - b_1) + x_1(b'_0 - b'_1) + x_0(y_0 - y_1) - x_1(y'_0 - y'_1)$, we can compute the values shown in the last two columns of Table 3.2. Since all of these values are −1 or 0, the corona $C_n \odot P_m$ is cordial for all $n \geq 4$ and $m \geq 4$. (Figs. 12–19,) illustrate $C_4 \odot P_5$, $C_5 \odot P_5$, $C_5 \odot P_7$, $C_6 \odot P_6$, $C_6 \odot P_7$, $C_7 \odot P_5$, $C_7 \odot P_6$ and $C_7 \odot P_7$, respectively. Thus the lemma follows. □

As a consequence of all previous lemmas and example 3.1, one can establish the following theorem.

Theorem 3.1. *The corona $C_n \odot P_m$ is cordial for all $n \geq 3$ and $m \geq 1$.*

References

[1] I. Cahit, On cordial and 3-equitable labelings of graphs, Utilitas Math. 37 (1990) 189–198.
[2] A. T. Diab, S. Nada, Some results on specific graphs, accepted for publication in Ars Combin.
[3] A.T. Diab, Study of some problems of cordial graphs, Ars Combin. 92 (2009) 255–261.
[4] A.T. Diab, On cordial labelings of the second power of paths with other graphs, Ars Combin. 97A (2010) 327–343.
[5] A.T. Diab, Generalization of some results on cordial graphs, Ars Combin. 99 (2009) 161–173.
[6] A.T. Diab, On cordial labelings of wheels with other graphs, Ars Combin. 100 (2011) 265–279.
[7] A.T. Diab, S.A.E. Mohammed, On cordial labelings of fans with other graphs,accepted for publication in Ars Combin.
[8] A.T. Diab, On cordial labelings of the second power of cycles with other graphs,accepted for publication in Utilitas Mathematica.
[9] G.M. Du, Cordiality of Complete k-Partite Graphs and Some Special Graphs, Neimenggu Shida Xuebao Ziran Kexue Hanwen Ban, 1997, pp. 9–12.
[10] A. Rosa, On certain valuations of the vertices of a graph, in: Theory of Graphs (Internat. Symposium, Rome, July 1966), Gordon and Breach, N. Y. and Dunod Paris, 1967, pp. 349–355.
[11] S.W. Golomb, in: R.C. Read (Ed.), How to Number a Graph in Graph Theory and Computing, Academic Press, New York, 1972, pp. 23–37.
[12] R.L. Graham, N.J.A. Sloane, On additive bases and harmonious graphs, SIAM J. Alg. Discrete Math. 1 (1980) 382–404.
[13] J.A. Gallian, A dynamic survey of graph labeling, Electron. J. Combin. DS6 (2014).
[14] R. Frucht, F. Harary, On the corona of two graphs, Aequationes Math. 4 (1970) 322–325.
[15] S. Nada, A.T. Diab, A. Elrokh, D.E. Sabra, The corona between paths and cycles, accepted in Ars Combin., 2015.