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Short Communication

Note on “$\mathcal{I}P$-separation axioms in ideal bitopological ordered spaces Π”

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Abstract In this note, we show that Examples 3.1, 3.3, 3.4, 3.5, 3.8, 3.9, 3.10 and 3.11 in [1] are incorrect, by giving remarks and comments on these examples. Finally, reasonable reasons to improve some of the incorrect examples have been mentioned.

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1. Preliminaries

In this section, we recall some basic notions in ideal and ideal bitopological ordered spaces.

Definition 1.1 [2]**.** A nonempty collection $\mathcal{I}$ of subsets of a set X is called an ideal on X, if it satisfies the following assertions:

1. $A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$, (finite additivity),
2. $A \in \mathcal{I}$ and $B \subseteq A \Rightarrow B \in \mathcal{I}$, (heredity).

Definition 1.2 [3]**.** Let (X, R) be a poset and $\mathcal{I}$ be an ideal on X. A set $A \subseteq X$ is said to be:

1. $\mathcal{I}$-decreasing if $Ra \cap A^c \in \mathcal{I}\ \forall a \in A$, where $Ra = \{b : bRa\}$ and A^c is the complement of A,
2. $\mathcal{I}$-increasing if $aR \cap A^c \in \mathcal{I}\ \forall a \in A$, where $aR = \{b : aRb\}$.

Definition 1.3 [4]**.** A space $(X, \tau_1, \tau_2, R, \mathcal{I})$ is called an ideal bitopological ordered space if (X, τ_1, τ_2, R) is a bitopological ordered space and $\mathcal{I}$ is an ideal on X.

Definition 1.4 [4]**.** An ideal bitopological ordered space $(X, \tau_1, \tau_2, R, \mathcal{I})$ is said to be:

1. $\mathcal{I}$-lower PT_1 ($\mathcal{I}LPT_1$, for short) ordered space if for every $a, b \in X$ such that $a\overline{R}b$, there exists an $\mathcal{I}$-increasing τ_i-open set U such that $a \in U$ and $b \notin U$, $i = 1$ or 2.
2. $\mathcal{I}$-upper PT_1 ($\mathcal{I}UPT_1$, for short) ordered space if for every $a, b \in X$ such that $a\overline{R}b$, there exists an $\mathcal{I}$-decreasing τ_i-open set V such that $b \in V$ and $a \notin V$, $i = 1$ or 2.
3. $\mathcal{I}PT_1$-ordered space if it is $\mathcal{I}LPT_1$ and $\mathcal{I}UPT_1$ ordered space.

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Definition 1.5 [1]. An ideal bitopological ordered space $(X, \tau_1, \tau_2, R, \mathcal{I})$ is said to be:

1. $\mathcal{I}$-lower pairwise regular ($\mathcal{I}LPR_2$, for short) ordered space if for every $\mathcal{I}$-decreasing τ_i-closed set F and for every $a \notin F$, there exist an $\mathcal{I}$-increasing τ_i-open set U and an $\mathcal{I}$-decreasing τ_j-open set V such that $a \in U$, $F - V \in \mathcal{I}$ and $U \cap V \in \mathcal{I}$.
2. $\mathcal{I}$-upper pairwise regular ($\mathcal{I}UPR_2$, for short) ordered space if for every $\mathcal{I}$-increasing τ_i-closed set F and for every $a \notin F$, there exist an $\mathcal{I}$-decreasing τ_i-open set U and an $\mathcal{I}$-increasing τ_j-open set V such that $a \in U$, $F - V \in \mathcal{I}$ and $U \cap V \in \mathcal{I}$.
3. $\mathcal{I}$-pairwise regular ($\mathcal{I}PR_2$, for short) ordered space if it is $\mathcal{I}LPR_2$ and $\mathcal{I}UPR_2$.

Definition 1.6 [1]. An ideal bitopological ordered space $(X, \tau_1, \tau_2, R, \mathcal{I})$ is called $\mathcal{I}PT_3$-ordered space if it is $\mathcal{I}PR_2$ and $\mathcal{I}PT_1$-ordered space.

Definition 1.7 [1]. Let $(X, \tau_1, \tau_2, R, \mathcal{I})$ be an ideal bitopological ordered space and $A, B \subseteq X$. Then A and B are said to be $\mathcal{I}P$-separated sets if $A \cap \tau_j - cl(B) \in \mathcal{I}$ and $\tau_i - cl(A) \cap B \in \mathcal{I}$.

Definition 1.8 [1]. An ideal bitopological ordered space $(X, \tau_1, \tau_2, R, \mathcal{I})$ is said to be $\mathcal{I}P$-completely normal ordered space ($\mathcal{I}PR_4$-ordered space, for short) if for any two $\mathcal{I}P$-separated subsets A and B of X such that A is $\mathcal{I}$-increasing set and B is $\mathcal{I}$-decreasing set there exist an $\mathcal{I}$-increasing τ_i-open set U and $\mathcal{I}$-decreasing τ_j-open set V such that $A \subseteq U$, $B \subseteq V$ and $U \cap V \in \mathcal{I}$.

2. Main results

Kandil et al. [Example 3.1, 1] claimed that $(X, \tau_1, \tau_2, R, \mathcal{I})$ is $\mathcal{I}LPR_2$-ordered space, but this is erroneous by the following remark.

Remark 2.1. The family of all $\mathcal{I}$-decreasing τ_1-closed sets is $\{X, \{2\}, \{2, 3, 4\}\}$, the collection of all $\mathcal{I}$-increasing τ_1-open sets is $\{X, \{4\}, \{1, 4\}, \{1, 3, 4\}\}$ and X is the only $\mathcal{I}$-decreasing τ_2-open set. Hence $F = \{2, 3, 4\}$ is $\mathcal{I}$-decreasing τ_1-closed set not containing 1, $U = X$ or $\{1, 4\}$ or $\{1, 3, 4\}$ is the only $\mathcal{I}$-increasing τ_1-open set containing 1 and $V = X$ is the only $\mathcal{I}$-decreasing τ_2-open set such that $F - V = \emptyset \in \mathcal{I}$ but $U \cap V \notin \mathcal{I}$.

Kandil et al. [Example 3.3, 1] claimed that $(X, \tau_1, \tau_2, R, \mathcal{I})$ is $\mathcal{I}PR_2$-ordered space, but this is incorrect by the following remark.

Remark 2.2. The family of all $\mathcal{I}$-decreasing τ_1-closed sets is $\{X, \{3\}, \{4\}, \{3, 4\}\}$, the collection of all $\mathcal{I}$-increasing τ_1-open sets is $\{X, \{1, 2, 3\}\}$ and $\{X, \{2, 3\}\}$ is the family of all $\mathcal{I}$-decreasing τ_2-open sets. Hence $F = \{3, 4\}$ is $\mathcal{I}$-decreasing τ_1-closed set not containing 1, $U = X$ or $\{1, 2, 3\}$ is the only $\mathcal{I}$-increasing τ_1-open set containing 1 and $V = X$ or $\{2, 3\}$ is the only $\mathcal{I}$-decreasing τ_2-open set such that $F - V \in \mathcal{I}$ but $U \cap V \notin \mathcal{I}$. Hence $(X, \tau_1, \tau_2, R, \mathcal{I})$ is not $\mathcal{I}LPR_2$-ordered space. As a result, $(X, \tau_1, \tau_2, R, \mathcal{I})$ is not $\mathcal{I}PR_2$-ordered space.

Kandil et al. [Example 3.4, 1] asserted that $(X, \tau_1, \tau_2, R, \mathcal{I})$ is $\mathcal{I}PT_3$-ordered space, but this is incorrect by the following remark.

Remark 2.3. The family of all $\mathcal{I}$-decreasing τ_1-closed sets is $\{X, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{2, 3, 4\}\}$, the collection of all $\mathcal{I}$-increasing τ_1-open sets is $\{X, \{3\}, \{1, 3\}, \{3, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}$ and $\{X, \{2, 3\}\}$ is the family of all $\mathcal{I}$-decreasing τ_2-open sets. Hence $F = \{2, 3, 4\}$ is $\mathcal{I}$-decreasing τ_1-closed set not containing 1, $U = X$ or $\{1, 3\}$ or $\{1, 2, 3\}$ or $\{1, 3, 4\}$ is the only $\mathcal{I}$-increasing τ_1-open set containing 1 and $V = X$ or $\{2, 3\}$ is the only $\mathcal{I}$-decreasing τ_2-open set such that $F - V \in \mathcal{I}$, but $U \cap V \notin \mathcal{I}$. Hence $(X, \tau_1, \tau_2, R, \mathcal{I})$ is not $\mathcal{I}LPR_2$-ordered space. As a result, $(X, \tau_1, \tau_2, R, \mathcal{I})$ is not $\mathcal{I}PR_2$-ordered space. It follows that it is not $\mathcal{I}PT_3$-ordered space.

Kandil et al. [Example 3.5, 1] asserted that $(X, \tau_1, \tau_2, R, \mathcal{I})$ is $\mathcal{I}PR_2$-ordered space, but this is incorrect by the following remark.

Remark 2.4. The family of all $\mathcal{I}$-decreasing τ_1-closed sets is $\{X, \{3\}, \{4\}, \{1, 4\}, \{3, 4\}, \{1, 3, 4\}\}$, the collection of all $\mathcal{I}$-increasing τ_1-open sets is $\{X, \{2, 3\}, \{1, 2, 3\}\}$ and $\{X, \{2, 3\}\}$ is the family of all $\mathcal{I}$-decreasing τ_2-open sets. Hence $F = \{1, 3, 4\}$ is $\mathcal{I}$-decreasing τ_1-closed set not containing 2, $U = X$ or $\{2, 3\}$ or $\{1, 2, 3\}$ is the only $\mathcal{I}$-increasing τ_1-open set containing 2 and $V = X$ or $\{2, 3\}$ are the only $\mathcal{I}$-decreasing τ_2-open set such that $F - V \in \mathcal{I}$, but $U \cap V \notin \mathcal{I}$. Hence $(X, \tau_1, \tau_2, R, \mathcal{I})$ is not $\mathcal{I}LPR_2$-ordered space. Thus $(X, \tau_1, \tau_2, R, \mathcal{I})$ is not $\mathcal{I}PR_2$-ordered space.

Note 2.5. It may be noted that [Example 3.5, 1] is not PR_2.

The following remark introduces suggestion to find possible real examples.

Remark 2.6. It may be noted that we can find correct examples if [Definition 3.1, 1] satisfied for $i \neq j$, $i, j = 1$ or 2.

Kandil et al. [Example 3.8, 1] asserted that the collection $\mathcal{I} = \{\emptyset, (1, \infty), (a, \infty), [a, \infty), (a, b), [a, b), (a, b], [a, b], \{c\}\}$, where $1 < a < b$, $1 < c < \infty$ is ideal and build their example on this assertion, but this is wrong by the following remark.

Remark 2.7. If $1 < a < b < c$ or $1 < c < a < b$, then $(a, b), [a, b), (a, b], [a, b], \{c\} \in \mathcal{I}$ but $(a, b) \cup \{c\}, [a, b) \cup \{c\}, (a, b] \cup \{c\}, [a, b] \cup \{c\} \notin \mathcal{I}$. As a consequence, $(\mathbb{R}, \tau_l, \tau_{\mathbb{U}}, R, \mathcal{I})$ is not ideal bitopological ordered space and the example is invalid.

The following remark shows that the collection $\mathcal{I} = \{\emptyset, (0, \infty), (a, \infty), [a, \infty), (a, b), [a, b), (a, b], [a, b], \{c\}\}$, where $0 \leq a < b$, $0 \leq c < \infty$ presented in [Examples 3.9 and 3.10, 1] is not ideal.

Remark 2.8. If $0 \leq a < b < c$ or $0 \leq c < a < b$, then $(a, b), [a, b), (a, b], [a, b], \{c\} \in \mathcal{I}$ but $(a, b) \cup \{c\}, [a, b) \cup \{c\}, (a, b] \cup \{c\}, [a, b] \cup \{c\} \notin \mathcal{I}$. As a consequence, $(\mathbb{R}, \tau_{\mathbb{U}}, \tau_l, R, \mathcal{I})$ and $(\mathbb{R}, \tau_{\mathbb{U}}, \tau_u, R, \mathcal{I})$ are not ideal bitopological ordered spaces and the examples are invalid. Moreover, the authors asserted in [Example 3.10, 1] that $A = (1, \infty)$ and $B = (-\infty, 0)$ are two P-separated sets but this is totally wrong as $\tau_u - cl(A) = \mathbb{R}$ and hence $\tau_u - cl(A) \cap B = (-\infty, 0) \neq \emptyset$.

Kandil et al. [Example 3.11, 1] claimed that the subsets $A = \{2, 3\}$ and $B = \{1\}$ are $\mathcal{I}P$-separated and construct their example on this assertion, but this is incorrect by the following remark.

Remark 2.9. $\tau_2 - cl(A) = X$ and hence $\tau_2 - cl(A) \cap B = \{1\} \notin \mathcal{I}$. That is A and B are not $\mathcal{I}P$-separated sets. As a result, the example is invalid.

Remark 2.10. It may be noted that Example 3.11 in [1] is correct if the authors stated that [Definition 3.5, 1] satisfied for $i \neq j,\ i, j = 1$ or 2.

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