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**ORIGINAL RESEARCH** **Open Access**

# Properties of neighborhood for certain classes associated with complex order and *m*-*q*-*p*-valent functions with higher order

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**Abstract**

In this paper, by using $q$-calculus (Jackson's $q$-derivative) $D_{q,p}$ we defined new operator $D^{\eta}_{\lambda,q,p}f^{(m)}(z)$. After that, we used this operator to introduce two new subclasses of multivalent analytic functions with complex order. Also, we obtained coefficients estimates and consequent inclusion relationships involving the $N^{p,q}_{j,\delta,m}(f)$-neighborhood of these classes

**Keywords:** Complex order, Coefficient estimates, Neighborhood, $p$-valent

**Mathematics Subject Classification:** 30C45

## Introduction

Let $\mathcal{A}_j(p)$ denote the class of functions in the form:

$$f(z) = z^p + \sum_{k=j+p}^{\infty} a_k z^k \quad (j, p \in \mathbb{N} = \{1, 2, \ldots\}), \tag{1.1}$$

which are analytic and $p$-valent in the open is open unit disk$\mathbb{U} = \{z : |z| < 1\}$.We note that $\mathcal{A}_1(p) = \mathcal{A}(p)$(see [13, 30]) and $\mathcal{A}_1(1) = \mathcal{A}$. Also let $T(p, j)$ denote the subclass of $\mathcal{A}_j(p)$which can express in the form:

$$f(z) = z^p - \sum_{k=j+p}^{\infty} a_k z^k \quad (a_k \geq 0,\ j, p \in \mathbb{N}). \tag{1.2}$$

In recent years, the topic of $q$-calculus had attracted the attention of several researchers (see, for example, [2, 15, 16, 23, 34, 43–45]). Quantum calculus is the modern name for the investigation of calculus without limits. The quantum calculus or $q$-calculus began with Jackson in the early twentieth century, but this kind of calculus had already been worked out by Euler and Jacobi. In the general run, the $q$-calculus is used in various fields of Mathematics and Physics. Also, $q$-calculus appeared the connection between

Mathematics and Physics. It had a lot of applications in different mathematical areas such as number theory, combinatorics, orthogonal polynomials, basic hypergeometric functions and other sciences quantum theory, mechanics and the theory of relativity. Several convolutional and fractional calculus $q$-operators were defined by many researchers. The generalization of derivative and integral in $q$-calculus is known as $q$-analogue derivative and $q$-analogue integral. Recently, many authors used the $q$- analogue derivative and $q$-analogue integral to generalize many classes and many operators in Geometric Function Theory (see, for example, [14, 33, 40, 42]).

For a function $f(z) \in A(p)$ given by (1.1) (with $j = 1$) Jackson's $q$-derivative (or $q$-difference) $D_{q,p}\,(0 < q < 1)$is defined as follows:

$$D_{q,p}f(z) = \begin{cases} \frac{f(z)-f(qz)}{(1-q)z} & for\ z \neq 0, \\ f'(0) & for\ z = 0, \end{cases} (f \in \mathcal{A}(p)) \tag{1.3}$$

provided that $f'(0)$exists. From (1.1) (with $j = 1$) and (1.3), we deduce that

$$D_{q,p}f(z) = [p]_q z^{p-1} + \sum_{k=p+1}^{\infty} [k]_q a_k z^{k-1}, \tag{1.4}$$

such that $q$-integer number $k\,[k]_q$ is defined by

$$[k]_q = \frac{1-q^k}{1-q} = 1 + \sum_{k=1}^{n-1} q^k,\ 0 < q < 1,\ [0]_q = 0. \tag{1.5}$$

We observe that

$$\lim_{q \to 1^-} D_{q,p}f(z) = \lim_{q \to 1^-} \frac{f(z) - f(qz)}{(1-q)z} = f'(z),$$

for a function $f$which is differentiable in a given subset of $\mathbb{C}$. For all $f(z) \in T(p,j)$,we find ( see [25] )

$$f^{(m)}(z) = \theta(p,m) z^{p-m} - \sum_{k=j+p}^{\infty} \theta(k,m) a_k z^{k-m}\ (p,j \in \mathbb{N},\ m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\},\ p > m), \tag{1.6}$$

where $\theta(p,m)$is defined by

$$\theta(p,m) = \frac{p!}{(p-m)!} = \begin{cases} 1 & , \quad m = 0, \\ p(p-1)\ldots(p-m+1), & m \neq 0. \end{cases} \tag{1.7}$$

For $f \in T(p,j)$, we introduce the operator $D^n_{\lambda,q,p} f^{(m)} : T(p,j) \to T(p,j)$ $(\lambda \geq 0,\ n, m \in \mathbb{N}_0, 0 < q < 1,\ j, p \in \mathbb{N},\ p > m)$as follows:

$$D^{n}_{\lambda,q,p}f^{(m)}(z) = f^{(m)}(z),$$

$$D^{1}_{\lambda,q,p}f^{(m)}(z) = D_{\lambda,q,p}\left(D^{0}_{\lambda,q,p}f^{(m)}(z)\right) = (1-\lambda)f^{(m)}(z) + \frac{\lambda}{[p-m]_q}zD_{q,p}(f^{(m)}(z))$$

$$= \theta(p,m)z^{p-m} - \sum_{k=j+p}^{\infty}\theta(k,m)\left[\frac{[p-m]_q+\lambda\left([k-m]_q-[p-m]_q\right)}{[p-m]_q}\right]a_k z^{k-m},$$

$$D^{2}_{\lambda,q,p}f^{(m)}(z) = D_{\lambda,q,p}\left(D^{1}_{\lambda,q,p}f^{(m)}(z)\right) = (1-\lambda)D^{1}_{\lambda,q,p}f^{(m)}(z) + \frac{\lambda}{[p-m]_q}zD_{q,p}(D^{1}_{\lambda,q,p}f^{(m)}(z))$$

$$= \theta(p,m)z^{p-m} - \sum_{k=j+p}^{\infty}\theta(k,m)\left[\frac{[p-m]_q+\lambda\left([k-m]_q-[p-m]_q\right)}{[p-m]_q}\right]^2 a_k z^{k-m} \vdots$$

$$\begin{aligned} D^{n}_{\lambda,q,p}f^{(m)}(z) &= D_{\lambda,q,p}\left(D^{n-1}_{\lambda,q,p}f^{(m)}(z)\right) \\ &= (1-\lambda)D^{n-1}_{\lambda,q,p}f^{(m)}(z) + \frac{\lambda}{[p-m]_q}zD_{q,p}(D^{n-1}_{\lambda,q,p}f^{(m)}(z))(n\in\mathbb{N}) \end{aligned} \tag{1.8}$$

From (1.2) and (1.8), we can obtain

$$= \theta(p,m)z^{p-m} - \sum_{k=j+p}^{\infty}\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k z^{k-m},$$

where

$$\Psi^{n,m}_{q,p}(k,\lambda) = \left[\frac{[p-m]_q+\lambda\left([k-m]_q-[p-m]_q\right)}{[p-m]_q}\right]^n \tag{1.9}$$
$$(\lambda \geq 0,\ m,n\in\mathbb{N}_0, 0<q<1,\ j,p\in\mathbb{N}, p>m).$$

We note that

(1) $D^{n}_{\lambda,q,p}f^{(0)}(z) = I^{n}_{q,p}(\lambda)f(z)$,(Aouf and Madian [14], with $\varrho = 0$]);
(2) $\lim_{q\to 1^-} D^{n}_{1,q,p}f^{(m)}(z) = D^{n}_{p}f^{(m)}(z)$,(Aouf [8, 9]);
(3) $\lim_{q\to 1^-} D^{n}_{1,q,p}f^{(0)}(z) = D^{n}_{p}f(z)$(see [11, 18], Cătaş [24], with $l = 0$] and [37]);
(4) $\lim_{q\to 1^-} D^{n}_{1,q,1}f^{(0)}(z) = D^{n}f(z)$(see ([26, 27], with $l = 0$));
(5) $\lim_{q\to 1^-} D^{n}_{\lambda,q,1}f^{(0)}(z) = D^{n}_{\lambda}f(z)$(see [1, 17, 21]);
(6) $D^{n}_{1,q,1}f^{(0)}(z) = D^{n}_{q}f(z)$(see [32] ), $\lim_{q\to 1^-} D^{n}_{q}f(z) = D^{n}f(z)$(see Sălă gean [39] see also [10, 12]);
(7) $\lim_{q\to 1^-} D^{n}_{1,q,p}f^{(m)}(z) = D^{n}_{p}f^{(m)}(z)$(see Aouf et al. [22]);
(8) $\lim_{q\to 1^-} D^{n}_{\lambda,q,p}f^{(m)}(z) = I^{n}_{\lambda,p}f^{(m)}(z)$

$$= \left\{ \begin{array}{c} f\in T(p,j) : I^{n}_{\lambda,p}f^{(m)}(z) = \theta(p,m)z^{p-m} - \sum\limits_{k=j+p}^{\infty}\left(\left[\frac{p-m+\lambda(k-p)}{p-m}\right]\right)^n\theta(k,m)a_k z^{k-m}, \\ n,m\in\mathbb{N}_0,\ j,p\in\mathbb{N}, \lambda\geq 0,\ p>m \end{array} \right\};$$

(9) $D^{n}_{1,q,p}f^{(m)}(z) = I^{n}_{q,p}f^{(m)}(z)$

$$= \left\{ \begin{array}{c} f \in T(p,j) : I^n_{q,p} f^{(m)}(z) = \theta(p,m) z^{p-m} - \sum_{k=j+p}^{\infty} \left( \frac{[k-m]_q}{[p-m]_q} \right)^n \theta(k,m) a_k z^{k-m}, \\ n,m \in \mathbb{N}_0, 0 < q < 1,\ j,p \in \mathbb{N}, p > m \end{array} \right\};$$

(10) $D^n_{1,q,p} f^{(0)}(z) = D^n_{q,p} f(z)$

$$= \left\{ \begin{array}{c} f \in T(p,j) : D^n_{q,p} f(z) = z^p - \sum_{k=j+p}^{\infty} \left( \frac{[k]_q}{[p]_q} \right)^n a_k z^k, \\ n \in \mathbb{N}_0,\ j,p \in \mathbb{N},\ 0 < q < 1 \end{array} \right\}.$$

Now by using $D^n_{\lambda,q,p} f^{(m)}(z)$,we defined the classes $F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$and $G^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$in below definitions:

**Definition 1** *Assume* $f(z) \in T(p,j)$,*then* $f(z) \in F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$*if it satisfies the following inequality:*

$$\left| \frac{1}{b} \left[ \frac{(1-\sigma) z D_{q,p}(D^n_{\lambda,q,p} f^{(m)}(z)) + \sigma z D_{q,p}(z D_{q,p}(D^n_{\lambda,q,p} f^{(m)}(z)))}{(1-\sigma) D^n_{\lambda,q,p} f^{(m)}(z) + \sigma z D_{q,p}(D^n_{\lambda,q,p} f^{(m)}(z))} - [p-m]_q \right] \right| < \beta$$

$$(b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1, \lambda \geq 0,\ 0 \leq \sigma \leq 1,\ 0 < \beta \leq 1, p > m). \tag{1.10}$$

We observe that:

(1) $\lim_{q \to 1^-} F^{n,m}_{q,p}(j,1,\sigma,b,\beta) = S_j(n,p,m,\sigma,b,\beta)$ see Aouf et al. [22];
(2) $F^{n,0}_{q,p}(j,\lambda,\sigma,b,\beta) = S^n_q(j,\lambda,p,\sigma,b,\beta)$see Aouf and madian [14], with $\varrho = 0$];
(3) $\lim_{q \to 1^-} F^{0,0}_{q,p}(j,\lambda,\sigma,b,\beta) = S_j(p,\sigma,b,\beta)$ see Aouf and Mostafa [19], with $b_k = 1$;
(4) $\lim_{q \to 1^-} F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta) = F^{n,m}_p(j,\lambda,\sigma,b,\beta)$

$$\Big\{ f \in T(p,j) : \left| \frac{1}{b} \left[ \frac{z(I^n_{\lambda,p} f^{(m)}(z))' + \sigma z^2 (I^n_{\lambda,p} f^{(m)}(z))''}{(1-\sigma) I^n_{\lambda,p} f^{(m)}(z) + \sigma z (I^n_{\lambda,p} f^{(m)}(z))'} - (p-m) \right] \right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N}, \lambda \geq 0,\ 0 \leq \sigma \leq 1,\ 0 < \beta \leq 1, p > m \Big\};$$

(5) $F^{n,m}_{q,p}(j,1,\sigma,b,\beta) = F^{n,m}_{q,p}(j,\sigma,b,\beta)$

$$\Big\{ f \in T(p,j) : \left| \frac{1}{b} \left[ \frac{(1-\sigma) z D_{q,p}(I^n_{q,p} f^{(m)}(z)) + \sigma z D_{q,p}(z D_{q,p}(I^n_{q,p} f^{(m)}(z)))}{(1-\sigma) I^n_{q,p} f^{(m)}(z) + \sigma z D_{q,p}(I^n_{q,p} f^{(m)}(z))} - [p-m]_q \right] \right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1,\ 0 \leq \sigma \leq 1,\ 0 < \beta \leq 1, p > m \Big\};$$

(6) $F^{n,0}_{q,p}(j,1,\sigma,b,\beta) = F^n_{q,p}(j,\sigma,b,\beta)$

$$\Big\{ f \in T(p,j) : \left| \frac{1}{b} \left[ \frac{(1-\sigma) z D_{q,p}(D^n_{q,p} f(z)) + \sigma z D_{q,p}(z D_{q,p}(D^n_{q,p} f(z)))}{(1-\sigma) D^n_{q,p} f(z) + \sigma z D_{q,p}(D^n_{q,p} f(z))} - [p]_q \right] \right| < \beta,$$
$$b \in \mathbb{C}^*, n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1,\ 0 \leq \sigma \leq 1,\ 0 < \beta \leq 1 \Big\};$$

(7) $\lim_{q \to 1^-} F^{n,m}_{q,p}(j,\lambda,0,b,\beta) = SF^{n,m}_p(j,\lambda,\sigma,b,\beta)$

$$\Big\{f \in T(p,j) : \left|\frac{1}{b}\left[\frac{z(I^n_{\lambda,p}f^{(m)}(z))'}{I^n_{\lambda,p}f^{(m)}(z)} - (p-m)\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N}, \lambda \geq 0,\ 0 < \beta \leq 1, p > m\Big\};$$

(8) $F^{n,m}_{q,p}(j,1,0,b,\beta) = SF^{n,m}_{q,p}(j,b,\beta)$

$$\Big\{f \in T(p,j) : \left|\frac{1}{b}\left[\frac{zD_{q,p}(I^n_{q,p}f^{(m)}(z))}{I^n_{q,p}f^{(m)}(z)} - [p-m]_q\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1,\ 0 < \beta \leq 1, p > m\Big\};$$

(9) $F^{n,m}_{q,p}(j,\lambda,0,b,\beta) = S^{n,m}_{q,p}(j,\lambda,b,\beta)$

$$\Big\{f \in T(p,j) : \left|\frac{1}{b}\left[\frac{zD_{q,p}(D^n_{\lambda,q,p}f^{(m)}(z))}{D^n_{\lambda,q,p}f^{(m)}(z)} - [p-m]_q\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1, \lambda \geq 0,\ 0 < \beta \leq 1, p > m\Big\};$$

(10) $\lim_{q\to 1^-} F^{n,m}_{q,p}(j,\lambda,1,b,\beta) = KF^{n,m}_{p}(j,\lambda,b,\beta)$

$$\Big\{f \in T(p,j) : \left|\frac{1}{b}\left[1 + \frac{z(I^n_{\lambda,p}f^{(m)}(z))''}{(I^n_{\lambda,p}f^{(m)}(z))'} - (p-m)\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N}, \lambda \geq 0,\ 0 < \beta \leq 1, p > m\Big\};$$

(11) $F^{n,m}_{q,p}(j,1,1,b,\beta) = KF^{n,m}_{q,p}(j,b,\beta)$

$$\Big\{f \in T(p,j) : \left|\frac{1}{b}\left[\frac{D_{q,p}(zD_{q,p}(I^n_{q,p}f^{(m)}(z)))}{D_{q,p}(I^n_{q,p}f^{(m)}(z))} - [p-m]_q\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1,\ 0 < \beta \leq 1, p > m\Big\};$$

(12) $F^{n,m}_{q,p}(j,\lambda,1,b,\beta) = K^{n,m}_{q,p}(j,\lambda,b,\beta)$

$$\Big\{f \in T(p,j) : \left|\frac{1}{b}\left[\frac{D_{q,p}(zD_{q,p}(D^n_{\lambda,q,p}f^{(m)}(z)))}{D_{q,p}(D^n_{\lambda,q,p}f^{(m)}(z))} - [p-m]_q\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1, \lambda \geq 0,\ 0 < \beta \leq 1, p > m\Big\}.$$

**Definition 2** *Assume* $f(z) \in T(p,j)$,*if it satisfies* (1.11), *then* $f(z) \in G^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$

$$\left|\frac{1}{b}\left\{(1-\sigma)\frac{D^n_{\lambda,q,p}f^{(m)}(z)}{z^{p-m}} + \sigma\frac{D_{q,p}(D^n_{\lambda,q,p}f^{(m)}(z))}{[p-m]_q z^{p-m-1}} - \theta(p,m)\right\}\right| < \beta$$

$$(b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1, \lambda \geq 0,\ 0 \leq \sigma \leq 1,\ 0 < \beta \leq 1, p > m). \tag{1.11}$$

We note that:

(1) $\lim_{q\to 1^-} G^{n,m}_{q,p}(j,1,\sigma,b,\beta) = K_j(n,p,m,\sigma,b,\beta)$ see Aouf et al. [22];
(2) $G^{n,0}_{q,p}(j,\lambda,\sigma,b,\beta) = K^n_q(j,\lambda,p,\sigma,b,\beta)$see Aouf and Madian [14], with $\varrho = 0$];
(3) $\lim_{q\to 1^-} G^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta) = G^{n,m}_{p}(j,\lambda,\sigma,b,\beta)$

$$\Big\{f \in T(p,j): \left|\frac{1}{b}\left[(1-\sigma)\frac{I^{n}_{\lambda,p}f^{(m)}(z)}{z^{p-m}} + \sigma\frac{(I^{n}_{\lambda,p}f^{(m)}(z))'}{(p-m)z^{p-m-1}} - \theta(p,m)\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N}, \lambda \geq 0,\ 0 \leq \sigma \leq 1,\ 0 < \beta \leq 1, p > m\Big\};$$

(4) $G^{n,m}_{q,p}(j,1,\sigma,b,\beta) = G^{n,m}_{q,p}(j,\sigma,b,\beta)$

$$\Big\{f \in T(p,j): \left|\frac{1}{b}\left\{(1-\sigma)\frac{I^{n}_{q,p}f^{(m)}(z)}{z^{p-m}} + \sigma\frac{D_{q,p}(I^{n}_{q,p}f^{(m)}(z))}{[p-m]_q z^{p-m-1}} - \theta(p,m)\right\}\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1, 0 \leq \sigma \leq 1,\ 0 < \beta \leq 1, p > m\Big\};$$

(5) $G^{n,0}_{q,p}(j,1,\sigma,b,\beta) = G^{n}_{q,p}(j,\sigma,b,\beta)$

$$\Big\{f \in T(p,j): \left|\frac{1}{b}\left\{(1-\sigma)\frac{D^{n}_{q,p}f(z)}{z^{p}} + \sigma\frac{D_{q,p}(D^{n}_{q,p}f(z))}{[p]_q z^{p-1}} - 1\right\}\right| < \beta,$$
$$b \in \mathbb{C}^*, n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1,\ 0 \leq \sigma \leq 1,\ 0 < \beta \leq 1\Big\};$$

(6) $\lim_{q\to 1^-} G^{n,m}_{q,p}(j,\lambda,1,b,\beta) = L^{n,m}_{p}(j,\lambda,b,\beta)$

$$\Big\{f \in T(p,j): \left|\frac{1}{b}\left[\frac{(I^{n}_{\lambda,p}f^{(m)}(z))'}{(p-m)z^{p-m-1}} - \theta(p,m)\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N}, \lambda \geq 0,\ 0 < \beta \leq 1, p > m\Big\};$$

(7) $G^{n,m}_{q,p}(j,1,1,b,\beta) = M^{n,m}_{q,p}(j,b,\beta)$

$$\Big\{f \in T(p,j): \left|\frac{1}{b}\left[\frac{D_{q,p}(I^{n}_{q,p}f^{(m)}(z))}{[p-m]_q z^{p-m-1}} - \theta(p,m)\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1,\ 0 < \beta \leq 1, p > m\Big\};$$

(8) $G^{n,0}_{q,p}(j,1,1,b,\beta) = G^{n}_{q,p}(j,b,\beta)$

$$\Big\{f \in T(p,j): \left|\frac{1}{b}\left[\frac{D_{q,p}(D^{n}_{q,p}f(z))}{[p]_q z^{p-1}} - 1\right]\right| < \beta$$
$$b \in \mathbb{C}^*, n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1,\ 0 \leq \sigma \leq 1,\ 0 < \beta \leq 1\Big\};$$

(9) $\lim_{q\to 1^-} G^{n,m}_{q,p}(j,\lambda,0,b,\beta) = O^{n,m}_{p}(j,\lambda,b,\beta)$

$$\Big\{f \in T(p,j): \left|\frac{1}{b}\left[\frac{I^{n}_{\lambda,p}f^{(m)}(z)}{z^{p-m}} - \theta(p,m)\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N}, \lambda \geq 0,\ 0 < \beta \leq 1, p > m\Big\};$$

(10) $G^{n,m}_{q,p}(j,1,0,b,\beta) = R^{n,m}_{q,p}(j,b,\beta)$

$$\Big\{f \in T(p,j): \left|\frac{1}{b}\left[\frac{I^{n}_{q,p}f^{(m)}(z)}{z^{p-m}} - \theta(p,m)\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1,\ 0 < \beta \leq 1, p > m\Big\};$$

(11) $G^{n,0}_{q,p}(j,1,0,b,\beta) = P^{n}_{q,p}(j,b,\beta)$

$$\Big\{f \in T(p,j) : \left|\frac{1}{b}\left[\frac{D^n_{q,p}f(z)}{z^p} - 1\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, n \in \mathbb{N}_0, p,j \in \mathbb{N},\ 0 < q < 1,\ 0 < \beta \le 1\Big\};$$

(12) $G^{n,m}_{q,p}(j,\lambda,1,b,\beta) = G^{n,m}_{q,p}(j,\lambda,b,\beta)$

$$\Big\{f \in T(p,j) : \left|\frac{1}{b}\left[\frac{D_{q,p}(D^n_{,\lambda,q,p}f^{(m)}(z))}{[p-m]_q z^{p-m-1}} - \theta(p,m)\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N}, \lambda \ge 0,\ 0 < q < 1,\ 0 < \beta \le 1, p > m\Big\};$$

(13) $G^{n,m}_{q,p}(j,\lambda,0,b,\beta) = GL^{n,m}_p(j,\lambda,b,\beta)$

$$\Big\{f \in T(p,j) : \left|\frac{1}{b}\left[\frac{I^n_{\lambda,p}f^{(m)}(z)}{z^{p-m}} - \theta(p,m)\right]\right| < \beta,$$
$$b \in \mathbb{C}^*, m,n \in \mathbb{N}_0, p,j \in \mathbb{N}, \lambda \ge 0,\ 0 < \beta \le 1, p > m\Big\}.$$

Now, as a results of Authors articles see ([3–7] [29, 31, 34, 35, 38]), we define the neighborhood $(j,\delta)$for $f \in T(p,j)$by

$$N^p_{j,\delta}(f) = \Big\{g : g \in T(p,j),\ g(z) = z^p - \sum_{k=j+p}^{\infty} b_k z^k \text{ and } \sum_{k=j+p}^{\infty} k|a_k - b_k| \le \delta\Big\}. \tag{1.12}$$

In specially, if

$$h(z) = z^p\ (p \in \mathbb{N}), \tag{1.13}$$

we obtain

$$N^p_{j,\delta}(h) = \left\{g : g \in T(p,j),\ g(z) = z^p - \sum_{k=j+p}^{\infty} b_k z^k \text{ and } \sum_{k=j+p}^{\infty} k|b_k| \le \delta\right\}. \tag{1.14}$$

Now, we define the $(q,j,\delta,m)$—neighborhood for $f \in T(p,j)$ by

$$N^{p,q}_{j,\delta,m}(f) = \Big\{g : g \in T(p,j),\ g(z) = z^p - \sum_{k=j+p}^{\infty} b_k z^k \text{ and } \sum_{k=j+p}^{\infty} [k-m]_q|a_k - b_k| \le \delta\Big\}. \tag{1.15}$$

In particular, if $h(z)$given by (1.12), we immediately have

$$N^{p,q}_{j,\delta,m}(h) = \Big\{g : g \in T(p,j),\ g(z) = z^p - \sum_{k=j+p}^{\infty} b_k z^k \text{ and } \sum_{k=j+p}^{\infty} [k-m]_q|b_k| \le \delta\Big\}. \tag{1.16}$$

We note that

(i) $N^{p,q}_{j,\delta,0}(f) = N^{p,q}_{j,\delta}(f)$and$N^{p,q}_{j,\delta,0}(h) = N^{p,q}_{j,\delta}(h)$(see [14, 20]);

(ii) $\lim_{q\to 1^-} N^{p,q}_{j,\delta,0}(f) = N^{p}_{j,\delta}(f)$ and $\lim_{q\to 1^-} N^{p,q}_{j,\delta,0}(h) = N^{p}_{j,\delta}(h)$ (see [20] and Aouf et al. [22] ).

## Preliminaries

On the other hand, we assume through the article that, $b \in \mathbb{C}^*,\ n,m \in \mathbb{N}_0,\ p,j \in \mathbb{N},\ \lambda \geq 0,\ 0 < q < 1,\ 0 \leq \sigma \leq 1$ $,0 < \beta \leq 1, p > m$ and $\Psi^{n,m}_{q,p}(k,\lambda)$ is given by (1.9). To prove the main outcomes in the article we need Lemmas 1 and 2 below.

**Lemma 1** *Let $f \in T(p,j)$ is given by (1.2), then $f \in F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$,*

*if and only if*

$$\sum_{k=j+p}^{\infty} ([k-m]_q+\beta\left|b\right|-[p-m]_q)[1+\sigma([k-m]_q-1)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k \leq\beta\left|b\right|\theta(p,m)[1+\sigma([p-m]_q-1)]. \tag{2.1}$$

***Proof***

*If $f \in F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$. Then we have*

$$\mathrm{Re}\left\{\frac{(1-\sigma)zD_{q,p}(D^{n}_{\lambda,q,p}f^{(m)}(z))+\sigma zD_{q,p}(zD_{q,p}(D^{n}_{\lambda,q,p}f^{(m)}(z)))}{(1-\sigma)D^{n}_{\lambda,q,p}f^{(m)}(z)+\sigma zD_{q,p}(D^{n}_{\lambda,q,p}f^{(m)}(z))} - [p-m]_q\right\} > -\beta\left|b\right|\ (z\in\mathbb{U}), \tag{2.2}$$

or, equivalently,

$$\mathrm{Re}\left\{\frac{-\sum\limits_{k=j+p}^{\infty}([k-m]_q-[p-m]_q)[1+\sigma([k-m]_q-1)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k\ z^{k-p}}{[1+\sigma([p-m]_q-1)]\theta(p,m)-\sum\limits_{k=j+p}^{\infty}[1+\sigma([k-m]_q-1)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k\ z^{k-p}}\right\} > -\beta\left|b\right|. \tag{2.3}$$

By setting $|z| = r(0 \leq r < 1)$ in (2.3), the term in the denominator of the left hand side of (2.3) is positive for $0 \leq r < 1$. Therefore, by Putting $r \longrightarrow 1$ through real values, (2.3) helps us to the desired assertion of Lemma 1.

Conversely, assume $|z| = 1$ and apply the hypothesis (2.1), from (2.3) we have

$$\left|\frac{(1-\sigma)zD_{q,p}(D^{n}_{\lambda,q,p}f^{(m)}(z))+\sigma zD_{q,p}(zD_{q,p}(D^{n}_{\lambda,q,p}f^{(m)}(z)))}{(1-\sigma)D^{n}_{\lambda,q,p}f^{(m)}(z)+\sigma zD_{q,p}(D^{n}_{\lambda,q,p}f^{(m)}(z))}-[p-m]_q\right|$$

$$=\left|\frac{\sum_{k=j+p}^{\infty}([k-m]_q-[p-m]_q)[1+\sigma([k-m]_q-1)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k\, z^{k-p}}{[1+\sigma([p-m]_q-1)]\theta(p,m)-\sum_{k=j+p}^{\infty}[1+\sigma([k-m]_q-1)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k\, z^{k-p}}\right|$$

$$\leq\frac{\sum_{k=j+p}^{\infty}([k-m]_q-[p-m]_q)[1+\sigma([k-m]_q-1)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k\, |z|^{k-p}}{[1+\sigma([p-m]_q-1)]\theta(p,m)-\sum_{k=j+p}^{\infty}[1+\sigma([k-m]_q-1)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k\, |z|^{k-p}}$$

$$\leq\frac{\sum_{k=j+p}^{\infty}([k-m]_q-[p-m]_q)[1+\sigma([k-m]_q-1)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k}{[1+\sigma([p-m]_q-1)]\theta(p,m)-\sum_{k=j+p}^{\infty}[1+\sigma([k-m]_q-1)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k}=\beta\left|b\right|.$$

So, we have $f(z)\in F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$by applying the maximum modulus theorem, which completes the proof of Lemma 1. □

***Remark 1***

*Letting $q\to 1^{-}$ and $n=m=0$ in Lemma 1, we obtain the result obtained by Aouf and Mostafa [19], Lemma 1, with $b_k=1$].*

The following lemma can be established similarly.

**Lemma 2** *Let $f\in T(p,j)$ is given by (1.2). Then$f\in G^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$,*

*if and only if*

$$\sum_{k=j+p}^{\infty}[[p-m]_q+\sigma([k-m]_q-[p-m]_q)]\theta(k,m)\Psi^{n,m}_{q,p}(k,\lambda)a_k\leq\beta\left|b\right|[p-m]_q. \quad (2.4)$$

## 3- Inclusion results

In this part, we showed inclusion relations for each of the classes $F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$and $G^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$including $(q,j,\delta,m)-$ neighborhood were defined by (1.15) and (1.16).

**Theorem 1** *Suppose$f\in T(p,j)$includes in $F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$, then*

$$F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)\subset N^{p,q}_{j,\eta,m}(h), \quad (3.1)$$

*since $h(z)$ is defined by (1.13) and $\eta$is given by*

$$\eta=\frac{[j+p-m]_q\beta\left|b\right|[1+\sigma([p-m]_q-1)]\theta(p,m)}{([j+p-m]_q+\beta\left|b\right|-[p-m]_q)[1+\sigma([j+p-m]_q-1)]\theta(j+p,m)\Psi^{n,m}_{q,p}(j+p,\lambda)}\ ([p-m]_q>\left|b\right|). \quad (3.2)$$

***Proof***

*Let* $f \in F_{q,p}^{n,m}(j,\lambda,\sigma,b,\beta)$*, then by using (2.1) of Lemma 1, we obtain*

$$([j+p-m]_q + \beta|b| - [p-m]_q)[1+\sigma([j+p-m]_q - 1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)\sum_{k=j+p}^{\infty} a_k$$
$$\leq \sum_{k=j+p}^{\infty}([k-m]_q + \beta|b| - [p-m]_q)[1+\sigma([k-m]_q - 1)]\theta(k,m)\Psi_{q,p}^{n,m}(k,\lambda)a_k$$
$$\leq \beta|b|[1+\sigma([p-m]_q - 1)]\theta(p,m), \tag{3.3}$$

which quickly gives

$$\sum_{k=j+p}^{\infty} a_k \leq \frac{\beta|b|[1+\sigma([p-m]_q-1)]\theta(p,m)}{([j+p-m]_q+\beta|b|-[p-m]_q)[1+\sigma([j+p-m]_q-1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)}. \tag{3.4}$$

Making use of (2.1) with (3.4), we obtain

$$[1+\sigma([j+p-m]_q - 1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)\sum_{k=j+p}^{\infty}[k-m]_q a_k \leq$$
$$\leq \beta|b|[1+\sigma([p-m]_q - 1)]\theta(p,m)$$
$$+([p-m]_q - \beta|b|)[1+\sigma([j+p-m]_q - 1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)\sum_{k=j+p}^{\infty} a_k$$
$$\leq \beta|b|[1+\sigma([p-m]_q - 1)]\theta(p,m) + \frac{([p-m]_q - \beta|b|)\beta|b|[1+\sigma([p-m]_q - 1)]\theta(p,m)}{([j+p-m]_q + \beta|b| - [p-m]_q)}$$
$$= \frac{[j+p-m]_q\beta|b|[1+\sigma([p-m]_q - 1)]\theta(p,m)}{[j+p-m]_q + \beta|b| - [p-m]_q}.$$

Hence

$$\sum_{k=j+p}^{\infty}[k-m]_q a_k \leq \frac{[j+p-m]_q\beta|b|[1+\sigma([p-m]_q-1)]\theta(p,m)}{([j+p-m]_q+\beta|b|-[p-m]_q)[1+\sigma([j+p-m]_q-1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)}$$
$$= \eta \; ([p-m]_q > |b|), \tag{3.5}$$

by means of (1.14), we obtained (3.1) which asserted by Theorem 1. □

***Remark 2***

*Letting* $q \to 1^-$ *and* $n = m = 0$*in Theorem 1, we obtain the result obtained by Aouf and Mostafa [19], Theorem 2, with* $b_k = 1$*].*

In a similar manner, we proved the following inclusion relationship by using (2.4) of Lemma 2 recompensed (2.1) of Lemma 1 on functions in $G_{q,p}^{n,m}(j,\lambda,\sigma,b,\beta)$.

**Theorem 2** *Assume* $f \in T(p,j)$*includes in* $G_{q,p}^{n,m}(j,\lambda,\sigma,b,\beta)$*, then*

$$G_{q,p}^{n,m}(j,\lambda,\sigma,b,\beta) \subset N_{j,\delta,m}^{p,q}(h), \tag{3.6}$$

*such that* $h(z)$ *is defined by* (1.13) *and* $\delta$ *is introduced by*

$$\delta = \frac{[j+p-m]_q \beta |b| [p-m]_q}{[[p-m]_q + \sigma([j+p-m]_q - [p-m]_q)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)} = . \tag{3.7}$$

## 4- Neighborhoods properties

In this section, we determine the neighborhood for each of the classes $F_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$ and $G_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$. If there exists a function $\rho(z) \in F_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$, satisfies (4.1), then $f(z) \in T(p,j)$ is said to be in the class $F_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$

$$\left|\frac{f(z)}{\rho(z)} - 1\right| < [p-m]_q - \gamma \quad (z \in \mathbb{U};\ 0 \le \gamma < [p-m]_q). \tag{4.1}$$

Analogously, if we find a function $\rho(z) \in G_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$ which the inequality (4.1) achieve, then we can say for $f(z) \in T(p,j)$, $f(z) \in G_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$.

**Theorem 3** *Let* $f \in T(p,j)$ *includes in* $F_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$ *and*

$$\gamma = [p-m]_q - \frac{\eta([j+p-m]_q + \beta|b| - [p-m]_q)[1+\sigma([j+p-m]_q - 1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)}{[j+p-m]_q\left\{([j+p-m]_q + \beta|b| - [p-m]_q)[1+\sigma([j+p-m]_q - 1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda) - \beta|b|[1+\sigma([p-m]_q - 1)]\right\}\theta(p,m)}, \tag{4.2}$$

*then*

$$N_{j,\eta,m}^{p,q}(h) \subset F_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta), \tag{4.3}$$

*where*

$$\eta \le [p-m]_q [j+p-m]_q \left\{1 - \frac{\beta|b|[1+\sigma([p-m]_q - 1)]\theta(p,m)}{([j+p-m]_q + \beta|b| - [p-m]_q)[1+\sigma([j+p-m]_q - 1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)}\right\}.$$

***Proof***

*Assume* $f \in N_{j,\eta,m}^{p,q}(h)$. *From (1.15) we find that*

$$\sum_{k=j+p}^{\infty} [k-m]_q |a_k - b_k| \le \eta, \tag{4.4}$$

which readily implies that

$$\sum_{k=j+p}^{\infty} |a_k - b_k| \le \frac{\eta}{[j+p-m]_q} \quad (p,\ j \in \mathbb{N}). \tag{4.5}$$

Next, since $\rho(z) \in F_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$, by using (3.4), we have

$$\sum_{k=j+p}^{\infty} b_k \le \frac{\beta|b|[1+\sigma([p-m]_q-1)]\theta(p,m)}{([j+p-m]_q+\beta|b|-[p-m]_q)[1+\sigma([j+p-m]_q-1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)}, \tag{4.6}$$

so that

$$\begin{aligned}\left|\frac{f(z)}{\rho(z)}-1\right| &\le \frac{\sum_{k=j+p}^{\infty}|a_k-b_k|}{1-\sum_{k=j+p}^{\infty}b_k}\\ &\le \frac{\eta([j+p-m]_q+\beta|b|-[p-m]_q)[1+\sigma([j+p-m]_q-1)]\theta(j+p,m)\Psi_{q,p}^{n,m}(j+p,\lambda)}{[j+p-m]_q\left\{([j+p-m]_q+\beta|b|-[p-m]_q)[1+\sigma([j+p-m]_q-1)]\Psi_{q,p}^{n,m}(j+p,\lambda)\theta(j+p,m)-\beta|b|[1+\sigma([p-m]_q-1)]\theta(p,m)\right\}}\\ &= [p-m]_q-\gamma,\end{aligned}$$

provided that $\gamma$ is given by (4.2) and by the above definition, $f \in F_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$, so the proof of Theorem 3 is finished. □

The proof of Theorem 4 below is similar to the proof of Theorem 3, we omit the details involved.

**Theorem 4** *Let $f \in T(p,j)$includes in $G_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$and*

$$\gamma = [p-m]_q - \frac{\delta[[p-m]_q+\sigma([j+p-m]_q-[p-m]_q)]\Psi_{q,p}^{n,m}(j+p,\lambda)\theta(j+p,m)}{[j+p-m]_q\left\{[[p-m]_q+\sigma([j+p-m]_q-[p-m]_q)]\Psi_{q,p}^{n,m}(j+p,\lambda)\theta(j+p,m)-\beta[p-m]_q|b|\right\}}, \tag{4.7}$$

*then*

$$N_{j,\delta,m}^{p,q}(h) \subset G_{q,p}^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta), \tag{4.8}$$

*where*

$$\delta \le [p-m]_q[j+p-m]_q\left\{1-\frac{\beta[p-m]_q|b|}{([p-m]_q+\sigma([j+p-m]_q-[p-m]_q))\Psi_{q,p}^{n,m}(j+p,\lambda)\theta(j+p,m)}\right\}. \tag{4.9}$$

***Remarks***

*(1) Taking $m=0$in our outcomes, we obtain the outcomes obtained by Aouf and Madian [14], with $\varrho=0$]; (2) Taking $q\to 1^-$ and $\lambda=1$in our outcomes, we obtain the outcomes obtained by Aouf et al. [16]; (3) Taking $q\to 1^-$and $n=0$in our outcomes, we obtain the outcomes obtained by El- El-Ashwah et al. [28], with $b_k=1$ and $m=0$]; (4) Taking $q\to 1^-$ in Theorems 1,2,3 and 4, respectively, we obtain new outcomes for the classes $F_p^{n,m}(j,\lambda,\sigma,b,\beta)$, $G_p^{n,m}(j,\lambda,\sigma,b,\beta)$, $F_p^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$and $G_p^{n,m(\gamma)}(j,\lambda,\sigma,b,\beta)$, respectively; (5) Taking (a) $\lambda=1$, (b) $\lambda=1$and $m=0$,(c) Letting $q\to 1^-$ and $\sigma=0$, (d)$\lambda=1$and $\sigma=0$,(E) $\sigma=0$,(F) Letting $q\to 1^-$ and $\sigma=1$,(G)$\lambda=1$and $\sigma=1$,(H) $\sigma=1$,in Lemma 1, Theorems 1 and 3, respectively, we get to new outcomes for the classes $F_{q,p}^{n,m}(j,\sigma,b,\beta)$, $F_{q,p}^{n}(j,\sigma,b,\beta)$, $SF_p^{n,m}(j,\lambda,\sigma,b,\beta)$, $SF_{q,p}^{n,m}(j,b,\beta)$, $S_{q,p}^{n,m}(j,\lambda,b,\beta)$, $KF_p^{n,m}(j,\lambda,b,\beta)$, $KF_{q,p}^{n,m}(j,b,\beta)$and$K_{q,p}^{n,m}(j,\lambda,b,\beta)$, respectively; (6) Taking (a) $\lambda=1$, (b) $\lambda=1$and $m=0$,(c) Letting $q\to 1^-$and $\sigma=1$,(d)$\lambda=1$and $\sigma=1$,(E) $\lambda=\sigma=1$and $m=0$,*

*(F) Letting* $q \to 1^-$ *and* $\sigma = 0$*,(G)*$\lambda = 1$*and* $\sigma = 0$*, (H)* $\lambda = 1$*and* $m = \sigma = 0$*,(I)* $\sigma = 1$*(J)* $\sigma = 0$*in Lemma 2, Theorems 2 and 4, respectively, we get to new outcomes for the classes* $G^{n,m}_{q,p}(j,\sigma,b,\beta)$, $G^{n}_{q,p}(j,\sigma,b,\beta)$, $L^{n,m}_{p}(j,\lambda,b,\beta)$, $M^{n,m}_{q,p}(j,b,\beta)$, $G^{n}_{q,p}(j,b,\beta)$, $O^{n,m}_{p}(j,\lambda,b,\beta)$,

$R^{n,m}_{q,p}(j,b,\beta)$, $P^{n}_{q,p}(j,b,\beta)$, $G^{n,m}_{q,p}(j,\lambda,b,\beta)$and $GL^{n,m}_{p}(j,\lambda,b,\beta)$,respectively.

## Conclusions

Throughout the paper, we defined new subclasses of complex order $F^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$and $G^{n,m}_{q,p}(j,\lambda,\sigma,b,\beta)$by using $D^{n}_{\lambda,q,p}f^{(m)}(z)$ operator. Also, we introduced coefficients estimates theorems and neighborhoods properties for this classes. This paper generalized many results for different authors. There was connection between q-analysis and (p,q)-analysis see Srivastava [41]. Srivastava [41], page 340] applied some obvious parametric, argument variations and considered $0 < q < p \le 1$, also translated the classical q-number $[n]_q$ to $[n]_{p,q}$as follows:

$$[n]_{p,q} = \begin{cases} \frac{p^n - q^n}{p-q} & if \ \ n \in \mathbb{N} \\ 0 & if \ \ n = 0 \end{cases} = p^{n-1}[n]_{\frac{q}{p}}.$$

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## Declarations

**Competing interests**
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