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SHORT COMMUNICATION

New traveling wave solutions of Drinefel'd–Sokolov–Wilson Equation using Tanh and Extended Tanh methods

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Abstract Traveling wave solutions are obtained by using a relatively new technique which is called Tanh and extended Tanh method for Drinefel'd–Sokolov–Wilson Equations. Solution procedure and obtained results re-confirm the efficiency of the proposed scheme.

MATHEMATICS SUBJECT CLASSIFICATION: 02.30.Jr; 05.45.Yv; 02.30.Ik

1. Introduction

Nonlinear partial differential equations are useful in describing the various phenomena in disciplines. The Drinefel'd–Sokolov–Wilson [1] equations are important partial differential equations of the nonlinear dispersive waves. Moreover, such equation arise frequently in number of scientific models including fluid mechanics, astrophysics, solid state physics, plasma physics, chemical kinematics, chemical chemistry, optical fiber and geochemistry. Solitary waves are wave packet or pulses which propagate in nonlinear dispersive media, see [2,3] and the references therein. Moreover, such equations play a major role [4,5] in the study of nonlinear dispersive waves, shallow water waves and ion acoustic plasma waves. It is an established fact that most of the physical phenomena are nonlinear in nature and hence their appropriate solutions are always more than essential. In the similar context, several numerical and analytical techniques including Homotopy Analysis (HAM) [6,7], Homotopy Perturbation (HPM) [8], Modified Adomian's Decomposition (MADM) [9,10], Variational Iteration (VIM) [11,12], Variation in Parameters [13,14], and Finite Difference [15,16] have been applied to tackle nonlinear problems of diversified physical nature. It is be highlighted that most of these techniques have their inbuilt deficiencies including evaluation of the so-called Adomian's polynomials, divergent results, discretization, successive applications of the integral operator, un-realistic assumptions, non-compatibility with the nonlinearity of physical problem and very lengthy calculations. It is worth mentioning that, recently, lot of attention is being given on solitary wave solutions because appearance of solitary wave in nature is rather frequent, especially in fluids, plasmas, solid state physics, condensed matter physics, optical fibers, chemical kinematics, electrical circuits, bio-genetics, elastic media, etc. The detailed study of literature reveals some important contributions in this field. Wang et al. [17] presented a reliable technique which is called (G'/G)-expansion method and obtained traveling wave

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solutions for the nonlinear evolution equations (NLEEs). In this method, second order linear ordinary differential equation with constant coefficients $G''(\eta)+\lambda G'(\eta)+\mu G(\eta)$, is used, as an auxiliary equation. In the subsequent work, the same has been used to obtain exact traveling wave solutions for the nonlinear differential equations, see [18–21] and the references therein. In the context of solitary solutions, there are many methods such as, Hirota's bilinear [22,23], Exp-function [24–26], Sine–cosine [27], Tanh function [28,29], general algebraic [30], extended Tanh function [31,32], (G'/G)-Expansion [17,33–35], F-expansion [36,37], homogeneous balance [38], Backlund transformation [39], and modified Exp-function [40]. Inspired and motivated by the ongoing research in this area, we apply a relatively new techniques which is called Tanh method and Extended Tanh method [41–43] to find traveling wave solutions of Drinefel'd–Sokolov–Wilson (DSW) Equation. It is to be highlighted that Drinefel'd–Sokolov–Wilson (DSW) Equation [1–5] arises frequently in plasma physics, surface physics, population dynamic, mathematical physics and applied sciences. The proposed scheme is fully compatible with the complexity of such problems and is very user-friendly. Numerical results are very encouraging.

2. Tanh method [41–43]

Consider the following nonlinear differential equations with two unknowns:

$$\begin{aligned} &P1(u,v,ux,vx,ut,vt,uxx,vxx,\ldots)=0,\\ &P2(u,v,ux,vx,ut,vt,uxx,vxx,\ldots)=0, \end{aligned} \tag{1}$$

where $P1$, $P2$ are polynomials of the variable u and v and its derivatives. If we consider $u(x,t)=u(\xi)$, $v(x,t)=v(\xi)$, $\xi=k(x-\lambda t)$ so that $u(x,t)=U(\xi)$, $v(x,t)=V(\xi)$, we can use the following changes:

$$\frac{\partial}{\partial t}=-k\lambda\frac{d}{d\xi},\quad \frac{\partial}{\partial x}=k\frac{d}{d\xi},\quad \frac{\partial^2}{\partial x^2}=k^2\frac{d^2}{d\xi^2},\quad \frac{\partial^3}{\partial x^3}=k^3\frac{d^3}{d\xi^3},$$

and so on, then Eq. (1) becomes an ordinary differential equation

$$\begin{aligned} &Q_1(U,V,U',V',U'',V'',U''',V'''\ldots)=0,\\ &Q_2(U,V,U',V',U'',V'',U''',V'''\ldots)=0, \end{aligned} \tag{2}$$

with $Q1$, $Q2$ being another polynomials form of their argument, which will be called the reduced ordinary differential equations of Eq. (2). Integrating Eq. (2) as long as all terms contain derivatives, the integration constants are considered to be zeros in view of the localized solutions. Now finding the traveling wave solutions to Eq. (1) is equivalent to obtaining the solution to the reduced ordinary differential Eq. (2). For the Tanh method, we introduce the new independent variable

$$Y(x,t)=\mathrm{Tanh}(\xi), \tag{3}$$

that leads to the change of variables:

$$\begin{aligned} &\frac{d}{d\xi}=(1-Y^2)\frac{d}{dY},\\ &\frac{d^2}{d\xi^2}=-2Y(1-Y^2)\frac{d}{dY}+(1-Y^2)^2\frac{d^2}{dY^2},\\ &\frac{d^3}{d\xi^3}=2(1-Y^2)(3Y^2-1)\frac{d}{dY}-6Y(1-Y^2)^2\frac{d^2}{dY^2}\\ &+(1-Y^2)^3\frac{d^3}{dY^3}. \end{aligned} \tag{4}$$

The next crucial step is that the solution we are looking for is expressed in the form

$$u(x,t)=U(\xi)=\sum_{i=1}^{m}aiYi,\quad v(x,t)=V(\xi)=\sum_{i=1}^{n}biYi, \tag{5}$$

where the parameters m and n can be found by balancing the highest-order linear term with the nonlinear terms in Eq. (2), and $k,\lambda,a0,a1,\ldots,am,b0,b1,\ldots,bm$ are to be determined. Substituting (5) into (2) will yield a set of algebraic equations for $k,\lambda,a0,a1,\ldots,am,b0,b1,\ldots,bm$ because all coefficients of Y^i have to vanish. From these relations, $k,\lambda,a0,a1,\ldots,am,b0,b1,\ldots,bm$ can be obtained. Having determined these parameters, knowing that m and n are positive integers in most cases, and using (5) we obtain analytic solutions $u(x,t)$, $v(x,t)$ in a closed form. The Tanh method seems to be powerful tool in dealing with coupled nonlinear physical models.

3. On using the Tanh method to solve Drinefel'd–Sokolov–Wilson Equations

Consider the following Drinefel'd–Sokolov–Wilson Equations

$$ut+pvvx=0, \tag{6}$$

$$vt+qvxxx+ruvx+suxv=0, \tag{7}$$

In order to implement the Tanh method, the starting point is the traveling wave hypothesis as given by

$$u(x,t)=U(\xi)=\sum_{i=1}^{m}aiYi, \tag{8}$$

$$v(x,t)=V(\xi)=\sum_{i=1}^{n}biYi, \tag{9}$$

where

$$Y(x,t)=\tanh(\xi), \tag{10}$$

$$\xi=k(x-\lambda t). \tag{11}$$

The nonlinear system of partial differential Eq. (6) and (7) is transformed to a system of ordinary differential equations

$$-\lambda kU'+pkVV'=0, \tag{12}$$

$$-\lambda kV'+qk^3V'''+rkUV'+skU'V=0. \tag{13}$$

We postulate the following Tanh series in Eqs. (8) and (9), and the transformation given in (10) and (11) the first Eq. (12) reduces to

$$-\lambda(1-Y^2)\frac{dU}{dY}+pV(1-Y^2)\frac{dV}{dY}=0. \tag{14}$$

The second equation in (13) reduces to

$$\begin{aligned} &-\lambda(1-Y^2)\frac{dV}{dY}+qk^2\Big[2(1-Y^2)(3Y^2-1)\frac{dV}{dY}\\ &-6Y(1-Y^2)^2\frac{d^2V}{dY^2}+(1-Y^2)^3\frac{d^3V}{dY^3}\Big]\\ &+rU(1-Y^2)\frac{dV}{dY}+sV(1-Y^2)\frac{dU}{dY}=0. \end{aligned} \tag{15}$$

Now, to determine the parameters m and n, we balance the linear term of highest-order with the highest-order nonlinear terms. So, in Eq. (12), we balance U' with VV', to obtain

$2 + m - 1 = 2 + n + n - 1 \Rightarrow m = 2n.$

While in Eq. (13) we balance V''' with UV', to obtain

$6 + n - 3 = 2 + m + n - 1 \Rightarrow m = 2, \quad n = 1.$

The Tanh method admits the use of the finite expansion for:

$$u(x,t) = U(Y) = a0 + a1\,Y + a2\,Y^2, \quad a2 \neq 0 \tag{16}$$

$$v(x,t) = V(Y) = b0 + b1\,Y, \quad b1 \neq 0. \tag{17}$$

Substituting U', VV' in Eq. (12), and then equating the coefficient of Yi, $i = 0, 1, 2$ leads to the following nonlinear system of algebraic equations:

$$\begin{aligned} &Y^0: -\lambda a1 + pb1b0 = 0, \\ &Y^1: -2\lambda a2 + pb1^2 = 0. \end{aligned} \tag{18}$$

Substituting V, U, V''', V', U' in Eq. (13), and again equating the coefficient of Yi, $i = 0, 1, 2, 3$ leads to the following nonlinear system of algebraic equations

$$\begin{aligned} &Y^0: -\lambda b1 + rb1a0 - 2qk^2b1 + sa1b0 = 0, \\ &Y^1: sa1b1 + 2sa2b0 + rb1a1 = 0, \\ &Y^2: -rb1a2 - 6qk^2b1 - 2sa2b1 = 0. \end{aligned} \tag{19}$$

Solving the nonlinear systems of Eqs. (18) and (19) we can get:

$$a0 = \frac{\lambda + 2k^2q}{r}, \quad a1 = 0, \quad a2 = -\frac{6k^2q}{r + 2s}, \quad b0 = 0,$$
$$b1 = 2\sqrt{-\frac{3\lambda q}{pr + 2ps}}k.$$

Then

$$\begin{aligned} u(x,t) &= \frac{\lambda + 2k^2q}{r} - \frac{6k^2q}{r + 2s}\mathrm{Tanh}^2(k(x - \lambda t)), \\ v(x,t) &= 2\sqrt{-\frac{3\lambda q}{pr + 2ps}}k\mathrm{Tanh}(k(x - \lambda t)). \end{aligned} \tag{20}$$

The solitary wave and behavior of the solutions $u(x, t)$ and $v(x, t)$ are shown in Figs. 1 and 2 respectively for some fixed values of the $\lambda = 1.5, k = 1, p = 2, q = -1, r = 3, s = 3$.

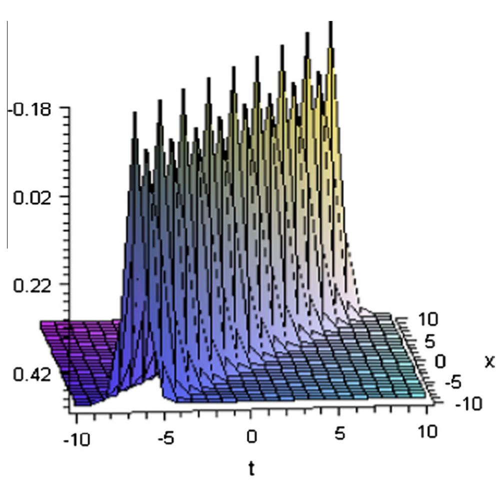

Figure 1 The soliton solution of $u(x, t)$.

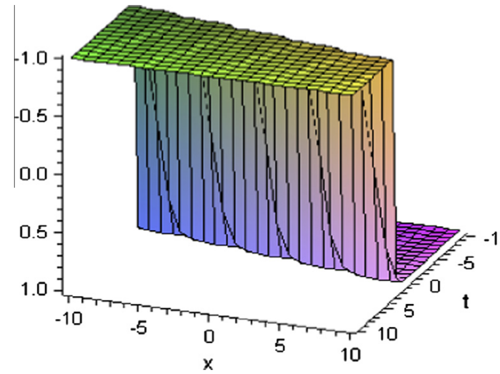

Figure 2 The kink solution of $v(x, t)$.

4. The Extended Tanh method

For a given system of nonlinear evolution equations, say, in two variables [41]

$$P(u, v, ux, vx, ut, vt, uxx, vxx, \ldots) = 0, \tag{21}$$

$$Q(u, v, ux, vx, ut, vt, uxx, vxx, \ldots) = 0. \tag{22}$$

We try to find the following traveling wave solutions:

$$u(x,t) = u(\xi), \quad v(x,t) = v(\xi), \quad \xi = x \pm ct,$$

so that $u(x, t) = U(\xi)$, $v(x, t) = V(\xi)$, which are of important physical significance, c is constants to be determined later. Then system (21) and (22) reduces to a system of nonlinear ordinary differential equations

$$P1(U, V, U', V', U'', V'', U''', V''', \ldots) = 0, \tag{23}$$

$$Q2(U, V, U', V', U'', V'', U''', V''', \ldots) = 0. \tag{24}$$

Introducing a new independent variables in the form

$$Y(x,t) = \mathrm{Tanh}(\mu\xi), \quad \xi = x \pm ct, \tag{25}$$

that leads to the change of derivatives

$$\begin{aligned} \frac{d}{d\xi} &= \mu(1 - Y^2)\frac{d}{dY}, \\ \frac{d^2}{d\xi^2} &= -2\mu^2Y(1 - Y^2)\frac{d}{dY} + \mu^2(1 - Y^2)^2\frac{d^2}{dY^2}, \\ \frac{d^3}{d\xi^3} &= 2\mu^3(1 - Y^2)(3Y^2 - 1)\frac{d}{dY} - 6\mu^3Y(1 - Y^2)^2\frac{d^2}{dY^2} \\ &\quad + \mu^3(1 - Y^2)^3\frac{d^3}{dY^3}. \end{aligned} \tag{26}$$

The extended Tanh method admits the use of the finite expansions:

$$\begin{aligned} u(x,t) &= U(\xi) = \sum_{i=1}^{M} aiY^i(\xi) + \sum_{i=1}^{M} biY^{-i}(\xi), \\ v(x,t) &= V(\xi) = \sum_{i=1}^{N} ciY^i(\xi) + \sum_{i=1}^{N} diY^{-i}(\xi), \end{aligned} \tag{27}$$

in which, ai, bi for $i = 0, \ldots, M$ and ci, di for $i = 0, \ldots, N$ are all real constants to be determined later, the balancing numbers M and N are positive integers which can be determined by balancing the highest order derivative terms with highest power nonlinear terms in Eqs. (23) and (24). We substitute

Eqs. (26) and (27) into Eqs. (23) and (24) with computerized symbolic computation, equating to zero the coefficients of all power $Y^{\pm i}$ yields a set of algebraic equations for ai, bi, ci, di and μ.

5. On using the Extended Tanh method to solve Drinefel'd–Sokolov–Wilson Equations

Consider the following Drinefel'd–Sokolov–Wilson Equations

$$ut + pvvx = 0, \tag{28}$$

$$vt + qvxxx + ruvx + suxv = 0. \tag{29}$$

Using the wave variable $\xi = x - ct$, carries Eqs. (28) and (29) into the ordinary differential equations.

$$-cU' + pVV' = 0, \tag{30}$$

$$-cV' + qV''' + rUV' + sU'V = 0. \tag{31}$$

We postulate the following Extended Tanh series in Eq. (27), and the transformation given in (25) the first Eq. (30) reduces to

$$-c\mu(1-Y^2)\frac{dU}{dY} + p\mu V(1-Y^2)\frac{dV}{dY} = 0. \tag{32}$$

The second equation in (31) reduces to

$$-c\mu(1-Y^2)\frac{dV}{dY} + q\left[2\mu^3(1-Y^2)(3Y^2-1)\frac{dV}{dY} - 6\mu^3Y(1-Y^2)^2\frac{d^2V}{dY^2} + \mu^3(1-Y^2)^3\frac{d^3V}{dY^3}\right] + r\mu U(1-Y^2)\frac{dV}{dY} + s\mu V(1-Y^2)\frac{dU}{dY} = 0. \tag{33}$$

Now, to determine the parameters M and N, we balance the linear term of highest-order with the highest-order nonlinear terms. So, in Eq. (32), we balance U' with VV', to obtain

$$2 + M - 1 = 2 + N + N - 1 \Rightarrow M = 2N.$$

While in Eq. (33) we balance V''' with UV', to obtain

$$6 + N - 3 = 2 + M + N - 1 \Rightarrow M = 2, N = 1.$$

The Extended Tanh method admits the use of the finite expansion for:

$$u(x,t) = U(Y) = a0 + a1Y + a2Y^2 + b1Y^{-1} + b2Y^{-2} = 0,$$
$$v(x,t) = V(Y) = c0 + c1Y + d1Y^{-1}. \tag{34}$$

Substituting U', VV' in Eq. (30), and then collecting the coefficients of Y, we obtain a system of algebraic equations for $a0$, $a1$, $a2$, $b1$, $b2$, $c0$, $c1$, $d1$ and μ.

$$2c\mu a_2 - p\mu c_1^2 = 0,$$

$$c\mu a_1 - p\mu c_0 c_1 = 0,$$

$$p\mu c_1^2 - 2c\mu a_2 = 0,$$

$$-c\mu a_1 + p\mu c_0 c_1 - c\mu b_1 + p\mu c_0 d_1 = 0,$$

$$-2c\mu b_2 - p\mu d_1^2 = 0,$$

$$c\mu b_1 - p\mu c_0 d_1 = 0,$$

$$-p\mu d_1^2 + 2c\mu b_2 = 0.$$

Substituting V, U, V''', V', U' in Eq. (31), and again collecting the coefficients of Y, we obtain a system of algebraic equations for $a0$, $a1$, $a2$, $b1$, $b2$, $c0$, $c1$, $d1$ and μ.

$$-2s\mu a_2 c_1 - 6q\mu^3 c_1 - r\mu a_2 c_1 = 0,$$

$$-s\mu a_1 c_1 - r\mu a_1 c_1 - 2s\mu c_0 a_2 = 0,$$

$$r\mu a_2 c_1 + c\mu c_1 - s\mu c_0 a_1 - r\mu a_0 c_1 - 2s\mu a_2 d_1 + r\mu a_2 d_1 + 2s\mu a_2 c_1 + 8q\mu^3 c_1 = 0,$$

$$-s\mu a_1 d_1 + s\mu b_1 c_1 - r\mu b_1 c_1 + r\mu a_1 c_1 + r\mu a_1 d_1 + s\mu a_1 c_1 + 2s\mu c_0 a_2 = 0,$$

$$-c\mu c_1 - 2q\mu^3 d_1 + 2s\mu b_2 c_1 - c\mu d_1 + r\mu a_0 d_1 + s\mu c_0 a_1 - r\mu b_2 c_1 + r\mu a_0 c_1 + s\mu c_0 b_1$$

$$-2q\mu^3 c_1 + 2s\mu a_2 d_1 - r\mu a_2 d_1 = 0,$$

$$s\mu b_1 d_1 + r\mu b_1 c_1 + s\mu a_1 d_1 + r\mu b_1 d_1 - s\mu b_1 c_1 + 2s\mu c_0 b_2 - r\mu a_1 d_1 = 0,$$

$$r\mu b_2 c_1 - r\mu a_0 d_1 + c\mu d_1 + r\mu b_2 d_1 + 2s\mu b_2 d_1 - s\mu c_0 b_1 + 8q\mu^3 d_1 - 2s\mu b_2 c_1 = 0,$$

$$-2s\mu c_0 b_2 - r\mu b_1 d_1 - s\mu b_1 d_1 = 0,$$

$$-r\mu b_2 d_1 - 2s\mu b_2 d_1 - 6q\mu^3 d_1 = 0.$$

The above equations are cumbersome to solve. Using a modern computer algebra system, say Maple, we obtain the four sets of solutions:

$$a_0 = \frac{c + 2q\mu^2}{r}, \quad a_1 = 0, \quad a_2 = \frac{-6q\mu^2}{r+2s}, \quad b_1 = 0,$$
$$b_2 = 0, \quad c_0 = 0, \quad c_1 = \sqrt[2]{-\frac{3qc}{2sp+rp}}\mu = d_1 = 0,$$

$$a_0 = \frac{c + 2q\mu^2}{r}, \quad a_1 = 0, \quad a_2 = 0, \quad b_1 = 0,$$
$$b_2 = \frac{-6q\mu^2}{r+2s}, \quad c_0 = 0, \quad c_1 = 0, \quad d_1 = \sqrt[2]{-\frac{3qc}{2sp+rp}}\mu,$$

$$a_0 = \frac{2sc + 16sq\mu^2 + rc - 4q\mu^2 r}{(r+2s)r}, \quad a_1 = 0, \quad a_2 = \frac{-6q\mu^2}{r+2s},$$
$$b_1 = 0, \quad b_2 = \frac{-6q\mu^2}{r+2s}, \quad c_0 = 0,$$
$$c_1 = \sqrt[2]{-\frac{3qc}{2sp+rp}}\mu, \quad d_1 = \sqrt[2]{-\frac{3qc}{2sp+rp}}\mu,$$

$$a_0 = \frac{2sc - 8sq\mu^2 + rc + 8q\mu^2 r}{(r+2s)r}, \quad a_1 = 0, \quad a_2 = \frac{-6q\mu^2}{r+2s},$$
$$b_1 = 0, \quad b_2 = \frac{-6q\mu^2}{r+2s}, \quad c_0 = 0,$$
$$c_1 = -\sqrt[2]{-\frac{3qc}{2sp+rp}}\mu, \quad d_1 = \sqrt[2]{-\frac{3qc}{2sp+rp}}\mu,$$

In view of this, we obtain the following solitons and kink solutions:

$$u_1(x,t)=\frac{c+2q\mu^2}{r}-\frac{6q\mu^2}{r+2s}\mathrm{Tanh}^2(\mu(x-ct)),$$

$$v_1(x,t)=\sqrt[2]{-\frac{3qc}{2sp+rp}}\mu\tanh(\mu(x-ct)),$$

$$u_2(x,t)=\frac{c+2q\mu^2}{r}-\frac{6q\mu^2}{r+2s}coth^2(\mu(x-ct)),$$

$$u_3(x,t)=\frac{2sc+16sq\mu^2+rc-4q\mu^2r}{(r+2s)r}-\frac{6q\mu^2}{r+2s}tanh^2(\mu(x-ct))$$

$$v_2(x,t)=\sqrt[2]{-\frac{3qc}{2sp+rp}}\mu\coth(\mu(x-ct)),$$

$$-\frac{6q\mu^2}{r+2s}\coth^2(\mu(x-ct)),$$

$$v_3(x,t)=\sqrt[2]{-\frac{3qc}{2sp+rp}}\mu\mathrm{Tanh}(\mu(x-ct))+\sqrt[2]{-\frac{3qc}{2sp+rp}}\mu\coth(\mu(x-ct)),$$

$$u_4(x,t)=\frac{2sc-8sq\mu^2+rc+8q\mu^2r}{(r+2s)r}-\frac{6q\mu^2}{r+2s}tanh^2(\mu(x-ct))-\frac{6q\mu^2}{r+2s}coth^2(\mu(x-ct)),$$

$$v_4(x,t)=-\sqrt[2]{-\frac{3qc}{2sp+rp}}\mu\tanh(\mu(x-ct))+\sqrt[2]{-\frac{3qc}{2sp+rp}}\mu\coth(\mu(x-ct)).$$

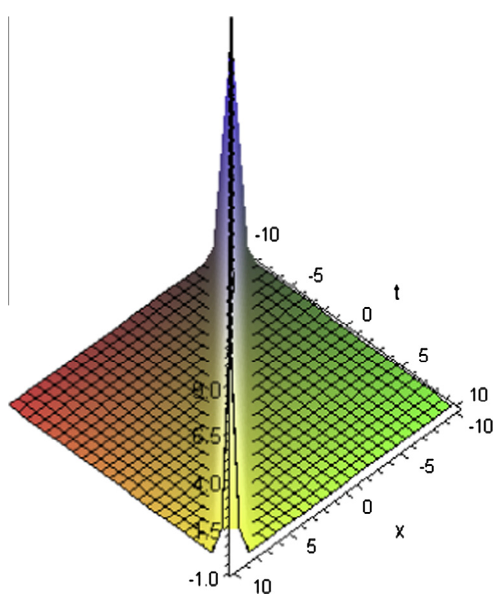

Figure 3 The soliton solution of $u_1(x,t)$.

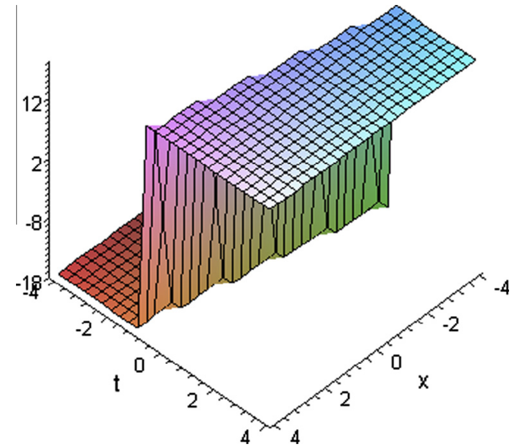

Figure 4 The kink solution of $v_1(x,t)$.

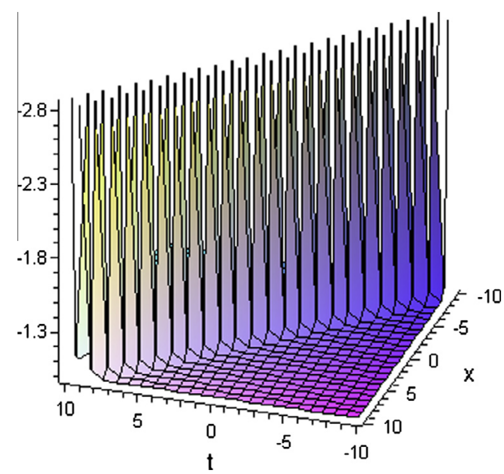

Figure 5 The soliton solution of $u_2(x,t)$.

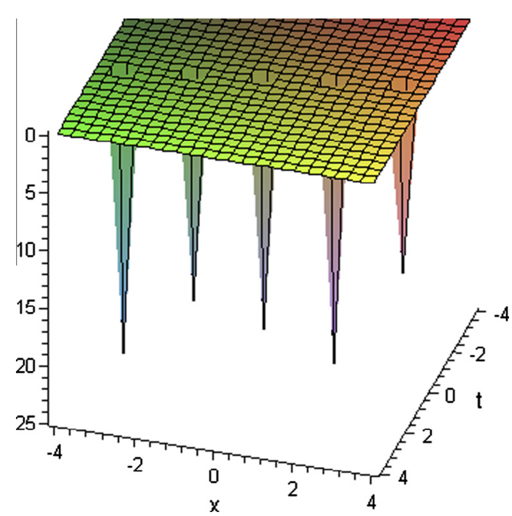

Figure 6 The soliton solution of $v_2(x,t)$.

The solitary wave and behavior of the solutions $u_1(x,t)$, $v_1(x,t)$ and $u_2(x,t)$, $v_2(x,t)$ are shown in Figs. 3–6 respectively.

6. Conclusion

Tanh and Extended Tanh method is quite efficient and practically well suited for use in calculating traveling wave solutions for Drinefel'd–Sokolov–Wilson Equation. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

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