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Original Article

Extended trial equation method for nonlinear coupled Schrodinger Boussinesq partial differential equations

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Abstract In this paper, we improve the extended trial equation method to construct the exact solutions for nonlinear coupled system of partial differential equations in mathematical physics. We use the extended trial equation method to find some different types of exact solutions such as the Jacobi elliptic function solutions, soliton solutions, trigonometric function solutions and rational, exact solutions to the nonlinear coupled Schrodinger Boussinesq equations when the balance number is a positive integer. The performance of this method is reliable, effective and powerful for solving more complicated nonlinear partial differential equations in mathematical physics. The balance number of this method is not constant as we have in other methods. This method allows us to construct many new types of exact solutions. By using the Maple software package we show that all obtained solutions satisfy the original partial differential equations.

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1. Introduction

The effort in finding exact solutions to nonlinear differential equations is important for the understanding of most nonlinear physical phenomena. For instance, the nonlinear wave phenomena observed in fluid dynamics, plasma and optical fibers are often modeled by the bell shaped sech solutions and the kink shaped tanh solutions. In recent years, the exact solutions of nonlinear PDEs have been investigated by many authors (see for example [1–30]) who are interested in non-linear physical

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phenomena. Many powerful methods have been presented by those authors such as the inverse scattering transform [1], the Backlund transform [2], Darboux transform [3], the generalized Riccati equation [4,5], the Jacobi elliptic function expansion method [6,7], Painlevé expansionsmethod [8], the extended tanh-function method [9,10], the F-expansion method [11,12], the exp-function expansion method [13,14], the sub-ODE method [15,16], the extended sinh–cosh and sine–cosine methods [17,18], the (G′/G)-expansion method [19,20] and so on. Also, there are many methods for finding the analytic approximate solutions for nonlinear partial differential equations such as the homotopy perturbation method [21,22], Adomain decomposition method [23,24], Variation iteration and homotopy analysis method [27,28]. There are many other methods for solving the nonlinear partial differential equations (see for example [29–37]). Recently Khan et al. [38–43] implemented the modified simple equation method and enhanced (G′/G) expansion method to construct the traveling wave solutions for nonlinear evolution equations in mathematics physics. Also Khan and Akbar [44] and Akter et al. [45] used the $\exp(-\phi(\xi))$ expansion method to find exact solutions for nonlinear partial differential equations. More recently Gurefe et al. [46] have presented a direct method, namely the extended trial equation method for solving the nonlinear partial differential equations. The main objective of this paper is to modify the extended trial equation method to construct a series of some new analytic exact solutions for some nonlinear partial differential equations in mathematical physics via nonlinear coupled Schrodinger Boussinesq equations. In this present paper, we will construct the exact solutions in many different types of the roots of the trial equation. We will obtain many different kinds of exact solutions in hyperbolic function solutions, trigonometric function solutions, Jacobi elliptic functions solutions and rational solutions.

2. Description of the extended trial equation method

Suppose that we have a nonlinear partial differential equation in the following form:

$$F(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0, \tag{2.1}$$

where $u = u(x, t)$ is an unknown function, F is a polynomial in $u = u(x, t)$ and its partial derivatives, in which the highest order derivatives and nonlinear terms are involved. Let us now give the main steps for solving Eq. (2.1) using the extended trial equation method as [46–48]:

Step 1. The traveling wave variable

$$u(x, t) = u(\xi), \quad \xi = x - \omega t, \tag{2.2}$$

where ω is a nonzero constant, the transformation (2.2) permits us reducing Eq. (2.1) to an ODE for $u = u(\xi)$ in the following form

$$P(u, -\omega u', u', \omega^2 u'', -\omega u'', u'', \ldots) = 0, \tag{2.3}$$

where P is a polynomial of $u = u(\xi)$ and its total derivatives.

Step 2. Suppose the trial solution is of the form:

$$u(\xi) = \sum_{i=0}^{\delta} \tau_i Y^i, \tag{2.4}$$

where Y satisfies the following nonlinear auxiliary equation:

$$(Y')^2 = \Lambda(Y) = \frac{\Phi(Y)}{\Psi(Y)} = \frac{\xi_\theta Y^\theta + \xi_{\theta-1} Y^{\theta-1} + \cdots + \xi_1 Y + \xi_0}{\zeta_\varepsilon Y^\varepsilon + \zeta_{\varepsilon-1} Y^{\varepsilon-1} + \cdots + \zeta_1 Y + \zeta_0} \tag{2.5}$$

where τ_i, ξ_i, ζ_j are constants to be determined later. Using (2.4) and (2.5), we have

$$u''(\xi) = \frac{\Phi'(Y)\Psi(Y) - \Phi(Y)\Psi'(Y)}{2\Psi^2(Y)} \left(\sum_{i=0}^{\delta} i\, \tau_i Y^{i-1} \right) + \frac{\Phi(Y)}{\Psi(Y)} \left(\sum_{i=0}^{\delta} i\,(i-1) \tau_i Y^{i-2} \right). \tag{2.6}$$

where $\Phi(Y)$, $\Psi(Y)$ are polynomials in Y.

Step 3. Balancing the highest derivative term with the nonlinear term we can find the relations between δ, θ and ε. We can calculate some values of δ, θ and ε.

Step 4. Substituting Eqs. (2.4)–(2.6) into (2.3) yields a polynomial $\Omega(y)$ as follows

$$\Omega(y) = \rho_s Y^s + \cdots + \rho_1 Y + \rho_0 = 0 \tag{2.7}$$

Step 5. Setting the coefficients of the polynomial $\Omega(y)$ yield to be zero, and then we have a set of algebraic equations

$$\rho_i = 0, \quad i = 0, \ldots, s. \tag{2.8}$$

Solving this system of algebraic equations to determine the values of $\xi_\theta, \xi_{\theta-1}, \ldots, \xi_1, \xi_0, \zeta_\varepsilon, \zeta_{\varepsilon-1}, \ldots, \zeta_1, \zeta_0$ and $\tau_\delta, \tau_{\delta-1}, \ldots, \tau_1, \tau_0$.

Step 6. Reduce Eq. (2.5) in the elemental integral form:

$$\pm(\xi - \eta_0) = \int \frac{dY}{\sqrt{\Lambda(y)}} = \int \sqrt{\frac{\Psi(Y)}{\Phi(Y)}} dY. \tag{2.9}$$

where η_0 is an arbitrary constant. Using a complete discrimination system for the polynomial to classify the roots of $\Phi(Y)$, we solve (2.9) with the help of software packages such as Maple or Mathematica and classify the exact solutions to Eq. (2.3). In addition, we can write the exact traveling wave solutions to (2.1).

3. Extended trial equation method for nonlinear coupled Schrodinger Boussinesq equations

We consider the coupled nonlinear Schrodinger Boussinesq equations

$$\begin{aligned} &iE_t + E_{xx} + \beta_1 E - NE = 0, \\ &3N_{tt} - N_{xxxx} + 3(N^2)_{xx} + \beta_2 N_{xx} - (|E|^2)_{xx} = 0, \end{aligned} \tag{3.1}$$

where β_1, β_2 are real constants and $N(x, t)$ is a real function while $E(x, t)$ is a complex function. The system (3.1) is known to describe various physical processes in Laser and plasma, such as formation, Langmuir field amplitude and intense electromagnetic waves and modulation instabilities [49,50]. The traveling wave variable

$$E(x, t) = u(x, t)e^{i\eta}, \quad \eta = kx + ct + c_0 \tag{3.2}$$

where $u(x,t)$ is a real function and k, c, c_0 are real arbitrary constants, permits us to convert (3.1) into the following nonlinear system of partial differential equations:

$$u_t + 2ku_x = 0, \tag{3.3}$$

$$u_{xx} - (c + k^2 - \beta_1)u - Nu = 0, \tag{3.4}$$

$$3N_{tt} - N_{xxxx} + 3(N^2)_{xx} + \beta_2 N_{xx} - (u^2)_{xx} = 0, \tag{3.5}$$

We suppose that the solutions of the system (3.3)–(3.5) has the form:

$$u = \phi(\xi), \quad N = \psi(\xi), \quad \xi = x - 2kt, \tag{3.6}$$

where ϕ, ψ are arbitrary functions of ξ. The transformations (3.6) lead to write Eqs. (3.3)–(3.5) in the following form:

$$\phi'' - (c + k^2 - \beta_1)\phi - \phi\psi = 0, \tag{3.7}$$

$$-\psi'' + (12k^2 + \beta_2)\psi + 3\psi^2 - \phi^2 + A = 0. \tag{3.8}$$

Suppose that the exact solutions of Eqs. (3.7) and (3.8) can be rewritten in the form:

$$\phi(\xi) = \sum_{i=0}^{\delta_1} \tau_i Y^i, \quad \psi(\xi) = \sum_{i=0}^{\delta_2} T_i Y^i, \tag{3.9}$$

where τ_i, T_i are arbitrary constants to be determined later, Y satisfies the auxiliary Eq. (2.5) and δ_1, δ_2 are arbitrary positive integers. Balancing the highest order derivative terms with the nonlinear terms in (3.7) and (3.8), we get the relations between δ_1, δ_2, θ and ε as follows

$$\delta_1 = \delta_2 = \theta - \varepsilon - 2 \tag{3.10}$$

Eq. (3.10) has infinity solutions, consequently we suppose some of these solutions as follows:

Case 1. If $\theta = 3$ and $\varepsilon = 0$, we get $\delta_1 = \delta_2 = 1$. In this case we have:

$$\begin{aligned} \phi(\xi) &= \tau_0 + \tau_1 Y, \\ (\phi')^2 &= \frac{\tau_1^2(\xi_3 Y^3 + \xi_2 Y^2 + \xi_1 Y + \xi_0)}{\zeta_0}, \\ \phi'' &= \frac{\tau_1(3\xi_3 Y^2 + 2\xi_2 Y + \xi_1)}{2\zeta_0}, \end{aligned} \tag{3.11}$$

and

$$\begin{aligned} \psi(\xi) &= T_0 + T_1 Y, \\ (\psi')^2 &= \frac{T_1^2(\xi_3 Y^3 + \xi_2 Y^2 + \xi_1 Y + \xi_0)}{\zeta_0}, \\ \psi'' &= \frac{T_1(3\xi_3 Y^2 + 2\xi_2 Y + \xi_1)}{2\zeta_0}, \end{aligned} \tag{3.12}$$

Substituting Eqs. (3.11, (3.12) into Eqs. (3.7), (3.8) and setting the coefficients of Y to be zero, we get a system of algebraic equations which can be solved by using the Maple software package to obtain the following results:

$$\begin{aligned} A &= -\frac{297}{25}k^4 + k^2\left(-\frac{49}{25}\beta_2 + \frac{6}{25}\beta_1 - \frac{6}{25}c\right) + \frac{1}{25}(3\beta_1^2 - 2\beta_2^2) \\ &\quad + \frac{1}{25}\beta_2(\beta_1 - c) + \frac{3}{25}c(c - 2\beta_1), \\ T_0 &= -\frac{1}{10\zeta_0^2}\left[4c\zeta_0^2 - 4\beta_1\zeta_0^2 + 16k^2\zeta_0^2 - 5\xi_2 + \beta_2\zeta_0^2\right], \\ T_1 &= \frac{1}{\sqrt{2}}\tau_1, \\ \tau_0 &= -\frac{\sqrt{2}}{10\zeta_0^2}\left[-6k^2\zeta_0^2 - 5\xi_2 + 6c\zeta_0^2 - 6\beta_1\zeta_0^2 - \beta_2\zeta_0^2\right], \\ \xi_3 &= \frac{\sqrt{2}}{3}\tau_1\zeta_0^2, \\ \xi_1 &= \frac{-\zeta_0^2}{25\tau_1}\{\beta_2^2 + \left[(12\mathrm{k}^2 - 12\mathrm{c}) + 12\beta_1\right]\beta_2 + 36\beta_1^2 \\ &\quad + (72\mathrm{k}^2 - 72\mathrm{c})\beta_1 - \frac{25\xi_2^2}{\zeta_0^4} + (36\mathrm{k}^4 + 36\mathrm{c}^2 - 72\mathrm{k}^2\mathrm{c})\} \end{aligned} \tag{3.13}$$

where ζ_0, ξ_2 and τ_1 are arbitrary constants. Substituting Eqs. (3.13) into Eqs. (2.5) and (2.9), we have

$$\pm(\xi - \eta_0) = L\int \frac{dY}{\sqrt{Y^3 + \frac{\xi_2}{\xi_3}Y^2 + \frac{\xi_1}{\xi_3}Y + \frac{\xi_o}{\xi_3}}}, \tag{3.14}$$

where $L = \sqrt{\frac{\zeta_0}{\xi_3}}$.

To integrate Eq. (3.14), we must discuss the following families:

Family 1. If $Y^3 + \frac{\xi_2}{\xi_3}Y^2 + \frac{\xi_1}{\xi_3}Y + \frac{\xi_o}{\xi_3}$, can be written in the following form:

$$\begin{aligned} &Y^3 + \frac{3\sqrt{2}\,\xi_2}{2\tau_1\,\zeta_0^2}Y^2 - \frac{3\sqrt{2}}{50\tau_1^2}\Big\{\beta_2^2 + \left[(12k^2 - 12c) + 12\beta_1\right]\beta_2 + 36\beta_1^2 \\ &\quad + (72k^2 - 72c)\beta_1 - \frac{25\xi_2^2}{\zeta_0^4} + (36k^4 + 36c^2 - 72k^2c)\Big\}Y \\ &\quad + \frac{3\sqrt{2}\xi_0}{2\tau_1\,\zeta_0^2} = (Y - \alpha_1)^3 \end{aligned} \tag{3.15}$$

where α_1 is an arbitrary nonzero constant.

From equating the coefficients of Y in both sides of Eq. (3.15), we get a system of algebraic equations in ξ_0, ξ_2, ζ_0, τ_1 and c, which can be solved by using the Maple software package to get the following results:

$$c = \beta_1 + k^2 + \frac{1}{6}\beta_2, \quad \xi_0 = -\alpha_1^3\zeta_0, \quad \xi_2 = -3\alpha_1\zeta_0, \quad \tau_1 = \frac{3\sqrt{2}}{2}. \tag{3.16}$$

Eqs. (3.16), (3.13) and (3.14) lead to get:

$$\begin{aligned} &A = -\frac{1}{12}\beta_2^2 - 12k^4 - 2\beta_2 k^2, \quad T_0 = -2k^2 - \frac{1}{6}\beta_2 - \frac{3}{2}\alpha_1, \\ &T_1 = \frac{3}{2}, \quad \xi_1 = 3\zeta_0\alpha_1^2, \quad \xi_3 = \zeta_0, \quad \tau_0 = -\frac{3\sqrt{2}\alpha_1}{2}, \end{aligned} \tag{3.17}$$

where ζ_0 is an arbitrary constant and

$$\pm(\xi - \eta_0) = \int \frac{dY}{(Y - \alpha_1)^{3/2}} = \frac{-2}{\sqrt{Y - \alpha_1}}. \tag{3.18}$$

or

$$Y = \alpha_1 + \frac{4}{(x - 2kt - \eta_0)^2}. \tag{3.19}$$

Substituting Eqs. (3.16), (3.17) and (3.19) into (3.11) and (3.12), we deduce that exact solutions of Eqs. (3.7) and (3.8) have the following form:

$$\phi(\xi) = \frac{6\sqrt{2}}{(x - 2kt - \eta_0)^2}, \tag{3.20}$$

and

$$\psi(\xi) = -2k^2 - \frac{1}{6}\beta_2 + \frac{6}{(x - 2kt - \eta_0)^2}. \tag{3.21}$$

Hence the exact solutions of nonlinear Schrodinger Boussinesq equations (3.1) take the following form:

$$E(x,t) = \frac{6\sqrt{2}}{(x - 2kt - \eta_0)^2} e^{i\left[kx + \left(\beta_1 + k^2 + \frac{1}{6}\beta_2\right)t + c_0\right]}, \tag{3.22}$$

and

$$N(x,t) = -2k^2 - \frac{1}{6}\beta_2 + \frac{6}{(x - 2kt - \eta_0)^2}. \tag{3.23}$$

Family 2. If $Y^3 + \frac{\xi_2}{\xi_3}Y^2 + \frac{\xi_1}{\xi_3}Y + \frac{\xi_0}{\xi_3}$, can be written in the following form:

$$\begin{aligned} Y^3 &+ \frac{3\sqrt{2}\,\xi_2}{2\tau_1\,\zeta_0^2}Y^2 - \frac{3\sqrt{2}}{50\tau_1^2}\Big\{\beta_2^2 + \left[(12k^2 - 12c) + 12\beta_1\right]\beta_2 + 36\beta_1^2 \\ &+ (72k^2 - 72c)\beta_1 - \frac{25\xi_2^2}{\zeta_0^4} + (36k^4 + 36c^2 - 72k^2c)\Big\}Y \\ &+ \frac{3\sqrt{2}\xi_0}{2\tau_1\,\zeta_0^2} = (Y - \alpha_1)^2(Y - \alpha_2) \end{aligned} \tag{3.24}$$

where α_1 and α_2 are arbitrary nonzero constants. From equating the coefficients of Y in both sides of Eq. (3.24), we get a system of algebraic equations in ξ_0, ξ_2, ζ_0, τ_1 and c which can be solved by using the Maple software package to get the following results:

$$\begin{aligned} &c = \beta_1 + k^2 + \frac{1}{6}\beta_2 + \frac{5}{6}(\alpha_2 - \alpha_1), \quad \xi_0 = -\alpha_1^2\alpha_2\zeta_0, \\ &\xi_2 = -2\alpha_1\zeta_0 - \alpha_2\zeta_0, \quad \tau_1 = \frac{3\sqrt{2}}{2}. \end{aligned} \tag{3.25}$$

Eqs. (3.13), (3.14) and (3.25) lead to get:

$$\begin{aligned} &A = \frac{1}{12}(\alpha_2^2 + \alpha_1^2) - \frac{1}{6}\alpha_2\alpha_1 - \frac{1}{12}\beta_2^2 - 12k^4 - 2\beta_2k^2, \\ &T_0 = -\frac{1}{6}\beta_2 - 2k^2 - \frac{3}{2}\alpha_1 - \frac{5}{6}\alpha_2, \quad T_1 = \frac{3}{2}, \\ &\xi_1 = \zeta_0(\alpha_1 + 2\alpha_2)\alpha_1, \quad \xi_3 = \zeta_0, \quad \tau_0 = -\frac{\sqrt{2}}{2}(\alpha_1 + 2\alpha_2), \end{aligned} \tag{3.26}$$

where ζ_0 is an arbitrary constant and if $\alpha_2 > \alpha_1$, we have

$$\pm(\xi - \eta_0) = \int \frac{dY}{(Y - \alpha_1)\sqrt{Y - \alpha_2}} = \frac{2}{\sqrt{\alpha_2 - \alpha_1}}\tan^{-1} \times \left[\frac{\sqrt{Y - \alpha_2}}{\sqrt{\alpha_2 - \alpha_1}}\right], \quad \alpha_2 > \alpha_1. \tag{3.27}$$

or

$$Y = \alpha_2 + (\alpha_2 - \alpha_1)\tan^2\left[\frac{\sqrt{\alpha_2 - \alpha_1}}{2}(\xi - \eta_0)\right], \quad \alpha_2 > \alpha_1. \tag{3.28}$$

Substituting Eqs. (3.28), (3.26) and (3.25) into (3.11) and (3.12), we get the exact solutions of Eqs. (3.7) and (3.8) take the form:

$$\phi(\xi) = -\frac{\sqrt{2}}{2}(\alpha_1 + 2\alpha_2) + \frac{3\sqrt{2}}{2} \times \left\{\alpha_2 + (\alpha_2 - \alpha_1)\tan^2\left[\frac{\sqrt{\alpha_2 - \alpha_1}}{2}(x - 2kt - \eta_0)\right]\right\}, \tag{3.29}$$

and

$$\psi(\xi) = -\frac{1}{6}\beta_2 - 2k^2 - \frac{3}{2}\alpha_1 - \frac{5}{2}\alpha_2 + \frac{3}{2} \times \left\{\alpha_2 + (\alpha_2 - \alpha_1)\tan^2\left[\frac{\sqrt{\alpha_2 - \alpha_1}}{2}(x - 2kt - \eta_0)\right]\right\}. \tag{3.30}$$

Hence the exact solutions of nonlinear Schrodinger Boussinesq equations (3.1) take the following form:

$$E(x,t) = \left\{-\frac{\sqrt{2}}{2}(\alpha_1 + 2\alpha_2) + \frac{3\sqrt{2}}{2}\left\{\alpha_2 + (\alpha_2 - \alpha_1)\tan^2 \times \left[\frac{\sqrt{\alpha_2 - \alpha_1}}{2}(x - 2kt - \eta_0)\right]\right\} e^{i\left[kx + \left(\beta_1 + k^2 + \frac{1}{6}\beta_2\right)t + c_0\right]}\right\}, \tag{3.31}$$

and

$$N(x,t) = -\frac{1}{6}\beta_2 - 2k^2 - \frac{3}{2}\alpha_1 - \frac{5}{2}\alpha_2 + \frac{3}{2}\left\{\alpha_2 + (\alpha_2 - \alpha_1)\tan^2\left[\frac{\sqrt{\alpha_2 - \alpha_1}}{2}(x - 2kt - \eta_0)\right]\right\}. \tag{3.32}$$

Also, when $\alpha_1 > \alpha_2$, we have

$$Y = \alpha_1 + (\alpha_1 - \alpha_2)\operatorname{csch}^2\left[\frac{\sqrt{\alpha_1 - \alpha_2}}{2}(\xi - \eta_0)\right], \quad \alpha_1 > \alpha_2. \tag{3.33}$$

Substituting Eqs. (3.33), (3.26) and (3.25) into (3.11) and (3.12), we get the exact solutions of Eqs. (3.7) and (3.8) take the form:

$$\phi(\xi) = -\frac{\sqrt{2}}{2}(\alpha_1 + 2\alpha_2) + \frac{3\sqrt{2}}{2} \times \left\{\alpha_1 + (\alpha_1 - \alpha_2)\operatorname{csch}^2\left[\frac{\sqrt{\alpha_1 - \alpha_2}}{2}(\xi - \eta_0)\right]\right\}, \tag{3.34}$$

and

$$\psi(\xi) = -\frac{1}{6}\beta_2 - 2k^2 - \frac{3}{2}\alpha_1 - \frac{5}{2}\alpha_2 + \frac{3}{2}\left\{\alpha_1 + (\alpha_1 - \alpha_2)\operatorname{csch}^2\left[\frac{\sqrt{\alpha_1 - \alpha_2}}{2}(\xi - \eta_0)\right]\right\}. \tag{3.35}$$

Hence the exact solutions of coupled nonlinear Schrodinger Boussinesq equations (3.1) take the form:

$$E(x,t) = \left\{-\frac{\sqrt{2}}{2}(\alpha_1+2\alpha_2) + \frac{3\sqrt{2}}{2}\left\{\alpha_1 + (\alpha_1-\alpha_2)\mathrm{csch}^2 \times \left[\frac{\sqrt{\alpha_1-\alpha_2}}{2}(x-2kt-\eta_0)\right]\right\} e^{i[kx+(\beta_1+k^2+\frac{1}{6}\beta_2)t+c_0]}\right\}, \tag{3.36}$$

and

$$N(x,t) = -\frac{1}{6}\beta_2 - 2k^2 - \frac{3}{2}\alpha_1 - \frac{5}{2}\alpha_2 + \frac{3}{2}\left\{\alpha_1 + (\alpha_1-\alpha_2)\mathrm{csch}^2\left[\frac{\sqrt{\alpha_1-\alpha_2}}{2}(x-2kt-\eta_0)\right]\right\}. \tag{3.37}$$

Family 3. If $Y^3 + \frac{\xi_2}{\xi_3}Y^2 + \frac{\xi_1}{\xi_3}Y + \frac{\xi_o}{\xi_3}$, can be written in the following form:

$$Y^3 + \frac{3\sqrt{2}\,\xi_2}{2\tau_1\,\zeta_0^2}Y^2 - \frac{3\sqrt{2}}{50\tau_1^2}\left\{\beta_2^2 + \left[(12k^2-12c)+12\beta_1\right]\beta_2 + 36\beta_1^2 + (72k^2-72c)\beta_1 - \frac{25\xi_2^2}{\zeta_0^4} + (36k^4+36c^2-72k^2c)\right\}Y + \frac{3\sqrt{2}\xi_0}{2\tau_1\,\zeta_0^2} = (Y-\alpha_1)(Y-\alpha_2)(Y-\alpha_3) \tag{3.38}$$

where α_1, $\alpha_2$andα_3 are nonzero arbitrary constants. Equating the coefficients of Y in both sides of Eq. (3.38), we get a system of algebraic equations in ξ_0, ξ_2, ζ_0, τ_1 and c which can be solved by using the Maple software package to get the following results:

$$c = D_i(i=1,2), \quad \xi_0 = -\alpha_1\alpha_2\alpha_3\zeta_0,$$
$$\xi_2 = -\alpha_1\zeta_0 - \alpha_2\zeta_0 - \alpha_3\zeta_0, \quad \tau_1 = \frac{3\sqrt{2}}{2}, \tag{3.39}$$

where $D_i(i=1,2)$are the roots of equation

$$36Z^2 + \left(-72\beta_1 - 12\beta_2 - 72k^2\right)Z + 25\alpha_2\alpha_3 + 25\alpha_1\alpha_3 + 25\alpha_1\alpha_2 + 36\beta_1^2 + 36k^4 - 25\alpha_1^2 - 25\alpha_2^2 - 25\alpha_3^2 + 12k^2\beta_2 + \beta_2^2 + 12\beta_1\beta_2 + 72k^2\beta_1 = 0, \tag{3.40}$$

which take the following form

$$D_1 = \beta_1 + \frac{1}{6}\beta_2 + k^2 + \frac{5}{6}\sqrt{\alpha_1^2+\alpha_2^2+\alpha_3^2-\alpha_2\alpha_3-\alpha_1\alpha_3-\alpha_1\alpha_2},$$
$$D_2 = \beta_1 + \frac{1}{6}\beta_2 + k^2 - \frac{5}{6}\sqrt{\alpha_1^2+\alpha_2^2+\alpha_3^2-\alpha_2\alpha_3-\alpha_1\alpha_3-\alpha_1\alpha_2}. \tag{3.41}$$

Eqs. (3.39), (3.13) and (3.14) lead to get:

$$A = -12k^4 - 2\beta_2k^2 - \frac{1}{12}\beta_2^2 - \frac{1}{12}(\alpha_2\alpha_3+\alpha_1\alpha_3+\alpha_1\alpha_2) + \frac{1}{12}(\alpha_1^2+\alpha_2^2+\alpha_3^2),$$
$$T_0 = -\frac{2}{5}D_i + \frac{2}{5}\beta_1 - \frac{8}{5}k^2 - \frac{1}{2}(\alpha_1+\alpha_2+\alpha_3) - \frac{1}{10}\beta_2, \quad T_1 = \frac{3}{2},$$
$$\xi_1 = \zeta_0(\alpha_2\alpha_3+\alpha_1\alpha_3+\alpha_1\alpha_2), \quad \xi_3 = \zeta_0,$$
$$\tau_0 = -\frac{\sqrt{2}}{10}(-6k^2 + 5(\alpha_1+\alpha_2+\alpha_3) + 6D_i - 6\beta_1 - \beta_2), \tag{3.42}$$

where ζ_0 is an arbitrary constant and

$$\pm(\xi-\eta_0) = \int\frac{dY}{\sqrt{(Y-\alpha_1)(Y-\alpha_2)(Y-\alpha_3)}} = -\frac{2}{\sqrt{\alpha_3-\alpha_1}} EllipticF\left[\frac{\sqrt{Y-\alpha_1}}{\sqrt{\alpha_2-\alpha_1}}, \sqrt{\frac{\alpha_1-\alpha_2}{\alpha_1-\alpha_3}}\right], \tag{3.43}$$

or

$$Y = \alpha_1 + (\alpha_2-\alpha_1)sn^2\left[\frac{\sqrt{\alpha_3-\alpha_1}}{2}(\eta-\eta_0), \sqrt{\frac{\alpha_1-\alpha_2}{\alpha_1-\alpha_3}}\right]. \tag{3.44}$$

Substituting Eqs. (3.39), (3.42) and (3.44) into (3.11) and (3.12), we get the exact solutions of Eqs. (3.7) and (3.8) take the form:

$$\phi(\xi) = -\frac{\sqrt{2}}{10}(-6k^2 + 5(\alpha_1+\alpha_2+\alpha_3) + 6D_i - 6\beta_1 - \beta_2) + \frac{3\sqrt{2}}{2} \times \left\{\alpha_1 + (\alpha_2-\alpha_1)sn^2\left[\frac{\sqrt{\alpha_3-\alpha_1}}{2}(\xi-\eta_0), \sqrt{\frac{\alpha_1-\alpha_2}{\alpha_1-\alpha_3}}\right]\right\} \tag{3.45}$$

and

$$\psi(\xi) = -\frac{2}{5}D_i + \frac{2}{5}\beta_1 - \frac{8}{5}k^2 - \frac{1}{2}(\alpha_1+\alpha_2+\alpha_3) - \frac{1}{10}\beta_2 + \frac{3}{2} \times \left\{\alpha_1 + (\alpha_2-\alpha_1)sn^2\left[\frac{\sqrt{\alpha_3-\alpha_1}}{2}(\xi-\eta_0), \sqrt{\frac{\alpha_1-\alpha_2}{\alpha_1-\alpha_3}}\right]\right\}, \tag{3.46}$$

Hence the exact solutions of the nonlinear Schrodinger Boussinesq equations (3.1) have the following form:

$$E(x,t) = \left\{-\frac{\sqrt{2}}{10}(-6k^2 + 5(\alpha_1+\alpha_2+\alpha_3) + 6D_i - 6\beta_1 - \beta_2 + \frac{3\sqrt{2}}{2}\left\{\alpha_1 + (\alpha_2-\alpha_1)sn^2\left[\frac{\sqrt{\alpha_3-\alpha_1}}{2}(x-2kt-\eta_0), \sqrt{\frac{\alpha_1-\alpha_2}{\alpha_1-\alpha_3}}\right]\right\} e^{i[kx+D_it+c_0]}, \tag{3.47}$$

and

$$N(x,t) = -\frac{2}{5}D_i + \frac{2}{5}\beta_1 - \frac{8}{5}k^2 - \frac{1}{2}(\alpha_1+\alpha_2+\alpha_3) - \frac{1}{10}\beta_2 + \frac{3}{2}\left\{\alpha_1 + (\alpha_2-\alpha_1)sn^2\left[\frac{\sqrt{\alpha_3-\alpha_1}}{2}(x-2kt-\eta_0), \sqrt{\frac{\alpha_1-\alpha_2}{\alpha_1-\alpha_3}}\right]\right\}. \tag{3.48}$$

Family 4. If $Y^3 + \frac{\xi_2}{\xi_3}Y^2 + \frac{\xi_1}{\xi_3}Y + \frac{\xi_o}{\xi_3}$, can be written in the following form:

$$Y^3 + \frac{3\sqrt{2}\,\xi_2}{2\tau_1\,\zeta_0^2}Y^2 - \frac{3\sqrt{2}}{50\tau_1^2}\{\beta_2^2 + [(12k^2-12c) + 12\beta_1]\beta_2 + 36\beta_1^2 + (72k^2-72c)\beta_1 - \frac{25\xi_2^2}{\zeta_0^4} + (36k^4+36c^2-72k^2c)\}Y + \frac{3\sqrt{2}\xi_0}{2\tau_1\,\zeta_0^2} = (Y-\alpha_1)(Y-(N_1+iN_2)(Y-(N_1-iN_2) \tag{3.49}$$

where α_1, N_1, N_2 are nonzero real numbers. From equating the coefficients of Y in both sides of Eq. (3.49), we get a system of algebraic equations in ξ_0, ξ_2, ζ_0, τ_1 and c which can be solved by using the Maple software package to get the following results:

$$c = D_i(i=1,2),\quad \xi_0 = -\alpha_1\zeta_0(N_1^2+N_2^2),$$
$$\xi_2 = -\alpha_1\zeta_0 - 2N_1\zeta_0,\quad \tau_1 = \frac{3\sqrt{2}}{2}, \tag{3.50}$$

where $D_i(i=1,2)$ are the roots of equation

$$36Z^2 + (-72\beta_1 - 12\beta_2 - 72k^2)Z + 72N_2^2 + 50\alpha_1 N_1 - 25N_1^2 + 36\beta_1^2 + 36k^4 + 12k^2\beta_2 + \beta_2^2 - 25\alpha_1^2 + 72k^2\beta_1 + 12\beta_1\beta_2 = 0, \tag{3.51}$$

which take the following form:

$$D_1 = \beta_1 + \frac{1}{6}\beta_2 + k^2 + \frac{5}{6}\sqrt{\alpha_1^2 + N_1^2 - 3N_3^2 - 2\alpha_1 N_1},$$
$$D_2 = \beta_1 + \frac{1}{6}\beta_2 + k^2 - \frac{5}{6}\sqrt{\alpha_1^2 + N_1^2 - 3N_3^2 - 2\alpha_1 N_1}. \tag{3.52}$$

Eqs. (3.50), (3.13) and (3.14) lead to get:

$$A = -12k^4 - 2\beta_2 k^2 - \frac{1}{12}\beta_2^2 - \frac{1}{4}N_2^2 - \frac{1}{6}\alpha_1 N_1 + \frac{1}{12}N_1^2 + \frac{1}{12}\alpha_1^2,$$
$$T_0 = -\frac{2}{5}D_i + \frac{2}{5}\beta_1 - \frac{8}{5}k^2 - \frac{1}{2}\alpha_1 - N_1 - \frac{1}{10}\beta_2,$$
$$T_1 = \frac{3}{2},\quad \xi_1 = \zeta_0(N_2^2 + 2\alpha_1 N_1 + N_1^2),\quad \xi_3 = \zeta_0,$$
$$\tau_0 = -\frac{\sqrt{2}}{10}(-6k^2 + 5\alpha_1 + 10N_1 + 6D_i - 6\beta_1 - \beta_2), \tag{3.53}$$

where ζ_0 is an arbitrary constant. With the help of Maple software package the integration of Eq. (3.14) in this family takes the following form:

$$\pm(\xi - \eta_0) = \int \frac{dY}{\sqrt{(Y-\alpha_1)(Y^2 - 2N_1 Y + N_1^2 + N_2^2)}} = \frac{2}{\sqrt{N_1 + iN_2 - \alpha_1}} EllipticF \times\left[\frac{\sqrt{Y-\alpha_1}}{\sqrt{N_1 - iN_2 - \alpha_1}}, \sqrt{\frac{N_1 - iN_2 - \alpha_1}{N_1 + iN_2 - \alpha_1}}\right], \tag{3.54}$$

or

$$Y = \alpha_1 + (N_1 - iN_2 - \alpha_1)sn^2\left[\frac{\sqrt{N_1 + iN_2 - \alpha_1}}{2}(\xi - \eta_0), \sqrt{\frac{N_1 - iN_2 - \alpha_1}{N_1 + iN_2 - \alpha_1}}\right]. \tag{3.55}$$

Substituting Eqs. (3.50), (3.53) and (3.55) into (3.11) and (3.12), we get the exact solutions of Eqs. (3.7) and (3.8) take the form:

$$\phi(\xi) = -\frac{\sqrt{2}}{10}(-6k^2 + 5\alpha_1 + 10N_1 + 6D_i - 6\beta_1 - \beta_2) + \frac{3\sqrt{2}}{2}\left\{\alpha_1 + (N_1 - iN_2 - \alpha_1)sn^2\left[\frac{\sqrt{N_1 + iN_2 - \alpha_1}}{2} \times(\xi - \eta_0), \sqrt{\frac{N_1 - iN_2 - \alpha_1}{N_1 + iN_2 - \alpha_1}}\right]\right\}, \tag{3.56}$$

and

$$\psi(\xi) = -\frac{2}{5}D_i + \frac{2}{5}\beta_1 - \frac{8}{5}k^2 - \frac{1}{2}\alpha_1 - N_1 - \frac{1}{10}\beta_2 + \frac{3}{2}\left\{\alpha_1 + (N_1 - iN_2 - \alpha_1)sn^2\left[\frac{\sqrt{N_1 + iN_2 - \alpha_1}}{2} \times(\xi - \eta_0), \sqrt{\frac{N_1 - iN_2 - \alpha_1}{N_1 + iN_2 - \alpha_1}}\right]\right\}. \tag{3.57}$$

Hence the exact solutions of nonlinear Schrodinger Boussinesq equations (3.1) take the following form:

$$E(x,t) = \left\{-\frac{\sqrt{2}}{10}(-6k^2 + 5\alpha_1 + 10N_1 + 6D_i - 6\beta_1 - \beta_2) + \frac{3\sqrt{2}}{2}\left\{\alpha_1 + (N_1 - iN_2 - \alpha_1)sn^2\left[\frac{\sqrt{N_1 + iN_2 - \alpha_1}}{2} \times(x - 2kt - \eta_0), \sqrt{\frac{N_1 - iN_2 - \alpha_1}{N_1 + iN_2 - \alpha_1}}\right]\right\}e^{i[kx + D_i t + c_0]} \tag{3.58}$$

and

$$N(x,t) = -\frac{2}{5}D_i + \frac{2}{5}\beta_1 - \frac{8}{5}k^2 - \frac{1}{2}\alpha_1 - N_1 - \frac{1}{10}\beta_2 + \frac{3}{2}\left\{\alpha_1 + (N_1 - iN_2 - \alpha_1)sn^2\left[\frac{\sqrt{N_1 + iN_2 - \alpha_1}}{2} \times(x - 2kt - \eta_0), \sqrt{\frac{N_1 - iN_2 - \alpha_1}{N_1 + iN_2 - \alpha_1}}\right]\right\}. \tag{3.59}$$

Case 2. If $\varepsilon = 0$ and $\theta = 4$, we get $\delta_1 = \delta_2 = 2$. In this case, we have

$$\phi(\xi) = \tau_0 + \tau_1 Y + \tau_2 Y^2,$$
$$(\phi')^2 = \frac{(\tau_1 + 2\tau_2 Y)^2(\xi_4 Y^4 + \xi_3 Y^3 + \xi_2 Y^2 + \xi_1 Y + \xi_0)}{\zeta_0},$$
$$\phi'' = \frac{\tau_1(4\xi_4 Y^3 + 3\xi_3 Y^2 + 2\xi_2 Y + \xi_1)}{2\zeta_0} + \frac{\tau_2}{\zeta_0}(6\xi_4 Y^4 + 5\xi_3 Y^3 + 4\xi_2 Y^2 + 3\xi_1 Y + 3\xi_0) \tag{3.60}$$

and

$$\psi(\xi) = T_0 + T_1 Y + T_2 Y^2,$$
$$\psi'^2(\xi) = \frac{(T_1 + 2T_2 Y)^2(\xi_4 Y^4 + \xi_3 Y^3 + \xi_2 Y^2 + \xi_1 Y + \xi_0)}{\zeta_0},$$
$$\psi''(\xi) = \frac{T_1(4\xi_4 Y^3 + 3\xi_3 Y^2 + 2\xi_2 Y + \xi_1)}{2\zeta_0} + \frac{T_2}{\zeta_0}(6\xi_4 Y^4 + 5\xi_3 Y^3 + 4\xi_2 Y^2 + 3\xi_1 Y + 3\xi_0), \tag{3.61}$$

Substituting Eqs. (3.60), (3.61) into Eqs. (3.7), (3.8) and setting the coefficient Y to be zero, we get a system of algebraic equations which can be solved to obtain the following results:

$$A = -\frac{297}{25}k^4 + k^2\left(-\frac{49}{25}\beta_2 + \frac{6}{25}\beta_1 - \frac{6}{25}c\right) + \frac{1}{25}(3\beta_1^2 - 2\beta_2^2) + \frac{1}{25}\beta_2(\beta_1 - c) + \frac{3}{25}c(c - 2\beta_1),$$
$$T_0 = \frac{\sqrt{2}}{2}\tau_0 - \frac{1}{5}(\beta_1 + \beta_2 - c + 11k^2),\quad T_1 = \frac{\sqrt{2}}{2}\tau_1,\quad T_2 = \frac{\sqrt{2}}{2}\tau_2,$$

$$\xi_0 = \frac{1}{480\tau_2^3}\Big[\zeta_0\Big(-30\sqrt{2}\tau_1^2\tau_2\tau_0 + \frac{5\sqrt{2}}{2}\tau_1^4 + 36k^2\tau_1^2\tau_2 + 6\tau_1^2\beta_2\tau_2 + 36\tau_1^2\beta_1\tau_2 - 36\tau_1^2c_2 - 288k^2\tau_0\tau_2^2 - 288\beta_1\tau_0\tau_2^2 + 288c\tau_0\tau_2^2 + 120\sqrt{2}\tau_0^2\tau_2^2 - 48\tau_0\tau_2^2\beta_2\Big)\Big],$$

$$\xi_1 = \frac{1}{120\tau_2^2}\Big[\tau_1\zeta_0\Big(30\sqrt{2}\tau_2\tau_0 - \frac{5\sqrt{2}}{2}\tau_1^2 - 36k^2\tau_2 - 6\beta_2\tau_2 - 36\beta_1\tau_2 + 36c\tau_2\Big)\Big],$$

$$\xi_2 = \frac{1}{40\tau_2}\Big[\zeta_0\Big(\frac{5\sqrt{2}}{2}\tau_1^2 - 10\sqrt{2}\tau_2\tau_0 - 12k^2\tau_2 - 2\beta_2\tau_2 - 12\beta_1\tau_2 + 12c\tau_2\Big)\Big],$$

$$\xi_3 = \frac{\sqrt{2}}{6}\tau_1\zeta_0, \quad \xi_4 = \frac{\sqrt{2}}{12}\tau_2\zeta_0, \tag{3.62}$$

where ζ_0, τ_2, τ_1 and τ_0 are arbitrary constants. Substituting these results (3.62) into Eqs. (2.5) and (2.9), we have

$$\pm(\xi - \eta_0) = L\int \frac{dY}{\sqrt{Y^4 + \frac{\xi_3}{\xi_4}Y^3 + \frac{\xi_2}{\xi_4}Y^2 + \frac{\xi_1}{\xi_4}Y + \frac{\xi_0}{\xi_4}}}, \tag{3.63}$$

where $L = \sqrt{\frac{\zeta_0}{\xi_4}}$. To integrate Eq. (3.63), we must discuss the following families:

Family 5. If $Y^4 + \frac{\xi_3}{\xi_4}Y^3 + \frac{\xi_2}{\xi_4}Y^2 + \frac{\xi_1}{\xi_4}Y + \frac{\xi_0}{\xi_4}$ can be written in the following form:

$$Y^4 + \frac{2\tau_1}{\tau_2}Y^3 + \frac{3\sqrt{2}}{20\tau_2^2}\Big[\Big(\frac{5\sqrt{2}}{2}\tau_1^2 - 10\sqrt{2}\tau_2\tau_0 - 12k^2\tau_2 - 2\beta_2\tau_2 - 12\beta_1\tau_2 + 12c\tau_2\Big)\Big]Y^2 + \frac{\sqrt{2}}{20\tau_2^3}\Big[\tau_1\Big(30\sqrt{2}\tau_2\tau_0 - \frac{5\sqrt{2}}{2}\tau_1^2 - 36k^2\tau_2 - 6\beta_2\tau_2 - 36\beta_1\tau_2 + 36c\tau_2\Big)\Big]Y + \frac{\sqrt{2}}{80\tau_2^4}\Big[\Big(-30\sqrt{2}\tau_1^2\tau_2\tau_0 + \frac{5\sqrt{2}}{2}\tau_1^4 + 36k^2\tau_1^2\tau_2 + 6\tau_1^2\beta_2\tau_2 + 36\tau_1^2\beta_1\tau_2 - 36\tau_1^2c_2 - 288k^2\tau_0\tau_2^2 - 288\beta_1\tau_0\tau_2^2 + 288c\tau_0\tau_2^2 + 120\sqrt{2}\tau_0^2\tau_2^2 - 48\tau_0\tau_2^2\beta_2\Big)\Big] = (Y-\alpha_1)^4. \tag{3.64}$$

where α_1 is an arbitrary constant. From equating the coefficients of Y in both sides of Eq. (3.64), we get a system of algebraic equations in ζ_0, τ_2, τ_1, τ_0 and c which can be solved by using the Maple software package to get the following results:

$$c = \beta_1 + k^2 + \frac{1}{6}\beta_2, \quad \tau_0 = 6\sqrt{2}\alpha_1^2,$$
$$\tau_1 = -12\sqrt{2}\alpha_1, \quad \tau_2 = 6\sqrt{2}. \tag{3.65}$$

Eqs. (3.65), (3.62) and (3.63) lead to get:

$$A = -\frac{1}{12}\beta_2^2 - 12k^4 - 2\beta_2k^2, \quad T_0 = -2k^2 - \frac{1}{6}\beta_2 + 6\alpha_1^2,$$
$$T_1 = -12\alpha_1, \quad T_2 = 6 \quad \xi_0 = \zeta_0\alpha_1^4, \quad \xi_1 = -4\zeta_0\alpha_1^3,$$
$$\xi_2 = 6\zeta_0\alpha_1^2, \quad \xi_3 = -4\zeta_0\alpha_1, \quad \xi_4 = \zeta_0, \tag{3.66}$$

where ζ_0 is an arbitrary constant and

$$\pm(\eta - \eta_0) = \int \frac{dY}{(Y-\alpha_1)^2} = \frac{-1}{Y-\alpha_1}, \tag{3.67}$$

or

$$Y = \alpha_1 \mp \frac{4}{(x - 2kt - \eta_0)}. \tag{3.68}$$

Substituting (3.68), (3.66) and (3.65) into (3.60) and (3.61), we get the exact solutions of Eqs. (3.7) and (3.8) take the form:

$$\phi(\xi) = \frac{6\sqrt{2}}{(x - 2kt - \eta_0)^2}, \tag{3.69}$$

and

$$\psi(\xi) = -2k^2 - \frac{1}{6}\beta_2 + \frac{6}{(x - 2kt - \eta_0)^2}. \tag{3.70}$$

Hence the exact solutions of nonlinear Schrodinger Boussinesq equations (3.1) have the following form:

$$E(x,t) = \frac{6\sqrt{2}}{(x - 2kt - \eta_0)^2}e^{i[kx + (\beta_1 + k^2 + \frac{1}{6}\beta_2)t + c_0]}, \tag{3.71}$$

$$N(x,t) = -2k^2 - \frac{1}{6}\beta_2 + \frac{6}{(x - 2kt - \eta_0)^2}, \tag{3.72}$$

Note that the solutions (3.71) and (3.72) are the same solutions (3.22) and (3.23) in Family 1.

Family 6. If $Y^4 + \frac{\xi_3}{\xi_4}Y^3 + \frac{\xi_2}{\xi_4}Y^2 + \frac{\xi_1}{\xi_4}Y + \frac{\xi_0}{\xi_4}$ can be written in the following form:

$$Y^4 + \frac{2\tau_1}{\tau_2}Y^3 + \frac{3\sqrt{2}}{20\tau_2^2}\Big[\Big(\frac{5\sqrt{2}}{2}\tau_1^2 - 10\sqrt{2}\tau_2\tau_0 - 12k^2\tau_2 - 2\beta_2\tau_2 - 12\beta_1\tau_2 + 12c\tau_2\Big)\Big]Y^2 + \frac{\sqrt{2}}{20\tau_2^3}\Big[\tau_1\Big(30\sqrt{2}\tau_2\tau_0 - \frac{5\sqrt{2}}{2}\tau_1^2 - 36k^2\tau_2 - 6\beta_2\tau_2 - 36\beta_1\tau_2 + 36c\tau_2\Big)\Big]Y + \frac{\sqrt{2}}{80\tau_2^4}\Big[\Big(-30\sqrt{2}\tau_1^2\tau_2\tau_0 + \frac{5\sqrt{2}}{2}\tau_1^4 + 36k^2\tau_1^2\tau_2 + 6\tau_1^2\beta_2\tau_2 + 36\tau_1^2\beta_1\tau_2 - 36\tau_1^2c_2 - 288k^2\tau_0\tau_2^2 - 288\beta_1\tau_0\tau_2^2 + 288c\tau_0\tau_2^2 + 120\sqrt{2}\tau_0^2\tau_2^2 - 48\tau_0\tau_2^2\beta_2\Big)\Big] = (Y-\alpha_1)^2(Y-\alpha_2)^2 \tag{3.73}$$

where α_1, α_2 are arbitrary constants and $\alpha_1 \neq \alpha_2$. From equating the coefficients of Y in both sides of Eq. (3.73), we get a system of algebraic equations in ζ_0, τ_2, τ_1, τ_0 and c which can be solved by using the Maple software package to get the following results:

$$c = \beta_1 + k^2 + \frac{1}{6}\beta_2 + \frac{5}{6}(\alpha_2 - \alpha_1)^2, \quad \tau_0 = 6\sqrt{2}\alpha_1\alpha_2,$$
$$\tau_1 = -6\sqrt{2}(\alpha_1 + \alpha_2), \quad \tau_2 = 6\sqrt{2}. \tag{3.74}$$

Eqs. (3.74), (3.62) and (3.63) lead to get:

$$A = -\frac{1}{12}\beta_2^2 - 12k^4 - 2\beta_2k^2 + \frac{1}{12}(\alpha_2 - \alpha_1)^4,$$
$$T_0 = -2k^2 - \frac{1}{6}\beta_2 + \frac{1}{6}\alpha_1^2 + \frac{1}{6}\alpha_2^2 + \frac{17}{6}\alpha_2\alpha_1,$$

$$T_1 = -6(\alpha_1+\alpha_2), \quad T_2 = 6, \quad \xi_0 = \zeta_0\alpha_1^2\alpha_2^2,$$
$$\xi_1 = -2\alpha_1\alpha_2(\alpha_1+\alpha_2)\zeta_0, \quad \xi_2 = \zeta_0(\alpha_1^2+4\alpha_1\alpha_2+\alpha_1^2),$$
$$\xi_3 = -2\zeta_0(\alpha_1+\alpha_2), \quad \xi_4 = \zeta_0, \tag{3.75}$$

where ζ_0 is an arbitrary constant and

$$\pm(\xi-\eta_0) = \int \frac{dY}{(Y-\alpha_1)(Y-\alpha_2)} = \frac{1}{\alpha_1-\alpha_2}\ln\left|\frac{Y-\alpha_1}{Y-\alpha_2}\right|, \tag{3.76}$$

or

$$Y = \frac{-\alpha_1+\alpha_2 e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}{-1+e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}. \tag{3.77}$$

Substituting (3.77), (3.75) and (3.74) into (3.60) and (3.61), we get the exact solutions of Eqs. (3.7) and (3.8) take the form:

$$\phi(\xi) = 6\sqrt{2}\alpha_1\alpha_2 - 6\sqrt{2}(\alpha_1+\alpha_2)\left[\frac{-\alpha_1+\alpha_2 e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}{-1+e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}\right] + 6\sqrt{2}\left[\frac{-\alpha_1+\alpha_2 e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}{-1+e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}\right]^2, \tag{3.78}$$

and

$$\psi(\xi) = -2k^2 - \frac{1}{6}\beta_2 + \frac{1}{6}\alpha_1^2 + \frac{1}{6}\alpha_2^2 + \frac{17}{6}\alpha_2\alpha_1 - 6(\alpha_1+\alpha_2) \times\left[\frac{-\alpha_1+\alpha_2 e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}{-1+e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}\right] + 6\left[\frac{-\alpha_1+\alpha_2 e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}{-1+e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}\right]^2. \tag{3.79}$$

Hence the exact solutions of nonlinear Schrodinger Boussinesq equations (3.1) have the following form:

$$E(x,t) = \left\{6\sqrt{2}\alpha_1\alpha_2 - 6\sqrt{2}(\alpha_1+\alpha_2)\left[\frac{-\alpha_1+\alpha_2 e^{\pm(\alpha_1-\alpha_2)(x-2kt-\eta_0)}}{-1+e^{\pm(\alpha_1-\alpha_2)(x-2kt-\eta_0)}}\right] + 6\sqrt{2}\left[\frac{-\alpha_1+\alpha_2 e^{\pm(\alpha_1-\alpha_2)(x-2kt-\eta_0)}}{-1+e^{\pm(\alpha_1-\alpha_2)(x-2kt-\eta_0)}}\right]^2 \times e^{i[kx+(\beta_1+k^2+\frac{1}{6}\beta_2+\frac{5}{6}(\alpha_2-\alpha_1)^2)t+c_0]}\right\}, \tag{3.80}$$

and

$$N(x,t) = -2k^2 - \frac{1}{6}\beta_2 + \frac{1}{6}\alpha_1^2 + \frac{1}{6}\alpha_2^2 + \frac{17}{6}\alpha_2\alpha_1 - 6(\alpha_1+\alpha_2) \times\left[\frac{-\alpha_1+\alpha_2 e^{\pm(\alpha_1-\alpha_2)(x-2kt-\eta_0)}}{-1+e^{\pm(\alpha_1-\alpha_2)(x-2kt-\eta_0)}}\right] + 6\left[\frac{-\alpha_1+\alpha_2 e^{\pm(\alpha_1-\alpha_2)(\xi-\eta_0)}}{-1+e^{\pm(\alpha_1-\alpha_2)(x-2kt-\eta_0)}}\right]^2. \tag{3.81}$$

Family 7. If $Y^4+\frac{\xi_3}{\xi_4}Y^3+\frac{\xi_2}{\xi_4}Y^2+\frac{\xi_1}{\xi_4}Y+\frac{\xi_0}{\xi_4}$ can be written in the following form:

$$Y^4 + \frac{2\tau_1}{\tau_2}Y^3 + \frac{3\sqrt{2}}{20\tau_2^2}\left[\left(\frac{5\sqrt{2}}{2}\tau_1^2 - 10\sqrt{2}\tau_2\tau_0 - 12k^2\tau_2 - 2\beta_2\tau_2 - 12\beta_1\tau_2 + 12c\tau_2\right)\right]Y^2$$
$$+\frac{\sqrt{2}}{20\tau_2^3}\left[\tau_1\left(30\sqrt{2}\tau_2\tau_0 - \frac{5\sqrt{2}}{2}\tau_1^2 - 36k^2\tau_2 - 6\beta_2\tau_2 - 36\beta_1\tau_2 + 36c\tau_2\right)\right]Y$$
$$+\frac{\sqrt{2}}{80\tau_2^4}\left[\left(-30\sqrt{2}\tau_1^2\tau_2\tau_0 + \frac{5\sqrt{2}}{2}\tau_1^4 + 36k^2\tau_1^2\tau_2 + 6\tau_1^2\beta_2\tau_2 + 36\tau_1^2\beta_1\tau_2 - 36\tau_1^2c_2 - 288k^2\tau_0\tau_2^2 - 288\beta_1\tau_0\tau_2^2 + 288c\tau_0\tau_2^2 + 120\sqrt{2}\tau_0^2\tau_2^2 - 48\tau_0\tau_2^2\beta_2\right)\right]$$
$$= (Y-\alpha_1)(Y-\alpha_2)(Y-\alpha_3)(Y-\alpha_4) \tag{3.82}$$

where α_1, α_2, α_3, α_4 are arbitrary different constants. From equating the coefficients of Y in both sides of Eq. (3.82), we get a system of algebraic equations in ζ_0, τ_2, τ_1, τ_0 and c which can be solved by using the Maple software package to get the following results:

$$c = D_i, (i=1,2), \quad \alpha_1 = \alpha_2+\alpha_3-\alpha_4,$$
$$\tau_0 = -\frac{\sqrt{2}}{10}(-20\alpha_3\alpha_4 + 20\alpha_4^2 - 20\alpha_2\alpha_4 - 30\alpha_2\alpha_3 - 5\alpha_2^2 - \beta_2 - 6k^2 - 5\alpha_3^2 + 6D_i - 6\beta_1),$$
$$\tau_1 = -6\sqrt{2}(\alpha_2+\alpha_3), \quad \tau_2 = 6\sqrt{2}, \tag{3.83}$$

where

$$D_i = \beta_1 + k^2 + \frac{1}{6}\beta_2 \pm \frac{5}{6}(\alpha_3^4 - 4\alpha_3^3\alpha_4 + 16\alpha_4^4 - 32\alpha_2\alpha_4^3 + 20\alpha_4^2\alpha_2^2 - 4\alpha_4\alpha_2^3 + \alpha_2^4 - 32\alpha_3\alpha_4^3 + 14\alpha_2^2\alpha_3^2 + 20\alpha_3^2\alpha_4^2 - 28\alpha_2\alpha_4\alpha_3^2 + 56\alpha_2\alpha_3\alpha_4^2 - 28\alpha_4\alpha_3\alpha_2^2)^{1/2} \tag{3.84}$$

Eqs. (3.62), (3.63) and (3.83) lead to get:

$$A = -\frac{1}{12}\beta_2^2 - 12k^4 - 2\beta_2k^2 + \frac{4}{3}\alpha_4^2 - \frac{8}{3}(\alpha_3+\alpha_2)\alpha_4^3 + \frac{5}{3}\alpha_4^2\left(\alpha_3^2+\alpha_2^2+\frac{14}{5}\alpha_3\alpha_2\right) - \frac{1}{3}\alpha_4(\alpha_3^3+\alpha_2^3+7\alpha_3\alpha_2^2+7\alpha_2\alpha_4^2) + \frac{1}{12}(\alpha_3^4+\alpha_2^4+14\alpha_3^2\alpha_2^2),$$
$$T_0 = -\frac{2}{5}D_i - \frac{8}{5}k^2 - \frac{1}{10}\beta_2 + \frac{2}{5}\beta_1 + 2\alpha_3\alpha_4 - 2\alpha_4^2 + 2\alpha_2\alpha_4 + 3\alpha_2\alpha_3 + \frac{1}{2}\alpha_2^2 + \frac{1}{2}\alpha_3^2,$$
$$T_1 = -6(\alpha_2+\alpha_3), \quad T_2 = 6, \quad \xi_0 = \zeta_0\alpha_2\alpha_3\alpha_4(\alpha_2+\alpha_3-\alpha_4),$$
$$\xi_1 = -(\alpha_3\alpha_4 - \alpha_4^2 + \alpha_2\alpha_4 + \alpha_2\alpha_3)(\alpha_2+\alpha_3)\zeta_0,$$
$$\xi_2 = \zeta_0(3\alpha_2\alpha_3 + \alpha_2\alpha_4 + \alpha_3^2 + \alpha_3\alpha_4 - \alpha_4^2 + \alpha_2^2),$$
$$\xi_3 = -2\zeta_0(\alpha_2+\alpha_3), \quad \xi_4 = \zeta_0, \tag{3.85}$$

where ζ_0 is an arbitrary constant and

$$\pm(\xi-\eta_0) = \int \frac{dY}{\sqrt{(Y-(\alpha_2+\alpha_3-\alpha_4))(Y-\alpha_2)(Y-\alpha_3)(Y-\alpha_4)}}$$
$$= -\frac{2}{(\alpha_2-\alpha_4)}EllipticF\left[\sqrt{\frac{(\alpha_2-\alpha_4)(Y-\alpha_4)}{(\alpha_2+\alpha_3-2\alpha_4)(Y-\alpha_3)}}, \sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right] \tag{3.86}$$

or

$$Y=\frac{\alpha_4^2-\alpha_2\alpha_4+(\alpha_2\alpha_3+\alpha_3^2-2\alpha_3\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)}{\alpha_4-\alpha_2+(\alpha_2+\alpha_3-2\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)} \tag{3.87}$$

Substituting (3.83), (3.87) and (3.85) into (3.60) and (3.61), we get the exact solutions of Eqs. (3.7) and (3.8) take the following form:

$$\begin{aligned}\phi(\xi)=&-\frac{\sqrt{2}}{10}(-20\alpha_3\alpha_4+20\alpha_4^2-20\alpha_2\alpha_4-30\alpha_2\alpha_3-5\alpha_2^2-\beta_2-6k^2-5\alpha_3^2+6D_i-6\beta_1)\\&-6\sqrt{2}(\alpha_2+\alpha_3)\left[\frac{\alpha_4^2-\alpha_2\alpha_4+(\alpha_2\alpha_3+\alpha_3^2-2\alpha_3\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)}{\alpha_4-\alpha_2+(\alpha_2+\alpha_3-2\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)}\right]\\&+6\sqrt{2}\left[\frac{\alpha_4^2-\alpha_2\alpha_4+(\alpha_2\alpha_3+\alpha_3^2-2\alpha_3\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)}{\alpha_4-\alpha_2+(\alpha_2+\alpha_3-2\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)}\right]^2\end{aligned} \tag{3.88}$$

and

$$\begin{aligned}\psi(\xi)=&-\frac{2}{5}D_i-\frac{8}{5}k^2-\frac{1}{10}\beta_2+\frac{1}{10}\beta_2+\frac{2}{5}\beta_1+2\alpha_3\alpha_4-2\alpha_4^2+2\alpha_2\alpha_4+3\alpha_2\alpha_3+\frac{1}{2}\alpha_2^2+\frac{1}{2}\alpha_3^2\\&-6(\alpha_2+\alpha_3)\left[\frac{\alpha_4^2-\alpha_2\alpha_4+(\alpha_2\alpha_3+\alpha_3^2-2\alpha_3\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)}{\alpha_4-\alpha_2+(\alpha_2+\alpha_3-2\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)}\right]\\&+6\left[\frac{\alpha_4^2-\alpha_2\alpha_4+(\alpha_2\alpha_3+\alpha_3^2-2\alpha_3\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)}{\alpha_4-\alpha_2+(\alpha_2+\alpha_3-2\alpha_4)sn^2\left(\frac{1}{2}(\alpha_2-\alpha_4)(\xi-\eta_0),\sqrt{\frac{(\alpha_2-\alpha_3)(\alpha_2+\alpha_3-2\alpha_4)}{(\alpha_4-\alpha_2)^2}}\right)}\right]^2\end{aligned} \tag{3.89}$$

Family 8. If $Y^4+\frac{\xi_3}{\xi_4}Y^3+\frac{\xi_2}{\xi_4}Y^2+\frac{\xi_1}{\xi_4}Y+\frac{\xi_0}{\xi_4}$ can be written in the following form:

$$\begin{aligned}&Y^4+\frac{2\tau_1}{\tau_2}Y^3+\frac{3\sqrt{2}}{20\tau_2^2}\left[\left(\frac{5\sqrt{2}}{2}\tau_1^2-10\sqrt{2}\tau_2\tau_0-12k^2\tau_2-2\beta_2\tau_2-12\beta_1\tau_2+12c\tau_2\right)\right]Y^2\\&\quad+\frac{\sqrt{2}}{20\tau_2^3}\left[\tau_1\left(30\sqrt{2}\tau_2\tau_0-\frac{5\sqrt{2}}{2}\tau_1^2-36k^2\tau_2-6\beta_2\tau_2-36\beta_1\tau_2+36c\tau_2\right)\right]Y\\&\quad+\frac{\sqrt{2}}{80\tau_2^4}\left[(-30\sqrt{2}\tau_1^2\tau_2\tau_0+\frac{5\sqrt{2}}{2}\tau_1^4+36k^2\tau_1^2\tau_2+6\tau_1^2\beta_2\tau_2+36\tau_1^2\beta_1\tau_2-36\tau_1^2c_2-288k^2\tau_0\tau_2^2\right.\\&\quad\left.-288\beta_1\tau_0\tau_2^2+288c\tau_0\tau_2^2+120\sqrt{2}\tau_0^2\tau_2^2-48\tau_0\tau_2^2\beta_2)\right]\\&=(Y-(N_1+iN_2))(Y-(N_1-iN_2))(Y-(N_3+iN_4))(Y-(N_3-iN_4)),\end{aligned} \tag{3.90}$$

where N_i, $(i=1,2,3,4)$ are arbitrary constants. From equating the coefficients of Y in both sides of Eq. (3.90), we get a system of algebraic equations in ζ_0, τ_2, τ_1, τ_0 and c which can be solved by using the Maple software package to get the following results:

$$\begin{aligned}&c=D_i,\ (i=1,2),\quad N_1=N_3,\\&\tau_0=\frac{\sqrt{2}}{10}(\beta_2+6\beta_1+6k^2-6D_i+20(3N_3^4+N_4^4+N_2^4)),\\&\tau_1=-12\sqrt{2}N_3,\quad \tau_2=6\sqrt{2},\end{aligned} \tag{3.91}$$

where $D_i=\frac{1}{6}\beta_2+\beta_1+k^2\pm\frac{10}{3}\sqrt{N_2^4+N_4^4-N_2^2N_4^2}$. Eqs. (3.91), (3.62) and (3.63) lead to get:

$$\begin{aligned}&A=-\frac{1}{12}\beta_2^2-12k^4-2\beta_2k^2-\frac{4}{3}N_2^2N_4^2+\frac{4}{3}N_4^4+\frac{4}{3}N_2^4\\&T_0=\frac{2}{5}\beta_1-\frac{2}{5}D_i-\frac{8}{5}k^2-\frac{1}{10}\beta_2+2(3N_3^2+N_4^2+N_2^2),\quad \xi_4=\zeta_0,\\&T_1=-12N_3,\quad T_2=6,\quad \xi_0=\zeta_0(N_3^2N_4^2+N_2^2N_3^2+N_2^2N_4^2+N_3^4),\\&\xi_1=-2(2N_3^2+N_4^2+N_2^2)N_3\zeta_0,\quad \xi_2=(6N_3^2+N_4^2+N_2^2)\zeta_0,\quad \xi_3=-4N_3\zeta_0,\end{aligned} \tag{3.92}$$

where ζ_0 is an arbitrary constant and

$$\pm(\xi-\eta_0)=\int\frac{dY}{\sqrt{(Y^2-2N_3Y+N_3^2+N_2^2)(Y^2-2N_3Y+N_3^2+N_4^2)}}$$
$$=\frac{2}{(N_2-N_4)}ElliplticF\left[\sqrt{\frac{(N_2-N_4)(-Y+N_3+iN_4)}{(N_2+N_4)(-Y+N_3-iN_4)}},\ \frac{(N_2+N_4)}{(N_2-N_4)}\right] \tag{3.93}$$

or

$$Y=\frac{(N_4-N_2)(N_3+iN_4)+(N_4+N_2)(N_3-iN_4)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4)}\right)}{(N_4-N_2)+(N_4+N_2)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4))}\right)} \tag{3.94}$$

Substituting (3.94), (3.92) and (3.91) into (3.60) and (3.61), we get the exact solutions of Eqs. (3.7) and (3.8) have the form:

$$\phi(\xi)=\frac{\sqrt{2}}{10}(\beta_2+6\beta_1+6k^2-6D_i+20(3N_3^4+N_4^4+N_2^4))$$
$$-12\sqrt{2}N_3\left[\frac{(N_4-N_2)(N_3+iN_4)+(N_4+N_2)(N_3-iN_4)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4)}\right)}{(N_4-N_2)+(N_4+N_2)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4)}\right)}\right]$$
$$+6\sqrt{2}\left[\frac{(N_4-N_2)(N_3+iN_4)+(N_4+N_2)(N_3-iN_4)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4)}\right)}{(N_4-N_2)+(N_4+N_2)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4)}\right)}\right]^2 \tag{3.95}$$

and

$$\psi(\xi)=\frac{2}{5}\beta_1-\frac{2}{5}D_i-\frac{8}{5}k^2-\frac{1}{10}\beta_2+2(3N_3^2+N_4^2+N_2^2)$$
$$-12N_3\left[\frac{(N_4-N_2)(N_3+iN_4)+(N_4+N_2)(N_3-iN_4)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4)}\right)}{(N_4-N_2)+(N_4+N_2)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4)}\right)}\right]$$
$$+6\left[\frac{(N_4-N_2)(N_3+iN_4)+(N_4+N_2)(N_3-iN_4)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4)}\right)}{(N_4-N_2)+(N_4+N_2)sn^2\left(\frac{1}{2}(N_2-N_4)(\xi-\eta_0),\ \frac{(N_2+N_4)}{(N_2-N_4)}\right)}\right]^2 \tag{3.96}$$

4. Conclusion

In this paper, we used the extended trial equation method to construct a series of some new analytic exact solutions for some nonlinear partial differential equations in mathematical physics when the balance number is a positive integer. We constructed the exact solutions in many different functions such as hyperbolic function solutions, trigonometric function solutions, Jacobi elliptic functions solutions and rational solutions for the nonlinear coupled nonlinear Schrodinger Boussinesq equations. This method is more powerful than other method for solving the nonlinear partial differential equations. This method can be used to solve many nonlinear partial differential equations in mathematical physics.

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