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ORIGINAL RESEARCH

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Variable demand model for periodically reviewing with allowing refunding parts of the orders

O. M. Hollah*

*Correspondence: eng_Hollah@Yahoo.com; osama.hollah@science.tanta.edu.eg
Higher Institute of Engineering and Technology, Tanta, Egypt

Abstract

Depending on a field study for one of the largest iron and paints warehouses in Egypt, this paper presents a new multi-item periodic review inventory model considering the refunding quantity cost. Through this field study, we found that the inventory level is monitored periodically at equal time intervals. Returning a part of the goods that were previously ordered is permitted. Also, a shortage is permissible to occur despite having orders, and it is a combination of the backorder and lost sales. This model has been applied in both crisp and fuzzy environments since the fuzzy case is more suitable for real-life than crisp. The Lagrange multiplier technique is used for solving the restricted mathematical model. Here, the demand is a random variable that follows the normal distribution with zero lead-time. Finally, the model is followed by a real application to clarify the model and prove its efficiency.

Keywords: Inventory system, Fuzzy environment, Periodic review model, Zero lead-time, Multi-item

Mathematics Subject Classification: 90B05, 90C70, 93A30

Introduction

Analysis of the inventory system has appeared from more than one hundred years ago. Due to the importance of this branch of sciences, it has attracted the researchers' attention. For instance, Iida [1] investigated the non-stationary periodic review production model with uncertain production capacity and uncertain demand. Silver and Robb [2] studied the periodic review inventory model with some thoughts regarding the optimal reorder period. Hollah and Fergany [3] introduced a constrained periodic review inventory model for deteriorating items when the lead-time is zero. Lin and Lin [4] presented the periodic review integrated model with the optimal ordering and recovery policy. Fergany [5] studied a multi-item continuous review model with varying mixture shortage cost under restrictions. Jaggi et al. [6] introduced the periodic review inventory model with controllable lead-time when the backorder rate depends on a protection interval. Thomas et al. [7] studied a periodic review inventory policy with the lost sales and zero lead-time. Fergany [8] introduced a periodic review inventory model with zero lead-time and varying ordering cost.

The cost parameters in real inventory systems and other parameters are uncertain in nature, such as prices, marketing, production, and inventory. In recent years, many researchers have contributed many articles by applying the fuzzy sets theory as a mathematical way to deal with these uncertainties. For example, Dey and Chakraborty [9] developed a fuzzy random periodic review model with variable lead-time. Rong et al. [10] studied a multi-objective inventory model with controllable lead-time and triangular fuzzy numbers. Jauhari et al. [11] introduced a fuzzy periodic review model involving stochastic demand. Xiaobin [12] developed a continuous review inventory model with variable lead time in a fuzzy environment. Sadjadi et al. [13] studied the fuzzy pricing and marketing planning model using a geometric programming approach. Biswajit and Amalendu [14] introduced the periodic review inventory model with variable lead time and fuzzy demand. Priyan and Uthayakumar [15] studied a multi-echelon inventory model under a service level constraint in a fuzzy cost environment. Khurdi et al. [16] introduced a fuzzy collaborative supply chain model for imperfect items and a service level constraint.

Based on a realistic study for one of the biggest irons and paints warehouses in Egypt, some adjustments have been made to the multi-item periodic review inventory model. In this model, the inventory level is reviewed periodically at equal time cycles. The warehouse allows the customers to return part of the goods they previously ordered; therefore, an extra cost is paid and added to the expected total cost (the refunding quantity cost). Shortage can occur despite having orders, and then a part of these orders is fulfilled in the next cycle at the same price at the request time (backorder), while the other part is lost forever and a penalty clause is paid. There is a constraint on the expected varying lost sales cost, for if this cost exceeds a certain limit, it may lead to loss or increase the expected total cost. The constrained problem is solved by using the Lagrange multiplier technique. The demand is a random variable that follows the normal distribution with zero lead-time. This model has been applied in both crisp and fuzzy environments since the fuzzy environment is closer to real-life than crisp. The main goal is to find the minimum expected annual total cost by finding the optimal maximum inventory level and the optimal time between reviews. The results in this paper have been derived by

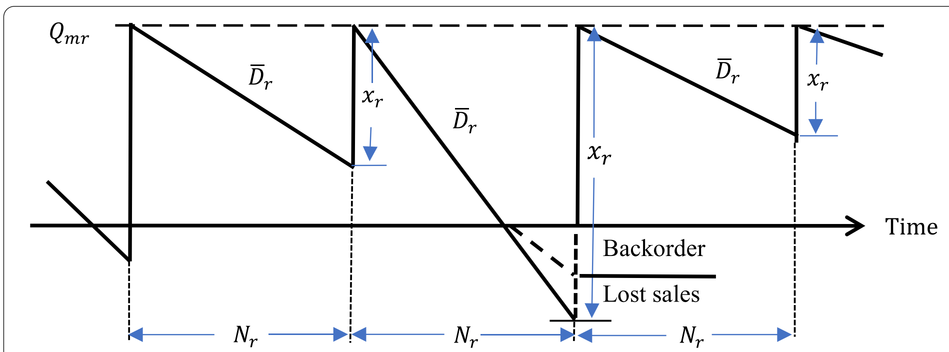

Fig. 1 Probabilistic periodic review inventory model

Mathematica program V. 12.0. Figure 1 shows the multi-item periodic review model with zero lead-time.

The following assumptions are made for developing the mathematical model

- The warehouse allows refunding a part of the goods that were previously ordered.
- The demand is a random variable, and the replenishment is instantaneous (lead-time is zero).
- The stock level decreases at a uniform rate over the cycle.
- $f(x_r)$ is the density function for the demand x_r.
- A fraction of unsatisfied demand that will be backorder is γ_r, while the remaining fraction $(1-\gamma_r)$ is completely lost.

The mathematical model for crisp environment

- The expected order cost for the cycle is given by

$$E(\mathrm{OC}) = \sum_{r=1}^{n} C_{or} \tag{1}$$

- The expected varying holding cost for the cycle is given by

$$E(\mathrm{HC}(N_r)) = \sum_{r=1}^{n} C_{hr}(N_r)\bar{I}_r = \sum_{r=1}^{n} C_{hr}N_r^{1-\beta}\left(Q_{mr} - \frac{\overline{D}_r N_r}{2} + (1-\gamma_r)\int_{Q_{mr}}^{\infty} (x_r - Q_{mr})f(x_r)\mathrm{d}x_r\right) \tag{2}$$

where the expected average amount in inventory is given by

$$\bar{I}_r = N_r\left(Q_{mr} - \frac{\overline{D}_r N_r}{2} + (1-\gamma_r)\int_{Q_{mr}}^{\infty} (x_r - Q_{mr})f(x_r)dx_r\right).$$

- The expected varying backorder cost for the cycle is given by

$$E(\mathrm{BC}(N_r)) = \sum_{r=1}^{n} C_{br}\ \gamma_r\ N_r^{\beta}\overline{S}(Q_{mr}) = \sum_{r=1}^{n} C_{br}\ \gamma_r\ N_r^{\beta}\int_{Q_{mr}}^{\infty} (x_r - Q_{mr})\, f(x_r)\ \mathrm{d}x_r \tag{3}$$

where $\overline{S}(Q_{mr})$ represents the expected shortage quantity.

- The expected varying lost sales cost for the cycle is given by

$$E(\mathrm{LC}) = \sum_{r=1}^{n} C_{Lr}(1-\gamma_r)N_r^{\beta}\overline{S}(Q_{mr}) = \sum_{r=1}^{n} C_{Lr}(1-\gamma_r)N_r^{\beta}\int_{Q_{mr}}^{\infty} (x_r - Q_{mr})f(x_r)\mathrm{d}x_r \tag{4}$$

And the expected varying refunding quantity cost for the cycle is given by

$$\begin{aligned} E(\mathrm{RQC}) &= \sum_{r=1}^{n} C_{\mathrm{RQ}r}\ N_r^{-\beta} \int_{0}^{Q_{mr}} x_r f(x_r)\ \mathrm{d}x_r \\ &= \sum_{r=1}^{n} C_{hr}\ \rho_r\ N_r^{-\beta} \int_{0}^{Q_{mr}} x_r f(x_r)\ \mathrm{d}x_r, \quad 0 < \rho_r < 1 \end{aligned} \tag{5}$$

The expected annual total cost will be the sum of the expected order cost, the expected varying holding cost, the expected varying backorder cost, the expected varying lost sales cost, and the expected varying refunding quantity cost

$$E(\mathrm{TC}(Q_{mr}, N_r)) = \sum_{r=1}^{n} \left[E(\mathrm{OC}_r) + E(\mathrm{HC}_r(N_r)) + E(\mathrm{BC}_r(N_r)) + E(\mathrm{LC}_r(N_r)) + E\big(\mathrm{RQC}_r(N_r)\big)\right]$$

Then from Eqs. (1), (2), (3), (4) and (5), the expected annual total cost is given by

$$\begin{aligned} E(\mathrm{TC}(Q_{mr},N_r)) = \sum_{r=1}^{n} \Bigg[& C_{or} + C_{hr} N_r^{1-\beta}\left(Q_{mr} - \frac{\overline{D}_r N_r}{2}\right) + C_{hr}\rho_r N_r^{-\beta} \int_{0}^{Q_{mr}} x_r f(x_r)\mathrm{d}x_r \\ & + \left(C_{br}\gamma_r N_r^{\beta} + (1-\gamma_r)\left(C_{Lr}N_r^{\beta} + C_{hr}N_r^{1-\beta}\right)\right) \int_{Q_{mr}}^{\infty} (x_r - Q_{mr}) f(x_r)\mathrm{d}x_r \end{aligned} \tag{6}$$

Note: Obviously, the expected order cost $\left(\sum_{r=1}^{n} C_{or}\right)$ is fixed, so it can be temporarily neglected in calculating the minimum expected annual total cost and eventually added to it.

Now, the main objective is to determine the optimal values Q_{mr}^{*} and N_r^{*} that minimize the expected annual total cost minE(TC). This paper puts a constraint on varying lost sales cost. The Karush–Kuhn–Tucker (KKT) conditions (Kuhn and Tucker [17]) are first-order necessary conditions for a solution of nonlinear programming to be optimal if some regularity conditions are satisfied. The Lagrange multiplier method is suitable to solve this constraint problem.

Consider a limitation on the expected varying lost sales cost, i.e.

$$\sum_{r=1}^{n} C_{Lr}(1-\gamma_r) N_r^{\beta} \int_{Q_{mr}}^{\infty} (x_r - Q_{mr}) f(x_r)\ \mathrm{d}x_r \le K_{Lr} \tag{7}$$

To solve this primal function which is a convex programming problem, Eqs. (6) and (7) can be written in the following form

$$\begin{aligned} \min E(\mathrm{TC}(Q_{mr},N_r)) = \sum_{r=1}^{n} \Bigg[& C_{hr} N_r^{1-\beta}\left(Q_{mr} - \frac{\overline{D}_r N_r}{2}\right) + C_{hr}\rho_r N_r^{-\beta} \int_{0}^{Q_{mr}} x_r f(x_r)\ \mathrm{d}x_r \\ & + \left(C_{br}\gamma_r N_r^{\beta} + (1-\gamma_r)\left(C_{Lr}N_r^{\beta} + C_{hr}N_r^{1-\beta}\right)\right) \int_{Q_{mr}}^{\infty} (x_r - Q_{mr}) f(x_r)\ \mathrm{d}x_r \Bigg] \end{aligned} \tag{8}$$

Subject to:

$$\sum_{r=1}^{n} C_{Lr}(1-\gamma_r) N_r^{\beta} \int_{Q_{mr}}^{\infty} (x_r - Q_{mr}) f(x_r)\ \mathrm{d}x_r \le K_{Lr} \tag{9}$$

To find optimal values Q^*_{mr} and N^*_r which minimize Eq. (8) under the constraint (Eq. (9)), the Lagrange multipliers function with the Kuhn-Tucker conditions is given by

$$\begin{aligned} L(Q_{mr},N_r,\lambda_{Lr}) = \sum_{r=1}^{n} \Bigg[& C_{hr}N_r^{1-\beta}\left(Q_{mr}-\frac{\overline{D}_rN_r}{2}\right)+C_{hr}\rho_rN_r^{-\beta}\int_0^{Q_{mr}} x_rf(x_r)\,\mathrm{d}x_r \\ & +\left(C_{br}\gamma_rN_r^{\beta}+(1-\gamma_r)\left(C_{Lr}N_r^{\beta}+C_{hr}N_r^{1-\beta}\right)\right)\int_{Q_{mr}}^{\infty}(x_r-Q_{mr})f(x_r)\,\mathrm{d}x_r \\ & +\lambda_{Lr}\left(C_{Lr}(1-\gamma_r)N_r^{\beta}\int_{Q_{mr}}^{\infty}(x_r-Q_{mr})f(x_r)\,\mathrm{d}x_r-K_{Lr}\right)\Bigg] \end{aligned} \tag{10}$$

where λ_{Lr} is a Lagrange multiplier.

The optimal values Q^*_{mr} and N^*_r can be calculated by setting the corresponding first partial derivatives of Eq. (10) equal to zero. Then we obtain:

$$\frac{\partial L(Q_{mr},N_r)}{\partial Q_{mr}}=0,\ \frac{\partial L(Q_{mr},N_r)}{\partial N_r}=0,\ \frac{\partial L(Q_{mr},N)}{\partial \lambda_{Lr}}=0$$

$$\int_{Q^*_{mr}}^{\infty} f(x_r)\mathrm{d}x_r = \frac{C_{hr}N_r^{*-\beta}\left(N_r^*+\rho_rQ^*_{mr}f\left(Q^*_{mr}\right)\right)}{\left(C_{br}\gamma_rN_r^{*\beta}+(1-\gamma_r)\left(C_{Lr}(1+\lambda_{Lr})N_r^{*\beta}+C_{hr}N_r^{*1-\beta}\right)\right)}, \tag{11}$$

$$\int_{Q^*_{mr}}^{\infty}(x_r-Q^*_{mr})f(x_r)\mathrm{d}x_r = \frac{C_{hr}\beta\rho_rN_r^{*-(1+\beta)}\int_0^{Q^*_{mr}}x_rf(x_r)\mathrm{d}x_r+\frac{1}{2}C_{hr}\overline{D}_rN_r^{*1-\beta}-C_{hr}(1-\beta)N_r^{*-\beta}\left(Q^*_{mr}-\frac{\overline{D}_rN_r^*}{2}\right)}{C_{br}\gamma_r\beta N_r^{*\beta-1}+C_{Lr}\beta(1-\gamma_r)(1+\lambda_{Lr})N_r^{*\beta-1}+C_{hr}(1-\gamma_r)(1-\beta)N_r^{*-\beta}} \tag{12}$$

and

$$C_{Lr}(1-\gamma_r)N_r^{*\beta}\int_{Q^*_{mr}}^{\infty}(x_r-Q^*_{mr})f(x_r)\,\mathrm{d}x_r=K_{Lr} \tag{13}$$

It can be determined the minimum expected annual total cost (min $E(\mathrm{TC})$), after finding the optimal values Q^*_{mr} and N^*_r, substituting these in Eq. (8), then adding the fixed value $\left(\sum_{r=1}^{n}C_{or}\right)$.

The mathematical model for fuzzy environment

The inventory cost coefficients and other coefficients are fuzzy in nature. Therefore, the decision variables and the objective function should be fuzzy as well. This model is resolved when the cost parameters are triangular fuzzy numbers (TFN), and the right and left shape functions of the objective function and its decision variables should be found by finding the upper and the lower bound of the optimal objective function, i.e., $\tilde{L}^U(\propto)$ and $\tilde{L}^V(\propto)$ (the left and right $\propto$ cuts of $\tilde{L}(\propto)$). For example, the approximated value of TFN of $\tilde{C}_{or}$, is observed in Fig. 2.

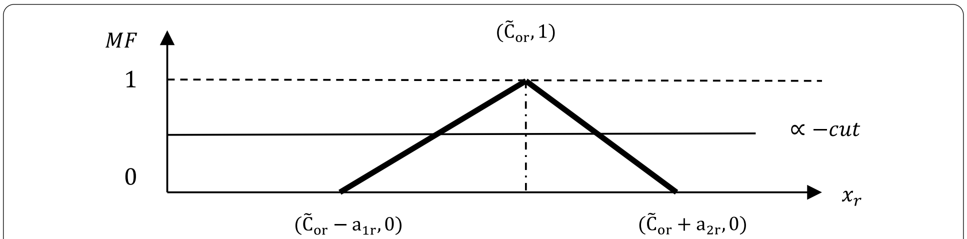

Fig. 2 Order cost as a triangular fuzzy number

Consider the model when all parameters are triangular fuzzy numbers (TFN) as given below

$$C_{or} = (C_{or} - a_{1r}, C_{or}, C_{or} + a_{2r}), \quad C_{hr} = (C_{hr} - a_{3r}, C_{hr}, C_{hr} + a_{4r}),$$
$$C_{br} = (C_{br} - a_{5r}, C_{br}, C_{br} + a_{6r}), \quad C_{Lr} = (C_{Lr} - a_{7r}, C_{Lr}, C_{Lr} + a_{8r})$$
$$\text{and} \quad \overline{D}_r = \left(\overline{D}_r - a_{9r}, \overline{D}_r, \overline{D}_r + a_{10r}\right).$$

where $a_{ir}, i = 1,2,\ldots,10$ are arbitrary positive numbers under the following restrictions:

$$0 \le a_{1r} \le C_{or}, \quad a_{2r} \ge 0, \qquad 0 \le a_{3r} \le C_{hr}, \quad a_{4r} \ge 0,$$
$$0 \le a_{5r} \le C_{br}, \quad a_{6r} \ge 0, \qquad 0 \le a_{7r} \le C_{Lr}, \quad a_{8r} \ge 0$$
$$\text{and} \quad 0 \le a_{9r} \le \overline{D}_r, \quad a_{10r} \ge 0$$

The left and right limits of $\propto$ cuts of $C_{or}, C_{hr}, C_{br}, C_{Lr}$ and $\overline{D}_r$ are given by

$$\tilde{C}_{or_U}(\propto) = C_{or} - (1 - \propto)a_{1r},$$
$$\tilde{C}_{or_V}(\propto) = C_{or} + (1 - \propto)a_{2r},$$
$$\tilde{C}_{hr_U}(\propto) = C_{hr} - (1 - \propto)a_{3r},$$
$$\tilde{C}_{hr_V}(\propto) = C_{hr} + (1 - \propto)a_{4r},$$
$$\tilde{C}_{br_U}(\propto) = C_{br} - (1 - \propto)a_{5r},$$
$$\tilde{C}_{br_V}(\propto) = C_{br} + (1 - \propto)a_{6r},$$
$$\tilde{C}_{Lr_U}(\propto) = C_{Lr} - (1 - \propto)a_{7r},$$
$$\tilde{C}_{Lr_V}(\propto) = C_{Lr} + (1 - \propto)a_{8r},$$

$$\text{and} \quad \tilde{\overline{D}}_{r_U}(\propto) = \overline{D}_r - (1 - \propto)a_{9r}, \quad \tilde{\overline{D}}_{r_V}(\propto) = \overline{D}_r + (1 - \propto)a_{10r}$$

Using the signed distance method, we have

$$\tilde{C}_{or} = \frac{1}{2}\int_0^1 \left(\tilde{C}_{or_U}(\propto) + \tilde{C}_{or_V}(\propto)\right)d\alpha \quad \tilde{C}_{hr} = \frac{1}{2}\int_0^1 \left(\tilde{C}_{hr_U}(\propto) + \tilde{C}_{hr_V}(\propto)\right)d\alpha$$
$$\tilde{C}_{br} = \frac{1}{2}\int_0^1 \left(\tilde{C}_{br_U}(\propto) + \tilde{C}_{br_V}(\propto)\right)d\alpha \quad \tilde{C}_{Lr} = \frac{1}{2}\int_0^1 \left(\tilde{C}_{Lr_U}(\propto) + \tilde{C}_{Lr_V}(\propto)\right)d\alpha$$
$$\text{and} \quad \tilde{\overline{D}}_r = \frac{1}{2}\int_0^1 \left(\tilde{D}_{r_U}(\propto) + \tilde{D}_{r_V}(\propto)\right)d\alpha$$

$$\text{hence}\quad \tilde{C}_{or} = C_{or} + \frac{1}{4}(a_{2r} - a_{1r}),\quad \tilde{C}_{hr} = C_{hr} + \frac{1}{4}(a_{4r} - a_{3r})$$
$$\tilde{C}_{br} = C_{br} + \frac{1}{4}(a_{6r} - a_{5r}),\quad \tilde{C}_{Lr} = C_{Lr} + \frac{1}{4}(a_{8r} - a_{7r})$$
$$\text{and}\quad \tilde{\overline{D}}_r = \overline{D}_r + \frac{1}{4}(a_{10r} - a_{9r})$$

The optimal values Q^*_{mr} and N^*_r for the fuzzy case can be determined as in crisp case except replacing the crisp costs C_{or},C_{hr},C_{br}, C_{Lr} and $\overline{D}_r$ by fuzzy costs $\tilde{C}_{or}$,$\tilde{C}_{hr}$,$\tilde{C}_{br}$, $\tilde{C}_{Lr}$ and $\tilde{\overline{D}}_r$.

The demand follows normal distribution

When the mean μ and the standard deviation σ, i.e.

$$f(x_r) = N(x_r;\, \mu_r, \sigma_r) = \frac{1}{\sigma_r\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x_r - \mu_r}{\sigma_r}\right)^2\right], -\infty < x_r < +\infty, -\infty < \mu_r < +\infty\,, \sigma_r > 0$$

μ_r, continuous location parameter, σ_r, continuous scale parameter $\sigma_r > 0$ where $E(x) = \mu_r,\ V(x) = \sigma_r^2,\ f(Q_{mr}) = \emptyset\left(\frac{Q_{mr}-\mu_r}{\sigma_r}\right)$, $\phi(z) = N(z; 0, 1)$ is the probability density function of the standard normal distribution, and $\Phi(z)$ is the cumulative distribution function of the standard normal distribution.

But

$$\int_{Q^*_{mr}}^{\infty} \phi\left(\frac{Q^*_{mr} - \mu_r}{\sigma_r}\right) dx_r = \int_0^{\infty} \phi\left(\frac{Q^*_{mr} - \mu_r}{\sigma_r}\right) dx_r - \int_0^{Q^*_{mr}} \phi\left(\frac{Q^*_{mr} - \mu_r}{\sigma_r}\right) dx_r$$

The optimal values Q^*_{mr} and N^*_r for crisp case can be calculated as follows:

$$1 - \Phi\left(\frac{Q^*_{mr} - \mu_r}{\sigma_r}\right) = \frac{C_{hr}N_r^{*-\beta}\left(N^*_r + \rho_r Q^*_{mr} f\left(Q^*_{mr}\right)\right)}{\left(C_{br}\gamma_r N_r^{*\beta} + (1-\gamma_r)\left(C_{Lr}(1+\lambda_{Lr})N_r^{*\beta} + C_{hr}N_r^{*1-\beta}\right)\right)}$$

and the expected number of shortages incurred per cycle is the solution of the following equation

$$\overline{S}(Q^*_{mr}) = \sigma_r\phi\left(\frac{Q^*_{mr} - \mu_r}{\sigma_r}\right) + \left(\mu_r - Q^*_{mr}\right)\left(1 - \Phi\left(\frac{Q^*_{mr} - \mu_r}{\sigma_r}\right)\right)$$
$$= \frac{C_{hr}\beta\rho_r N_r^{*-(1+\beta)} \int_0^{Q^*_{mr}} x_r f(x_r) dx_r + \frac{1}{2}C_{hr}\overline{D}_r N_r^{*1-\beta} - C_{hr}(1-\beta)N_r^{*-\beta}\left(Q^*_{mr} - \frac{\overline{D}_r N^*_r}{2}\right)}{C_{br}\gamma_r\beta N_r^{*\beta-1} + C_{Lr}\beta(1-\gamma_r)(1+\lambda_{Lr})N_r^{*\beta-1} + C_{hr}(1-\gamma_r)(1-\beta)N_r^{*-\beta}}$$

The decision variables and minimum expected annual total cost for a fuzzy case can be determined by the same way except replacing the crisp costs by fuzzy costs.

Application

A large store for iron and paints that sells its products wholesale follows a policy of reviewing all items periodically. Three items were selected (Tanner I, Lacquered II and, Plastic III). This store allows refunding a part of the goods previously ordered. The

Table 1 The crisp values of the cost parameters

Parameters	C_{or}	C_{br}	C_{Lr}	C_{hr}
Item I	47.78	3.3	4.12	0.824
Item II	134.66	15.48	17.8	2.32
Item III	83.52	9.6	11.04	1.44

Table 2 The fuzzy values of the cost parameters

Parameters	$\tilde{C}_{or}$	$\tilde{C}_{br}$	$\tilde{C}_{Lr}$	$\tilde{C}_{hr}$
Item I	(45.28, 47.8, 50.78)	(3, 3.3, 3.5)	(6.741, 4.12, 7.541)	(0.724, 0.824, 0.874)
Item II	(130.66, 134.66, 137.16)	(14.98, 15.48, 15.98)	(19.124, 17.8, 20.924)	(2.02, 2.32, 2.72)
Item III	(80.02, 83.52, 87.52)	(9.3, 9.6, 9.8)	(11.78, 11.04, 13.08)	(1.39, 1.44, 1.54)

Table 3 The crisp and fuzzy values of the average demand

Parameters	$\overline{D}_r$	$\tilde{D}_r$
Item I	5245.861	(5225.86, 5245.861, 5260.86)
Item II	7868.056	(7833.06, 7868.056, 7908.06)
Item III	10,491.64	(10,431.6, 10,491.64, 10,551.6)

Table 4 The maximum cost allowed (the limitations) for lost sales and its fraction

Parameters	K_{Lr}	γ_r	$(1-\gamma_r)$
Item I	685	0.65	0.35
Item II	2490	0.70	0.30
Item III	2900	0.60	0.40

Table 5 The parameters values for three items

Parameter	Item I	Item II	Item III
σ_r	773.591	1160.434	1547.257
μ_r	5245.86	7868.06	10,491.64
ρ_r	0.21	0.19	0.2

parameters with 36 samples indicate to the demand during the period 2016–2018 are estimated in Table 8 when $\alpha = 0.05$ (see the "Appendix"). However, for some unexpected reasons in some cycles, the store faces shortage and pays at least 7% for backorder and 6% for a lost sale. Table 4 shows the allowable cost of lost sales K_{Lr}. The store manager wishes to establish the optimal values Q_{mr}^* and N_r^* that achieve the minimum expected

Table 6 The results of crisp and fuzzy values for the normal distribution

β	**Item**	Q_{mr}	N_r	$E(\mathrm{TC}_r)$	$C_{or} + (E(\mathrm{TC}_r)/Q_{mr})$
0.1	Item I	4847.67	0.875517	3533.53	48.51
	Item II	7269.02	0.875276	16,155	136.88
	Item III	9657	0.872035	13,765.9	84.95
0.3	Item I	4824.91	0.757536	3603.84	48.53
	Item II	7234.77	0.757303	16,448.2	136.93
	Item III	9608.93	0.754288	14,004	84.98
0.5	Item I	4765.65	0.605767	3747.14	48.57
	Item II	7145.63	0.60554	17,039.1	137.05
	Item III	9484.28	0.60272	14,481.2	85.05
0.7	Item I	4599.64	0.404855	4000.75	48.65
	Item II	6895.85	0.404627	18,074.4	137.28
	Item III	9134.18	0.401906	15,314.1	85.20
0.9	Item I	3316.37	0.115159	3807.9	48.93
	Item II	4952.58	0.114599	17,181	138.13
	Item III	6089.13	0.105624	15,151.7	86.01
β	Item	Q_{mr}	N_r	$E(\widetilde{\mathrm{TC}}_r)$	$\tilde{C}_{or} + E((\widetilde{\mathrm{TC}}_r)/\tilde{Q}_{mr})$
0.1	Item I	4847.68	0.875727	3495.53	48.63
	Item II	7267.81	0.874992	16,261.3	136.52
	Item III	9655.66	0.871915	13,827.2	85.08
0.3	Item I	4824.94	0.75772	3564.71	48.64
	Item II	7233.48	0.757047	16,558.5	136.57
	Item III	9607.5	0.754176	14,067.4	85.11
0.5	Item I	4765.7	0.605917	3705.81	48.68
	Item II	7144.12	0.605315	17,156.4	136.69
	Item III	9482.64	0.602615	14,548.6	85.18
0.7	Item I	4599.73	0.40496	3955.57	48.77
	Item II	6893.73	0.404439	18,202.5	136.93
	Item III	9131.89	0.401805	15,387.9	85.33
0.9	Item I	3317.17	0.115214	3766.01	49.04
	Item II	4937.53	0.114233	17,299.8	137.79
	Item III	6064.97	0.105205	15,220.4	86.15

Table 7 The minimum expected annual total cost of crisp and fuzzy values

Distributions	β	Q_{mr}^*	N_r^*	Min $E(\mathrm{TC}_r)$	Min $E(\mathrm{TC})$
Normal distribution	0.1	4847.67	0.875517	48.51	270.34
		7269.02	0.875276	136.88	
		9657	0.872035	84.95	
		Q_{mr}^*	N_r^*	Min $E\left(\widetilde{\mathrm{TC}}_r\right)$	Min $E\left(\widetilde{\mathrm{TC}}\right)$
		4847.68	0.875727	48.63	
		7267.81	0.874992	136.52	270.23
		9655.66	0.871915	85.08	

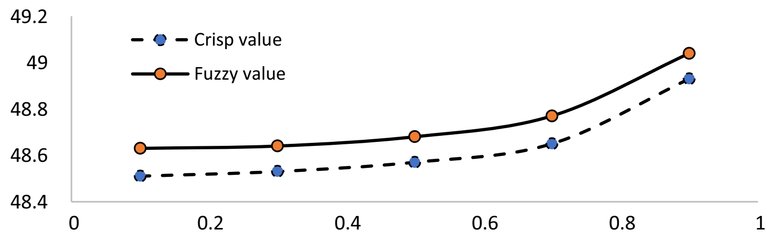

Fig. 3 The optimal crisp and fuzzy values of $E(\mathrm{TC}_1)$ against β for item I

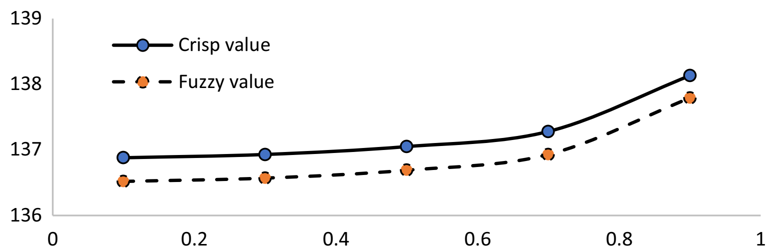

Fig. 4 The optimal crisp and fuzzy values of $E(\mathrm{TC}_2)$ against β for item II

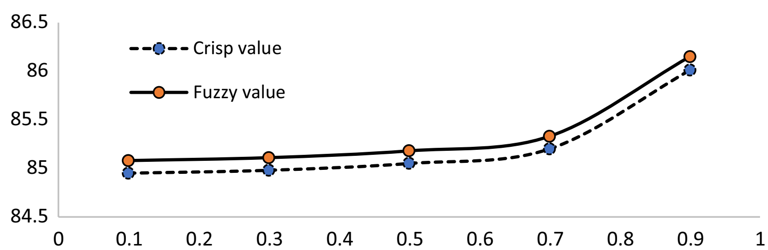

Fig. 5 The optimal crisp and fuzzy values of $E(\mathrm{TC}_3)$ against β for item III

annual total cost for different values of $\beta \in (0, 1)$ when the demand follows the normal distribution $-\infty < x_r < +\infty$.

Results and discussion

Tables 1 and 2 show the crisp and fuzzy values of the cost parameters. Table 3 represents the crisp and fuzzy values of the average demand. Table 4 shows the maximum cost allowed (the limitations) for lost sales and its fraction. Table 5 represents parameter values for normal distribution. In Table 6, the results of crisp and fuzzy values for the normal distribution are calculated. Table 7 presents the optimal values of Q^*_{mr}, N^*_r, and the minimum expected annual total cost for crisp and fuzzy values. Table 8 shows the demand during the period 2016–2018 for 36 samples. By using the SPSS program, Table 9 shows One-Sample Kolmogorov–Smirnov Test.

The optimal values Q^*_{mr}, N^*_r, and the minimum expected annual total cost min $E(\text{TC})$ for three items are deduced in Table 7. The results are calculated for the crisp and fuzzy environment. Figures 3, 4 and 5 are displayed to illustrate the crisp and fuzzy values of the expected annual total cost for the three items against the different values β.

Conclusion

By conducting a realistic study of a large warehouse for iron and paints in Egypt, this paper introduced a new multi-item inventory model where the warehouse allows the customers to return a part of the orders they previously ordered. The inventory level of the warehouse is monitored periodically at equal time cycles. Shortages can occur despite having orders, which are a mixture of backorder and lost sales. The demand is a random variable that follows the normal distribution with zero lead-time. It can be concluded that there is a restriction on the expected varying lost sales cost, for if this cost exceeds a certain limit, it may lead to loss or increase the expected total cost. After solving the model in a crisp environment, it resolved in a fuzzy sense, where the fuzzy environment is more suitable for real-life than crisp. Increasing the value of the varying β leads to a loss or an increase in the expected annual total cost. The minimum expected annual total cost is achieved at the minimum value of β ($\beta = 0.1$).

Appendix

See Tables 8 and 9.

Table 8 The demand during the period 2016–2018 for 36 samples

Month	D1			D2			D3		
	2016	**2017**	**2018**	**2016**	**2017**	**2018**	**2016**	**2017**	**2018**
Jen	4320	5198	5670	6480	7769	8505	8640	10,395	11,340
Feb	3780	5040	5280	5670	7560	7920	7560	10,080	10,560
March	4725	5670	6480	7088	8505	9720	9450	11,340	12,960
April	4550	4725	6318	6825	7088	9477	9100	9450	12,636
May	3640	4550	5200	5460	6825	7800	7280	9100	10,400
Jun	4725	4253	6156	7088	6379	9234	9450	8505	12,312
July	4253	5280	5760	6379	7920	8640	8505	10,560	11,520
Aug	4680	5460	6480	7020	8190	9720	9360	10,920	12,960
Sep	5460	5670	6318	8190	8505	9477	10,920	11,340	12,636
Oct	4725	5200	6480	7088	7800	9720	9450	10,400	12,960
Nov	4680	5460	6240	7020	8190	9360	9360	10,920	12,480
Dec	4725	5760	5940	7088	8640	8910	9450	11,520	11,880

Each demand unit contains a carton of 6 kg

Table 9 One-sample Kolmogorov–Smirnov test of the demands

		D1	D2	D3
N		36	36	36
Normal parameters[a,b]	Mean	5245.86	7868.056	10491.64
	SD	773.591	1160.434	1547.257
Most extreme differences	Absolute	.139	.138	.138
	Positive	.139	.138	.138
	Negative	−.075	−.075	−.075
Test statistic		.139	.138	.138
Asymp. sig. (2-tailed)		.078[c]	.080[c]	.078[c]

[a] Test distribution is normal

[b] Calculated from data

[c] Lilliefors significance correction

Abbreviations

Q_{mr}: The maximum inventory level of the rth item per period (decision variable); N_r: The time between reviews for the rth item (decision variable); x_r: The demand for the rth item during the period N_r(random variable); $\overline{D}_r$: The expected average demand for the rth item during the period; C_{or}: The order cost of the rth item per period; $C_{hr}(N)$: The varying holding cost of the rth item per period $= C_{hr}N_r^{-\beta}$; $C_{br}(N)$: The varying backorder cost of the rth item $= C_{br}N^{\beta}$; $C_{Lr}(N)$: The varying lost sales cost of the rth item $= C_{Lr}N^{\beta}$; $C_{\text{RQ}r}(N)$: The varying refunding quantity cost of the rth item per period $= C_{\text{RQ}r}N_r^{-\beta}$; Q^*_{mr}: The optimal maximum inventory level of the rth item per period; N^*_r: The optimal time between reviews for the rth item (the period); $E(\text{TC}_r)$: The expected annual total cost function of the rth item; $E(\text{TC})$: The expected annual total cost function $E(\text{TC}(Q_{mr}, N_r))$ of the whole items; $\min E(\text{TC})_r$: The minimum expected annual total cost function of the rth item; $\min E(\text{TC})$: The minimum expected annual total cost function of the whole items; $\tilde{C}_{or}$: The fuzzy order cost of the rth item per period; $\tilde{C}_{hr}(N)$: The fuzzy varying holding cost of the rth item per period $= \tilde{C}_{hr}N_r^{-\beta}$; $\tilde{C}_{br}(N)$: The fuzzy varying backorder cost $= \tilde{C}_{br}N^{\beta}$; $\tilde{C}_{Lr}(N)$: The fuzzy varying lost sales cost $= \tilde{C}_{Lr}N^{\beta}$; $\tilde{C}_{\text{RQ}r}(N)$: The fuzzy varying refunding quantity cost of the rth item per period $= \tilde{C}_{\text{RQ}r}N^{-\beta}$; $\tilde{\overline{D}}_r$: The fuzzy expected average demand for the rth during the period; $\min E\left(\widetilde{\text{TC}}_r\right)$: The fuzzy minimum expected annual total cost function of the rth item; $\min E\left(\widetilde{\text{TC}}\right)$: The fuzzy minimum expected annual total cost function of the whole items; k_{Lr}: The goal associated with the available lost sales of the rth item; β: A constant real number selected to provide the best fit of estimated expected cost function such that $0 < \beta < 1$.

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Authors' contributions

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Availability of data and materials

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Declarations

Competing interests

The author declares that he has no competing interests.

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