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ORIGINIAL ARTICLE

Asymptotical state estimation of fuzzy cellular neural networks with time delay in the leakage term and mixed delays: Sample-data approach

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Abstract In this paper, the sampled measurement is used to estimate the neuron states, instead of the continuous measurement, and a sampled-data estimator is constructed. Leakage delay is used to unstable the neuron states. It is a challenging task to develop delay dependent condition to estimate the unstable neuron states through available sampled output measurements such that the error-state system is globally asymptotically stable. By constructing Lyapunov–Krasovskii functional (LKF), a sufficient condition depending on the sampling period is obtained in terms of linear matrix inequalities (LMIs). Moreover, by using the free-weighting matrices method, simple and efficient criterion is derived in terms of LMIs for estimation.

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1. Introduction

Cellular neural networks (CNNs), proposed by Chua and Yang in [1,2] have been extensively studied both in theory and in applications. Based on traditional CNN, the fuzzy cellular neural networks (FCNNs) have been introduced at the first time in 1996, proposed by Yang in [3,4]. The FCNN is a fuzzy neural networks which integrates fuzzy logic into the structure of traditional CNN. It is a very useful tool in image processing and pattern recognition. However, the existence of time delays may lead to the instability or bad performance of systems [5–7]. So, it is of prime importance to consider the delay effects on the dynamical behavior of systems. Recently, FCNNs with various types of delay have been widely investigated by many authors; see [8–12] and references therein. However, so far, there has been very little existing work on FCNNs with time delay in the leakage (or "forgetting") term [13–17]. In fact, time delay in the leakage term also has great impact on the dynamics of FCNNs. As pointed out by

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Gopalsamy [18], time delay in the stabilizing negative feedback term has a tendency to destabilize a system. Moreover, the time delay involving in the first term of the state variable of dynamical networks known as leakage delay on FCNNs cannot be ignored.

It is well known that the knowledge about the system state is necessary to solve many control theory problems; for example, stabilizing a system using state feedback. In most practical cases, the physical state of the system cannot be determined by direct observation. Instead, indirect effects of the internal state are observed by the way of the system outputs. A simple example is that of vehicles in a tunnel: the rates and velocities at which vehicles enter and leave the tunnel can be observed directly, but the exact state inside the tunnel can only be estimated. If a system is observable, it is possible to fully reconstruct the system state from its output measurements using the state observer. Also, the state estimation problem for neural networks has attracted some attention in the recent years, see [19–21].

For the state estimation problem, normally the periodic type constant vector is used in the existing literatures for getting the unstable behavior in the system. Without having such constant vector the given system should be stable, see for example [19–21]. In this regard there is no meaningful idea behind for designing the estimator gain matrix H. Motivating this reason, in this paper, leakage delay in the leakage term is used to unstable the neuron states without constant vector. On the other hand, the sampled-data control technology has developed largely as the rapid development of computer hardware. The measurements used to estimate the neuron states are sampled by samplers. Based on the sampled measurements, in this paper a sampled-data estimator is constructed. By converting the sampling period into a time-varying but bounded delay, the error dynamics of the considered FCNN is derived in terms of a differential equation with two different time-delays [22]. To the best of authors' knowledge, there was no results available in any of the existing literature dealing the state estimation for FCNNs with time delay in the leakage term, discrete and unbounded distributed delays based on sample-data.

Motivated by the above discussion, in this paper leakage delay in the leakage term is used to unstable the neuron states. It is challenging to develop delay dependent condition to estimate the unstable neuron states through available sampled output measurements such that the error-state system is globally asymptotically stable. Based on the LKF which contains a triple-integral term, an improved delay-dependent stability criterion is derived in terms of LMIs. However using the free-weighting matrices method, simple and efficient criterion is derived in terms of LMIs for estimation. Finally, numerical examples and its simulations are provided to demonstrate the effectiveness and merits of the derived result.

Notations $\mathbb{R}^n$ denotes the n-dimensional Euclidean Space. For any matrix $A = [a_{ij}]_{n\times n}$, let A^T and A^{-1} denote the transpose and the inverse of A, respectively. $|A| = [|a_{ij}|]_{n\times n}$. Let $A > 0$ $(A < 0)$ denotes the positive-definite (negative-definite) symmetric matrix, respectively. I denotes the identity matrix of appropriate dimension. $\Lambda = \{1, 2, \ldots, n\}$ and $\Xi = \{1, 2, \ldots, m\}$. $*$ denotes the symmetric terms in a symmetric matrix.

2. Model formulation and preliminaries

Consider the following FCNNs with leakage delay, discrete and unbounded distributed delays

$$\begin{cases} \dot{x}_i(t) = -a_i x_i(t-\sigma) + \sum_{j=1}^{n} b_{0_{ij}} g_j(x_j(t)) \\ \quad + \sum_{j=1}^{n} b_{1_{ij}} g_j(x_j(t-\tau_1(t))) + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \quad + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds, \quad i \in \Lambda, \\ x_i(s) = u_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{1}$$

where $u_i(\cdot) \in C((-\infty, 0], \mathbb{R})$; α_{ij} and β_{ij} are the elements of fuzzy feedback MIN template, fuzzy feedback MAX template, respectively; $b_{0_{ij}}$ and $b_{1_{ij}}$ are the elements of feedback template; $\bigwedge$, $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; x_i denotes the state of the ith neuron; a_i is a diagonal matrix, a_i represents the rates with which the ith neuron will reset their potential to the resting state in isolation when disconnected from the networks and external inputs; g_j represents the neuron activation function; $k_i(s) \geqslant 0$ is the feedback kernel and satisfies

$$\int_0^{\infty} k_i(s) ds = 1, \quad i \in \Lambda. \tag{2}$$

(A_1) The transmission delay $\tau_1(t)$ is a time varying delay, and it satisfies $0 \leqslant \tau_1(t) \leqslant \tau_1$, where τ_1 is a positive constant;
(A_2) The leakage delay satisfies $\sigma \geqslant 0$. Also, it is assumed that the neuron activation function $g(\cdot)$ satisfies the following Lipschitz condition

$$|g(x) - g(y)| \leqslant |L(x - y)|, \tag{3}$$

where $L \in \mathbb{R}^{n\times n}$ is a known constant matrix.

Our aim in this paper is to investigate an efficient estimation algorithm in order to observe the neuron states from the available network outputs. Therefore, the network measurements are assumed to satisfy $y_l(t) = c_{lj} x_i(t)$, $l \in \Xi$, $i, j \in \Lambda$, where $y_l \in \mathbb{R}^m$ is the measurement output of the 1^{th} neuron and c_{lj} is the element of a known constant matrix with appropriate dimension.

In this paper, the measurement output is sampled before it enters the estimator. The sampled measurement is assumed to be generalized by a zero-order hold function with a sequence of hold times $0 = t_0 < t_1 < \cdots < t_k < \cdots$

$$y_l(t_k) = c_{lj} x_i(t_k), \quad t_k \leqslant t < t_{k+1}, \tag{4}$$

where t_k denotes the sampling instant and satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots < \lim_{k\to+\infty} t_k = +\infty$. Moreover, the sampling period under consideration is assumed to be bounded by a known constant τ_2, that is $t_{k+1} - t_k \leqslant \tau_2$ for $k \geqslant 0$.

The full order state estimation of system (1) is given as follows

$$\begin{cases} \dot{\hat{x}}_i(t) = -a_i\hat{x}_i(t-\sigma) + \sum_{j=1}^{n} b_{0_{ij}} g_j(\hat{x}_j(t)) + \sum_{j=1}^{n} b_{1_{ij}} g_j(\hat{x}_j(t-\tau_1(t))) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds \\ \qquad + h_{il}[y_l(t_k) - c_{lj}\hat{x}_i(t_k)], \quad l \in \Xi,\ i,j \in \Lambda, \\ \hat{x}_i(s) = v_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{5}$$

where $v_i(\cdot) \in C((-\infty,0],\mathbb{R}); \hat{x}_i(t)$ is the estimation of the i^{th} neuron state; h_{il} is the element of an estimator gain matrix to be designed.

Define the error $e_i(t) = x_i(t) - \hat{x}_i(t)$, $\phi_j(t) = g_j(x_j(t)) - g_j(\hat{x}_j(t))$, $i,j \in \Lambda$; then it follows from (1), (4), and (5) that

$$\begin{cases} \dot{e}_i(t) = -a_i e_i(t-\sigma) - h_{il}c_{lj}e_i(t_k) + \sum_{j=1}^{n} b_{0_{ij}} \phi_j(t) \\ \qquad + \sum_{j=1}^{n} b_{1_{ij}} \phi_j(t-\tau_1(t)) + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \qquad - \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \qquad - \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds, \\ \qquad i,j \in \Lambda,\ l \in \Xi, \\ e_i(s) = u_i(s) - v_i(s) = \varphi_i(s), \quad s \in (-\infty, 0]. \end{cases} \tag{6}$$

Clearly, it is difficult to analyze the state estimation for FCNNs based on error system (6) because of the discrete term $e(t_k)$. Therefore, the input delay approach [23] is applied, so that $\tau_2(t) = t - t_k, t_k \leqslant t < t_{k+1}$. It is easily seen that $0 \leqslant \tau_2(t) < \tau_2$. Therefore, the error estimator takes the following form

$$e_i(t_k) := e_i(t - \tau_2(t)), \quad t_k \leqslant t < t_{k+1}, \quad i \in \Lambda. \tag{7}$$

Consequently, connecting (7) to system (6) yields

$$\begin{cases} \dot{e}_i(t) = -a_i e_i(t-\sigma) - h_{il}c_{lj}e_i(t-\tau_2(t)) \\ \qquad + \sum_{j=1}^{n} b_{0_{ij}} \phi_j(t) + \sum_{j=1}^{n} b_{1_{ij}} \phi_j(t-\tau_1(t)) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \qquad - \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \qquad - \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds, \\ \qquad i,j \in \Lambda,\ l \in \Xi, \\ e_i(s) = \varphi_i(s), \quad s \in (-\infty, 0]. \end{cases} \tag{8}$$

Using a simple transformation, system (8) leads the following equivalent form

$$\begin{cases} \frac{d}{dt}\left[e_i(t) - a_i\int_{t-\sigma}^{t} e_i(s)ds\right] = -a_i e_i(t) - h_{il}c_{lj}e_i(t-\tau_2(t)) + \sum_{j=1}^{n} b_{0_{ij}} \phi_j(t) \\ \qquad + \sum_{j=1}^{n} b_{1_{ij}} \phi_j(t-\tau_1(t)) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \qquad - \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \qquad - \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds, \\ \qquad i,j \in \Lambda,\ l \in \Xi, \\ e_i(s) = \varphi_i(s), \quad s \in (-\infty, 0]. \end{cases} \tag{9}$$

It is clear from (3) that

$$\begin{aligned} \phi^T(t)\phi(t) &= |g(x(t)) - g(\hat{x}(t))|^2 \leqslant |Le(t)|^2 \\ &= e^T(t)L^TLe(t). \end{aligned} \tag{10}$$

3. Main results

Theorem 3.1. *Assume that assumptions $(A_1) - (A_2)$ and the Lipschitz condition (3) hold. The error dynamical system (8) is globally asymptotically stable, if there exist $n \times n$ positive diagonal matrices P, Q, some $n \times n$ positive definite symmetric matrices R, W, N, M_1, M_2, three scalars $\mu > 0$, $\epsilon_1 > 0$, $\epsilon_2 > 0$, and a $2n \times 2n$ matrix $\begin{pmatrix} T_{11} & T_{12} \\ * & T_{22} \end{pmatrix} > 0$, $\begin{pmatrix} V_{11} & V_{12} \\ * & V_{22} \end{pmatrix} > 0$ such that the following LMI has feasible solution*

$$\Omega = \begin{bmatrix} \Psi & \Gamma^T \\ * & -\mu n^{-1} I \end{bmatrix} < 0, \tag{11}$$

where $(\Psi)_{13\times13}$ with

$$\begin{aligned}
&\Psi_{1,1} = -2PA + P + W + \sigma^2 N - 2M_1 - 2M_2 + \epsilon_1 L^T L, \\
&\Psi_{1,3} = T_{12}^T, \quad \Psi_{1,4} = -RC + V_{12}^T, \\
&\Psi_{1,6} = A^T PA, \quad \Psi_{1,7} = \frac{2}{\tau_1} M_1, \quad \Psi_{1,8} = \frac{2}{\tau_1} M_1, \\
&\Psi_{1,9} = \frac{2}{\tau_2} M_2, \quad \Psi_{1,10} = \frac{2}{\tau_2} M_2, \quad \Psi_{1,11} = PB_0, \\
&\Psi_{1,12} = PB_1, \quad \Psi_{2,2} = -W, \quad \Psi_{2,5} = -A^T P^T, \\
&\Psi_{3,3} = \tau_1 T_{11} - 2T_{12}^T + \epsilon_2 L^T L, \quad \Psi_{4,4} = \tau_2 V_{11} - 2V_{12}^T, \\
&\Psi_{4,5} = -C^T R^T, \quad \Psi_{4,6} = C^T R^T A, \quad \Psi_{5,5} = -2P + \tau_1 T_{22} \\
&\qquad + \tau_2 V_{22} + \frac{\tau_1^2}{2} M_1 + \frac{\tau_2^2}{2} M_2, \quad \Psi_{5,11} = PB_0, \\
&\Psi_{5,12} = PB_1, \quad \Psi_{6,6} = A^T PA - N, \quad \Psi_{6,11} = -A^T PB_0, \\
&\Psi_{6,12} = -A^T PB_1, \quad \Psi_{7,7} = -\frac{2}{\tau_1^2} M_1, \\
&\Psi_{7,8} = -\frac{2}{\tau_1^2} M_1, \quad \Psi_{8,8} = -\frac{2}{\tau_1^2} M_1, \quad \Psi_{9,9} = -\frac{2}{\tau_2^2} M_2, \\
&\Psi_{9,10} = -\frac{2}{\tau_2^2} M_2, \quad \Psi_{10,10} = -\frac{2}{\tau_2^2} M_2,
\end{aligned}$$

$$\Psi_{11,11}=Q-\epsilon_1,\quad \Psi_{12,12}=-\epsilon_2,\quad \Psi_{13,13}=2nS^TPS+\mu I-Q,$$

$$|\alpha|_s=\mathrm{diag}\left\{\sum_{i=1}^{n}|\alpha_{i1}|,\sum_{i=1}^{n}|\alpha_{i2}|,\ldots,\sum_{i=1}^{n}|\alpha_{in}|\right\},$$

$$|\beta|_s=\mathrm{diag}\left\{\sum_{i=1}^{n}|\beta_{i1}|,\sum_{i=1}^{n}|\beta_{i2}|,\ldots,\sum_{i=1}^{n}|\beta_{in}|\right\},$$

$$S=|\alpha|_s+|\beta|_s,$$

$$\Gamma^T=\begin{bmatrix}0 & 0 & 0 & 0 & (PS)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T.$$

Moreover, the estimation gain is $H=P^{-1}R$.

Proof. Consider the following LKFs

$$V(t)=\sum_{i=1}^{7}V_i(t), \tag{12}$$

where

$$V_1(t)=\left[e(t)-A\int_{t-\sigma}^{t}e(s)ds\right]^TP\left[e(t)-A\int_{t-\sigma}^{t}e(s)ds\right]$$
$$=\sum_{i=1}^{n}p_i\left(e_i(t)-a_i\int_{t-\sigma}^{t}e_i(s)ds\right)^2,$$

$$V_2(t)=\int_{t-\sigma}^{t}e^T(s)We(s)ds,\ V_3(t)=\sigma\int_{t-\sigma}^{t}\int_{\theta}^{t}e^T(s)Ne(s)dsd\theta,$$

$$V_4(t)=\sum_{j=1}^{n}q_j\int_0^{\infty}k_j(\theta)\int_{t-\theta}^{t}\phi_j^2(s)dsd\theta,$$

$$V_5(t)=\int_0^t\int_{\theta-\tau_1(\theta)}^{\theta}\begin{bmatrix}e(\theta-\tau_1(\theta))\\ \dot e(s)\end{bmatrix}^T\begin{bmatrix}T_{11} & T_{12}\\ * & T_{22}\end{bmatrix}\times\begin{bmatrix}e(\theta-\tau_1(\theta))\\ \dot e(s)\end{bmatrix}dsd\theta$$
$$+\int_0^t\int_{\theta-\tau_2(\theta)}^{\theta}\begin{bmatrix}e(\theta-\tau_2(\theta))\\ \dot e(s)\end{bmatrix}^T\begin{bmatrix}V_{11} & V_{12}\\ * & V_{22}\end{bmatrix}\begin{bmatrix}e(\theta-\tau_2(\theta))\\ \dot e(s)\end{bmatrix}dsd\theta,$$

$$V_6(t)=\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot e^T(s)T_{22}\dot e(s)dsd\theta+\int_{-\tau_2}^{0}\int_{t+\theta}^{t}\dot e^T(s)V_{22}\dot e(s)dsd\theta,$$

$$V_7(t)=\int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot e^T(s)M_1\dot e(s)dsd\lambda d\theta+\int_{-\tau_2}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot e^T(s)M_2\dot e(s)dsd\lambda d\theta.$$

By calculating the time derivation of $V_i(t)$ along the trajectory of system (9), one can obtain

$$\dot V_1(t)=2\sum_{i=1}^{n}p_i\left(e_i(t)-a_i\int_{t-\sigma}^{t}e_i(s)ds\right)\times\frac{d}{dt}\left[e_i(t)-a_i\int_{t-\sigma}^{t}e_i(s)ds\right], \tag{13}$$

$$\dot V_2(t)=e^T(t)We(t)-e^T(t-\sigma)We(t-\sigma), \tag{14}$$

$$\dot V_3(t)\leqslant\sigma^2e^T(t)Ne(t)-\int_{t-\sigma}^{t}e^T(s)ds\,N\int_{t-\sigma}^{t}e(s)ds, \tag{15}$$

$$\dot V_4(t)=\phi^T(t)Q\phi(t)-\left(\int_{-\infty}^{t}K(t-s)\phi(s)ds\right)^T\times Q\left(\int_{-\infty}^{t}K(t-s)\phi(s)ds\right), \tag{16}$$

$$\dot V_5(t)\leqslant e^T(t-\tau_1(t))\left[\tau_1T_{11}-2T_{12}^T\right]e(t-\tau_1(t))+2e^T(t)T_{12}^Te(t-\tau_1(t))+\int_{t-\tau_1}^{t}\dot e^T(s)T_{22}\dot e(s)ds$$
$$+e^T(t-\tau_2(t))\left[\tau_2V_{11}-2V_{12}^T\right]e(t-\tau_2(t))+2e^T(t)V_{12}^Te(t-\tau_2(t))+\int_{t-\tau_2}^{t}\dot e^T(s)V_{22}\dot e(s)ds, \tag{17}$$

$$\dot V_6(t)=\tau_1\dot e^T(t)T_{22}\dot e(t)-\int_{t-\tau_1}^{t}\dot e^T(s)T_{22}\dot e(s)ds+\tau_2\dot e^T(t)V_{22}\dot e(t)-\int_{t-\tau_2}^{t}\dot e^T(s)V_{22}\dot e(s)ds, \tag{18}$$

$$\dot V_7(t)=\frac{\tau_1^2}{2}\dot e^T(t)M_1\dot e(t)-\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot e^T(s)M_1\dot e(s)dsd\theta+\frac{\tau_2^2}{2}\dot e^T(t)M_2\dot e(t)-\int_{-\tau_2}^{0}\int_{t+\theta}^{t}\dot e^T(s)M_2\dot e(s)dsd\theta, \tag{19}$$

On the other hand, it is clear from (10) that the following is true for $\epsilon_j>0,\ j=1,2$

$$0\leqslant\epsilon_1\left[e^T(t)L^TLe(t)-\phi^T(t)\phi(t)\right], \tag{20}$$
$$0\leqslant\epsilon_2\left[e^T(t-\tau_1(t))L^TLe(t-\tau_1(t))-\phi^T(t-\tau_1(t))\phi(t-\tau_1(t))\right]. \tag{21}$$

Hence, from (13)–(21) we have

$$\dot V(t)\leqslant\xi^T(t)[\Psi+\Gamma^T\mu^{-1}n\Gamma]\ \xi(t)=\xi^T(t)\ \Omega\ \xi(t), \tag{22}$$

where

$$\xi(t)=\Big[e^T(t),\ e^T(t-\sigma),\ e^T(t-\tau_1(t)),\ e^T(t-\tau_2(t)),\ \dot e^T(t),\ \int_{t-\sigma}^{t}e^T(s)ds,\ \int_{t-\tau_1(t)}^{t}e^T(s)ds,$$
$$\int_{t-\tau_1}^{t-\tau_1(t)}e^T(s)ds,\ \int_{t-\tau_2(t)}^{t}e^T(s)ds,\ \int_{t-\tau_2}^{t-\tau_2(t)}e^T(s)ds,\ \phi^T(t),\ \phi^T(t-\tau_1(t)),$$
$$\int_{-\infty}^{t}K(t-s)\phi^T(s)ds\Big]^T,$$

$$\Omega=\Psi+\Gamma^T\mu^{-1}n\Gamma.$$

By (11), it yields $\dot V(t)\leqslant-\xi^T(t)\ \Omega^\star\ \xi(t),\ t>0,$ where $\Omega^\star=-\Omega>0$. Thus, it can be deduced that

$$V(t)+\int_0^t\xi^T(s)\Omega^\star\xi(s)ds\leqslant V(0)<\infty,\quad t\geqslant0, \tag{23}$$

where

$$V(0)\leqslant\Big\{2\lambda_{max}(P)\ (1+\sigma^2\max_{i\in\Lambda}a_i)+\sigma\lambda_{max}(W)+\sigma^3\lambda_{max}(N)$$
$$+\sum_{j=1}^{n}q_jk_j\max_{j\in\Lambda}l_j^2\int_0^{\infty}\theta k_j(\theta)d\theta+\tau_1^2\lambda_{max}(T_{22})+\tau_2^2\lambda_{max}(V_{22})$$
$$+\tau_1^3\lambda_{max}(M_1)+\tau_2^3\lambda_{max}(M_2)\Big\}\|\varphi_e\|^2<\infty.$$

From the definition of $V_2(t)$ and Jensen's inequality lemma [24], we have $\left\|\int_{t-\sigma}^{t}e(s)ds\right\|^2\leqslant\frac{\sigma}{\lambda_{\min}(W)}V(t)\leqslant\frac{\sigma}{\lambda_{\min}(W)}V(0)$, which together with the definition of $V_1(t)$ yields

$$\|e(t)\|\leqslant\left\|A\int_{t-\sigma}^{t}e(s)ds\right\|+\sqrt{\frac{V_1(t)}{\lambda_{\min}(P)}}$$
$$\leqslant\left\{\sqrt{\sum_{i=1}^{n}a_i\frac{\sigma}{\lambda_{\min}(W)}}+\sqrt{\frac{1}{\lambda_{\min}(P)}}\right\}\sqrt{V(0)}.$$

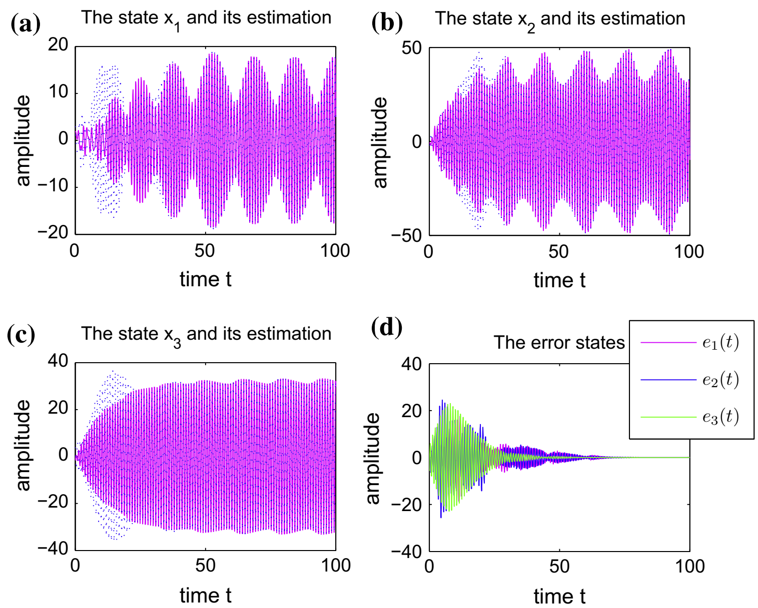

Fig. 1 (a) the true state $x_1(t)$ and its estimate state $\hat{x}_1(t)$ when $\sigma = 0.2$, with estimator gain matrix (31), (b) the true state $x_2(t)$ and its estimate state $\hat{x}_2(t)$ when $\sigma = 0.2$, with estimator gain matrix (31), (c) the true state $x_3(t)$ and its estimate state $\hat{x}_3(t)$ when $\sigma = 0.2$, with estimator gain matrix (31), and (d) the error trajectories of system (8) when $\sigma = 0.2$ with estimator gain matrix (31).

This implies that the error system (8) is locally stable. Next, one can prove that $\|e(t)\| \to 0$ *as* $t \to \infty$. First, for any constant $\theta \in [0, 1]$, it follows from (12) and Jensen's inequality lemma [24] that

$$\|e(t+\theta) - e(t)\|^2 \leqslant \frac{1}{\lambda_{\min}(\Omega^\star)} \int_t^{t+1} \xi^T(s)\Omega^\star \xi(s)ds \to 0 \quad \text{as } t \to \infty,$$

which implies that for any $\epsilon > 0, \theta \in [0, 1]$, there exists a $T_1 = T_1(\epsilon) > 0$ such that

$$\|e(t+\theta) - e(t)\| < \frac{\epsilon}{2}, \quad t > T_1. \tag{24}$$

On the other hand, from (12) we have

$$\left\| \int_t^{t+1} e(s)ds \right\|^2 \leqslant \frac{1}{\lambda_{\min}(\Omega^\star)} \int_t^{t+1} \xi^T(s)\Omega^\star \xi(s)ds \to 0 \quad \text{as } t \to \infty,$$

which implies that for any $\epsilon > 0$, there exists a $T_2 = T_2(\epsilon) > 0$ such that $\left\| \int_t^{t+1} e(s)ds \right\| < \frac{\epsilon}{2}$, $t > T_2$. Note that $e(s)$ is continuous on $[t, t+1]$, $t > 0$. Applying the integral mean value theorem, there exists a vector $\delta_t = (\delta_{t1}, \delta_{t2}, \cdots, \delta_{tn})^T \in \mathbb{R}^n$, $\delta_{tj} \in [t, t+1]$, such that

$$\|e(\delta_t)\| = \left\| \int_t^{t+1} e(s)ds \right\| < \frac{\epsilon}{2}, \quad t > T_2. \tag{25}$$

By (24), (25) and for any $\epsilon > 0$, there exists a $T = \max\{T_1, T_2\} > 0$ such that $t > T$ implies

$$\|e(t)\| \leqslant \|e(t) - e(\delta_t)\| + \|e(\delta_t)\| \leqslant \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

This proves that $\|e(t)\| \to 0$ as $t \to \infty$. Therefore, one can conclude that the error dynamical system (8) is globally asymptotically stable. As a result, the full order sampled state estimation FCNNs with time delay in the leakage term, discrete and unbounded distributed delays (5) is globally estimated with the FCNNs (1). This completes the proof. □

Remark 3.1. It is evident from the Fig. 2(a)–(c), the true state $x_i(t)$ is stable when $\sigma = 0$ and its estimated state is $\hat{x}_i(t)$, $i = 1, 2, 3$. Further through Fig. 1(a)–(c), it is clear that the true state $x_i(t)$ is unstable when $\sigma = 0.2$ and its estimated state is $\hat{x}_i(t)$, $i = 1, 2, 3$. Moreover, the error trajectories of Figs. 1(d) and 2(d) converges to 0. The time delays which is called leakage delay σ exists in the negative feedback term of system (1), which is different from the time-varying delays in other terms. It has been shown in [18,25] that the time delay in the leakage term has great impact on the dynamics of neural networks and often has a quick tendency to destabilize a system. This motivates to consider the leakage delay effects on the state estimation of FCNNs with discrete time-varying delays and continuously unbounded distributed delays. However, this paper deals for the constant leakage delay; to improve and extend the results for time-varying leakage delay may lead a challenging problem. In the near future, some further research on this topic will be investigated.

When there is no time delay in the leakage term, that is $\sigma = 0$, the error dynamical FCNNs (8) becomes the following

$$\begin{cases} \dot{e}_i(t) = -a_i e_i(t) - h_{il} c_{lj} e_i(t - \tau_2(t)) + \sum_{j=1}^n b_{0_{ij}} \phi_j(t) + \sum_{j=1}^n b_{1_{ij}} \phi_j(t - \tau_1(t)) \\ \qquad + \bigwedge_{j=1}^n \alpha_{ij} \int_{-\infty}^t k_j(t-s) g_j(x_j(s))ds - \bigwedge_{j=1}^n \alpha_{ij} \int_{-\infty}^t k_j(t-s) g_j(\hat{x}_j(s))ds \\ \qquad + \bigvee_{j=1}^n \beta_{ij} \int_{-\infty}^t k_j(t-s) g_j(x_j(s))ds - \bigvee_{j=1}^n \beta_{ij} \int_{-\infty}^t k_j(t-s) g_j(\hat{x}_j(s))ds, \\ \quad i, j \in \Lambda,\ l \in \Xi, \\ e_i(s) = \varphi_i(s), \quad s \in (-\infty, 0]. \end{cases} \tag{26}$$

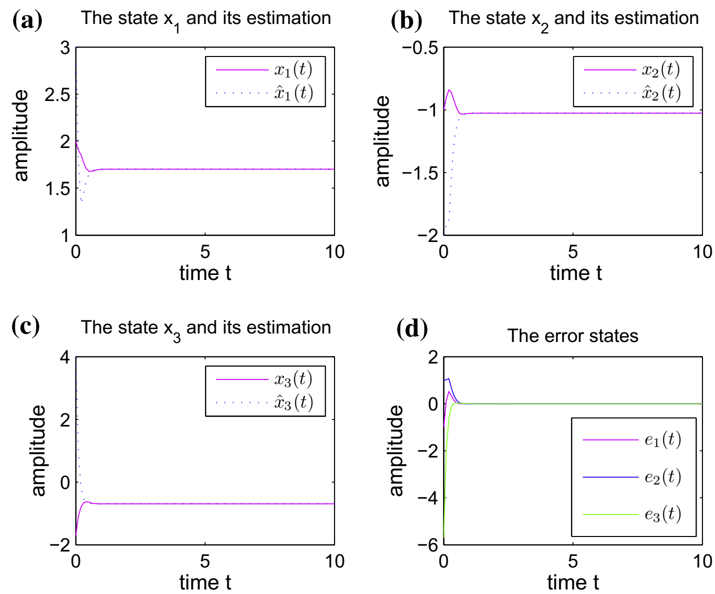

Fig. 2 (a) the true state $x_1(t)$ and its estimate state $\hat{x}_1(t)$ when $\sigma = 0$, with estimator gain matrix (34), (b) the true state $x_2(t)$ and its estimate state $\hat{x}_2(t)$ when $\sigma = 0$, with estimator gain matrix (34), (c) the true state $x_3(t)$ and its estimate state $\hat{x}_3(t)$ when $\sigma = 0$, with estimator gain matrix (34), and (d) the error trajectories of system (26) with estimator gain matrix (34).

In the following Corollary 3.1, global asymptotic stability criteria for error dynamical FCNNs (26) is discussed.

Corollary 3.1. *Assume that assumptions $(A_1)-(A_2)$, the Lipschitz condition (3) hold. The error dynamical system (26) is globally asymptotically stable, if there exist $n\times n$ positive diagonal matrices P, Q, some $n \times n$ positive definite symmetric matrices R, M_1, M_2, three scalars $\mu > 0$, $\epsilon_1 > 0$, $\epsilon_2 > 0$, and a $2n \times 2n$ matrices $\begin{pmatrix} T_{11} & T_{12} \\ * & T_{22}\end{pmatrix} > 0$, $\begin{pmatrix} V_{11} & V_{12} \\ * & V_{22}\end{pmatrix} > 0$, such that the following LMI has feasible solution*

$$\Omega = \begin{bmatrix} \Psi & \Gamma^T \\ * & -\mu n^{-1} I \end{bmatrix} < 0, \tag{27}$$

where $(\Psi)_{11\times 11}$ with

$$\Psi_{1,1} = -2PA + P - 2M_1 - 2M_2 + \epsilon_1 L^T L,\ \Psi_{1,2} = T_{12}^T,$$
$$\Psi_{1,3} = -RC + V_{12}^T,\ \Psi_{1,4} = -A^T P,$$
$$\Psi_{1,5} = \frac{2}{\tau_1} M_1,\ \Psi_{1,6} = \frac{2}{\tau_1} M_1,\ \Psi_{1,7} = \frac{2}{\tau_2} M_2,$$
$$\Psi_{1,8} = \frac{2}{\tau_2} M_2,\ \Psi_{1,9} = PB_0,\ \Psi_{1,10} = PB_1,$$
$$\Psi_{2,2} = \tau_1 T_{11} - 2T_{12}^T + \epsilon_2 L^T L,\ \Psi_{3,3} = \tau_2 V_{11} - 2V_{12}^T,$$
$$\Psi_{3,4} = -C^T R^T,$$
$$\Psi_{4,4} = -2P + \tau_1 T_{22} + \tau_2 V_{22} + \frac{\tau_1^2}{2} M_1 + \frac{\tau_2^2}{2} M_2, \Psi_{4,9} = PB_0,$$
$$\Psi_{4,10} = PB_1,\ \Psi_{5,5} = -\frac{2}{\tau_1^2} M_1,$$
$$\Psi_{5,6} = -\frac{2}{\tau_1^2} M_1,\ \Psi_{6,6} = -\frac{2}{\tau_1^2} M_1,\ \Psi_{7,7} = -\frac{2}{\tau_2^2} M_2,$$
$$\Psi_{7,8} = -\frac{2}{\tau_2^2} M_2,\ \Psi_{8,8} = -\frac{2}{\tau_2^2} M_2,$$
$$\Psi_{9,9} = Q - \epsilon_1,\ \Psi_{10,10} = -\epsilon_2,\ \Psi_{10,11} = 0,$$
$$\Psi_{11,11} = nS^T PS + \mu I - Q,$$
$$|\alpha|_s = \mathrm{diag}\left\{\sum_{i=1}^n |\alpha_{i1}|, \sum_{i=1}^n |\alpha_{i2}|, \ldots, \sum_{i=1}^n |\alpha_{in}|\right\},$$
$$|\beta|_s = \mathrm{diag}\left\{\sum_{i=1}^n |\beta_{i1}|, \sum_{i=1}^n |\beta_{i2}|, \ldots, \sum_{i=1}^n |\beta_{in}|\right\},$$
$$S = |\alpha|_s + |\beta|_s,\ \Gamma^T = \begin{bmatrix} 0 & 0 & 0 & (PS)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T.$$

Moreover, the estimation gain is $H = P^{-1}R$.

Proof. Consider the following LKFs

$$V(t) = \sum_{i=1}^5 V_i(t), \tag{28}$$

where

$$V_1(t) = e^T(t)Pe(t) = \sum_{i=1}^n p_i e_i^2(t),$$
$$V_2(t) = \sum_{j=1}^n q_j \int_0^\infty k_j(\theta) \int_{t-\theta}^t \phi_j^2(s) ds d\theta,$$
$$V_3(t) = \int_0^t \int_{\theta-\tau_1(\theta)}^\theta \begin{bmatrix} e(\theta-\tau_1(\theta)) \\ \dot{e}(s)\end{bmatrix}^T \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22}\end{bmatrix} \begin{bmatrix} e(\theta-\tau_1(\theta)) \\ \dot{e}(s)\end{bmatrix} ds d\theta$$
$$+ \int_0^t \int_{\theta-\tau_2(\theta)}^\theta \begin{bmatrix} e(\theta-\tau_2(\theta)) \\ \dot{e}(s)\end{bmatrix}^T \begin{bmatrix} V_{11} & V_{12} \\ * & V_{22}\end{bmatrix} \begin{bmatrix} e(\theta-\tau_2(\theta)) \\ \dot{e}(s)\end{bmatrix} ds d\theta,$$
$$V_4(t) = \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{e}^T(s) T_{22} \dot{e}(s) ds d\theta + \int_{-\tau_2}^0 \int_{t+\theta}^t \dot{e}^T(s) V_{22} \dot{e}(s) ds d\theta,$$
$$V_5(t) = \int_{-\tau_1}^0 \int_\theta^0 \int_{t+\lambda}^t \dot{e}^T(s) M_1 \dot{e}(s) ds d\lambda d\theta$$
$$+ \int_{-\tau_2}^0 \int_\theta^0 \int_{t+\lambda}^t \dot{e}^T(s) M_2 \dot{e}(s) ds d\lambda d\theta.$$

The proof of this Corollary 3.1 is immediately follows from Theorem 3.1. □

Remark 3.2. In this paper, delay rate independent stability conditions have been derived without involving the time-varying delay $\tau_1(t)$ in the LKFs. Moreover, the conditions that the time-varying delay is differentiable and the derivative is bounded or smaller than one are not required.

4. Numerical examples

Example 4.1. Consider the following simple three-dimensional FCNNs with leakage delay, discrete and unbounded distributed delays

$$\begin{cases} \dot{x}_i(t) = -a_i x_i(t-\sigma) + \sum_{j=1}^{n} b_{0_{ij}} g_j(x_j(t)) + \sum_{j=1}^{n} b_{1_{ij}} g_j(x_j(t-\tau_1(t))) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds, \quad i \in \Lambda, \\ x_i(s) = u_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{29}$$

with parameters defined as $\sigma = 0.2$, $\tau_1(t) = 0.1\,|\sin(t)|$, $u(s) = (2, -1, -1.7)^T$, $s \in (-\infty, 0]$, and $g_j(x_j) = \frac{1}{2}(|x_j + 1| - |x_j - 1|)$, $j = 1, 2, 3$, which satisfy the Lipschitz condition (3), we get $L = I$,

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 5 & -1.2 & -1 \\ -3 & 1.1 & -4 \\ -0.32 & 1.7 & 0.95 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1.5 & -0.7 & -2.6 \\ -3.3 & 1.2 & -0.5 \\ -0.9 & 1.5 & -2.3 \end{bmatrix},$$

$$\alpha = \begin{bmatrix} 1/31 & -1/31 & 1/31 \\ 1/31 & -1/31 & 1/31 \\ 1/31 & -1/31 & 1/31 \end{bmatrix},$$

$$\beta = \begin{bmatrix} -1/31 & 1/31 & 1/31 \\ 1/31 & -1/31 & 1/31 \\ 1/31 & 1/31 & -1/31 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

The corresponding full order sampled state estimation of system (29) is defined as follows

$$\begin{cases} \dot{\hat{x}}_i(t) = -a_i \hat{x}_i(t-\sigma) + \sum_{j=1}^{n} b_{0_{ij}} g_j(\hat{x}_j(t)) + \sum_{j=1}^{n} b_{1_{ij}} g_j(\hat{x}_j(t-\tau_1(t))) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds \\ \qquad + h_{il}[y_l(t_k) - c_{lj}\hat{x}_i(t_k)], \quad l \in \Xi, i \in \Lambda, \\ \hat{x}_i(s) = v_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{30}$$

where $y_l(t_k)$ is given by (4) and the initial condition is $v(s) = (3, -2, 4)^T$, $s \in (-\infty, 0]$. Moreover, the sampling period is taken as $\tau_2 = 0.05$. By using the Matlab LMI toolbox to solve the LMI (11) in Theorem 3.1, it can be found that the LMI is feasible. Consequently, the estimator gain matrix H is designed as follows

$$H = P^{-1}R = \begin{bmatrix} 0.0865 & 0.0235 & -0.0010 \\ 0.0264 & 0.0611 & -0.0165 \\ -0.0007 & -0.0095 & 0.0925 \end{bmatrix}. \tag{31}$$

By Theorem 3.1, systems (29) and (30) are asymptotically estimated. The simulation results are depicted in Fig. 1(a)–(d) by applying the estimator designed in (31) for this Example 4.1 by choosing the time step size $h = 0.1$, and time segment $T = 100$.

Example 4.2. Consider the following simple three-dimensional FCNNs without time delay in the leakage term, discrete and unbounded distributed delays

$$\begin{cases} \dot{x}_i(t) = -a_i x_i(t) + \sum_{j=1}^{n} b_{0_{ij}} g_j(x_j(t)) + \sum_{j=1}^{n} b_{1_{ij}} g_j(x_j(t-\tau_1(t))) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds, \quad i \in \Lambda, \\ x_i(s) = u_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{32}$$

with parameters defined as in Example 4.1 and $\sigma = 0$. The corresponding full order sampled state estimation of system (32) is defined as follows

$$\begin{cases} \dot{\hat{x}}_i(t) = -a_i \hat{x}_i(t) + \sum_{j=1}^{n} b_{0_{ij}} g_j(\hat{x}_j(t)) + \sum_{j=1}^{n} b_{1_{ij}} g_j(\hat{x}_j(t-\tau_1(t))) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(\hat{x}_j(s)) ds \\ \qquad + h_{il}\left[y_l(t_k) - c_{lj}\hat{x}_i(t_k)\right], \quad l \in \Xi, \; i \in \Lambda, \\ \hat{x}_i(s) = v_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{33}$$

By using the Matlab LMI toolbox to solve the LMI (27) in Corollary 3.1, it can be found that the LMI is feasible. Consequently, the estimator gain matrix H is designed as follows

$$H = P^{-1}R = \begin{bmatrix} 0.0523 & 0.0187 & -0.0127 \\ 0.0228 & 0.0471 & -0.0275 \\ -0.0078 & -0.0138 & 0.0603 \end{bmatrix}. \tag{34}$$

By Corollary 3.1, systems (32) and (33) are asymptotically estimated.

The simulation results are depicted in Fig. 2(a)–(d) by using the above estimator (34) for this Example 4.2 by choosing the time step size $h = 0.1$, and time segment $T = 10$.

5. Conclusion

In this paper, state estimation for FCNNs is considered with time delay in the leakage term, discrete and unbounded distributed delays based on sampled-data. The sampled measurements have been used to estimate the neuron states. Also simple and efficient estimation criterion is derived in terms of LMIs by constructing the LKF which contains a triple-integral term and the free-weighting matrices method. Further, the differentiability of the time-varying delay $\tau_1(t)$ is not required in this paper. Finally, the effectiveness of the proposed sampled-data estimation approach has been verified by demonstrating numerical simulations of the derived results.

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