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# Coupled fixed point results for mappings without mixed monotone property in partially ordered G-metric spaces

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**Abstract** In this paper, we prove coupled fixed point results for mappings without mixed monotone property in partially ordered G-metric spaces. Also we showed that if $(X, G)$ is regular, these fixed point results holds.

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## 1. Introduction

In 2005, Mustafa and Sims introduced a new class of generalized metric spaces (see [10,11]), which are called G-metric spaces, as generalization of a metric space $(X, d)$ (see [2–4,6–8]). In [4], coupled fixed point results in partially ordered metric spaces where established by T. Gnana Bhaskar and V. Lakshmikantham. After the publication of this work, several coupled fixed point and coincidence point results have appeared in recent literatures. Works noted in [12] are some examples of these works.

In this work, we establish coupled fixed point results for mappings without mixed monotone property in partially ordered G-metric spaces. In recent years, coupled fixed point results for mappings without mixed monotone property have been studied in many papers (see [1,13,14]). Before stating and proving our results, we shall recall some preliminaries.

## 2. Preliminaries

We recall some basic definitions and results which we need in the sequel. For details on the following notations we refer to [11]. First we give the definition of a $G$-metric.

**Definition 1.1** [11]. Let $X$ be a nonempty set and let $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following properties:

(G1) $G(x, y, z) = 0$ if $x = y = z$,
(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,
(G3) $G(x, x, y) \leqslant G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,
(G4) $G(x, y, z) = G(x, y, z) = G(y, z, x) = \ldots$, symmetry in all three variables,
(G5) $G(x, y, z) \leqslant G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

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Then the function $G$ is called a generalized metric, or, more specifically, a $G$-metric on $X$, and the pair $(X, G)$ is called a G-metric space.

**Definition 1.2** [11]. Let $(X, G)$ be a $G$-metric space, and let $\{x_n\}$ be a sequence of points of $X$. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$, if $\lim_{n,m\to+\infty} G(x, x_n, x_m) = 0$, and we say that the sequence $\{x_n\}$ is G- convergent to $x$ or $\{x_n\}$ G-convergent to $x$.

Thus, $x_n \to x$ in a G-metric space (X,G) if for any $\epsilon > 0$, there exist $k \in \mathbb{N}$ such that $G(x, x_n, x_m) < \epsilon$ for all $m, n \geqslant k$.

We have the following useful result.

**Proposition 1.3** [11]. *Let (X,G) be a G-metric space. Then the following are equivalent:*

(1) $\{x_n\}$ *is G-convergent to* $x$;
(2) $G(x_n, x_n, x) \to 0$ *as* $n \to +\infty$;
(3) $G(x_n, x, x) \to 0$ *as* $n \to +\infty$;
(4) $G(x_n, x_m, x) \to 0$ *as* $n, m \to +\infty$.

**Definition 1.4** [11]. Let (X,G) be a G-metric space, a sequence $\{x_n\}$ is called G-Cauchy if for every $\epsilon > 0$, there is $k \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \epsilon$, for all $n, m, l \geqslant k$; that is $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to +\infty$.

**Proposition 1.5** [11]. *Let (X,G) be a G-metric space. Then the following are equivalent:*

(1) *the sequence* $\{x_n\}$ *is G-Cauchy;*
(2) *for every* $\epsilon > 0$, *there is* $k \in \mathbb{N}$ *such that* $G(x_n, x_m, x_m) < \epsilon$, *for all* $n, m \geqslant k$.

**Definition 1.6** [11]. Let (X,G) and $(X', G')$ be G-metric spaces, and let $f : (X, G) \to (X', G')$ be a function. Then $f$ is said to be G-continuous at a point $a \in X$ if and only if for every $\epsilon > 0$, there is $\delta > 0$ such that $x, y \in X$ and $G(a, x, y) < \delta$ implies $G'(f(a), f(x), f(y)) < \epsilon$. A function $f$ is G-continuous at X if and only if it is G-continuous at all $a \in X$.

**Definition 1.7** [11]. A G-metric space $(X, G)$ is called G-complete if every G-Cauchy sequence is G-convergent in $(X, G)$.

**Definition 1.8** [11]. Let $(X, G)$ be a G-metric space. A mapping $F : X \times X \to X$ is said to be continuous if for any two G-convergent sequences $x_n$ and $y_n$ converging to $x$ and $y$ respectively, $F(x_n, y_n)$ is G-convergent to $F(x, y)$.

Let $(X, \preceq)$ be a partially ordered set and $F : X \to X$ be a mapping. The mapping $F$ is said to be non-decreasing if for all $x, y \in X$, $x \preceq y$ implies $F(x) \preceq F(y)$. Similarly, $F$ is said to be non-increasing, if for all $x, y \in X$, $x \preceq y$ implies $F(x) \succeq F(y)$.

Bhaskar and Lakshmikantham [4] introduced the following notions of mixed monotone mapping and coupled fixed point.

**Definition 1.9.** Let $(X, \preceq)$ be a partially ordered set and $F : X \to X$. The mapping $F$ is said to have the mixed monotone property if $F$ is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument, that is, for all $x_1, x_2 \in X$, $x_1 \preceq x_2$ implies $F(x_1, y) \preceq F(x_2, y)$, for any $y \in X$ and for all $y_1, y_2 \in X$, $y_1 \preceq y_2$ implies $F(x, y_1) \succeq F(x, y_2)$, for any $x \in X$.

The concept of the mixed monotone property is generalized by Lakshmikantham and Ćirić [9] as follows.

**Definition 1.10.** Let $(X, \preceq)$ be a partially ordered set and $F : X \times X \to X$ and $g : X \to X$. The mapping $F$ is said to have the mixed $g$-monotone property if $F$ is monotone $g$-non-decreasing in its first argument and is monotone $g$-non-increasing in its second argument, that is, for all $x_1, x_2 \in X$, $g(x_1) \preceq g(x_2)$ implies $F(x_1, y) \preceq F(x_2; y)$, for any $y_2 \in X$ and for all $y_1, y_2 \in X$, $g(y_1) \preceq g(y_2)$ implies $F(x, y_1) \succeq F(x, y_2)$, for any $x \in X$.

**Definition 1.11** [5]. An element $(x, y) \in X \times X$ is called a coupled fixed point of a mapping $F : X \times X \to X$ if

$F(x, y) = x$ and $F(y, x) = y$.

**Definition 1.12** [9]. An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings $F : X \times X \to X$ and $g : X \to X$ if

$F(x, y) = gx$ and $F(y, x) = gy$.

**Definition 1.13** [5]. Let $(X, d)$ be a metric space and let $g : X \to X, F : X \times X \to X$. The mappings $g$ and $F$ are said to be compatible if

$$\lim_{n\to\infty} d(gF(x_n, y_n), F(gx_n, gy_n)) = 0 \text{ and}$$
$$\lim_{n\to\infty} d(gF(y_n, x_n), F(gy_n, gx_n)) = 0,$$

hold whenever $x_n$ and $y_n$ are sequences in $X$ such that $\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} gx_n$ and $\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} gy_n$.

If elements $x, y$ of a partially ordered set $(X, \preceq)$ are comparable (i.e. $x \preceq y$ or $y \preceq x$holds) we will write $x \asymp y$.

## 3. Main result

Now, we will prove our main result.

**Theorem 3.1.** *Let* $(X, \preceq)$ *be a partially ordered set, G be a G-metric on X such that* $(X, G)$ *is a complete G-metric space,* $g : X \to X$, $F : X \times X \to X$, *are continuous and* $g(X)$ *is closed. Suppose that the following hold:*

(*i*) $F(X \times X) \subset g(X)$, *g and F are compatible.*
(*ii*) *If* $x, y, u, v \in X$ *are such that* $gx \asymp F(x, y) = gu$ *then* $F(x, y) \asymp F(F(x, y), v)$.
(*iii*) *There exists* $x_0, y_0 \in X$ *such that* $gx_0 \asymp F(x_0, y_0)$ *and* $gy_0 \asymp F(y_0, x_0)$.
(*iv*) *There exists* $k \in [0, 1)$ *such that*

$$G(F(x, y), F(u, v), F(w, z)) \leqslant \frac{k}{2}[G(gx, gu, gw) + G(gy, gv, gz)], \qquad (1)$$

*for all* $x, y, u, v \in X$ *satisfying* $gx \asymp gu, gy \asymp gv$.

(v) *For all* $(x, y), (w, z) \in X \times X$, *there exists* $(u, v) \in X \times X$ *such that* $(F(u, v), F(v, u))$ *is comparable to both* $(F(x, y), F(y, x))$ *and* $(F(w, z), F(z, w))$.

Then there exist $(x, y) \in X \times X$ such that $gx = F(x, y)$ and $gy = F(y, x)$, that is, $g$ and $F$ have a unique coupled coincidence point.

**Proof.** Let $x_0, y_0 \in X$ such that $gx_0 \asymp F(x_0, y_0)$ and $gy_0 \asymp F(y_0, x_0)$. Since $F(X \times X) \subset g(X)$, we can choose $x_1, y_1 \in X$ such that $gx_1 = F(x_0, y_0)$ and $gy_1 = F(y_0, x_0)$. Again since $F(X \times X) \subset g(X)$, we can choose $x_2, y_2 \in X$ such that $gx_2 = F(x_1, y_1)$ and $gy_2 = F(y_1, x_1)$. Continuing this process, we can construct two sequence $(x_n)$ and $(y_n)$ in $X$ such that

$$gx_n = F(x_{n-1}, y_{n-1}) \text{ and } gy_n = F(y_{n-1}, x_{n-1}) \text{ for } n = 1, 2, \ldots \tag{2}$$

Therefore $gx_0 \asymp F(x_0, y_0) = gx_1$. Condition (ii) implies that $gx_1 = F(x_0, y_0) \asymp F(x_1, y_1) = gx_2$. Continuing this process, we have $gx_{n-1} \asymp gx_n$. Similarly, $gy_{n-1} \asymp gy_n$ for each $n \in \mathbb{N}$. Hence by using (1)

$$\begin{aligned} G(gx_n, gx_n, gx_{n+1}) &= G(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\ &\leqslant \frac{k}{2}[G(gx_{n-1}, gx_{n-1}, gx_n) \\ &\quad + G(gy_{n-1}, gy_{n-1}, gy_n)]. \\ G(gy_n, gy_n, gy_{n+1}) &= G(F(y_{n-1}, y_{n-1}, x_{n-1}), F(y_{n-1}, y_{n-1}, x_{n-1}), \\ &\quad (y_n, y_n, x_n)) \\ &\leqslant \frac{k}{2}[G(gy_{n-1}, gy_{n-1}, gy_n) \\ &\quad + G(gx_{n-1}, gx_{n-1}, gx_n)]. \end{aligned}$$

Hence

$$\begin{aligned} &G(gx_n, gx_n, gx_{n+1}) + G(gy_n, gy_n, gy_{n+1}) \\ &\quad \leqslant k[G(gy_{n-1}, gy_{n-1}, gy_n) + G(gx_{n-1}, gx_{n-1}, gx_n)] \end{aligned}$$

for each $n \in \mathbb{N}$. Then we have

$$\begin{aligned} &G(gx_n, gx_n, gx_{n+1}) + G(gy_n, gy_n, gy_{n+1}) \\ &\quad \leqslant k^n[G(gy_0, gy_0, gy_1) + G(gx_0, gx_0, gx_1)]. \end{aligned}$$

Therefore

$$G(gx_m, gx_m, gx_n) \leqslant \frac{k^m}{1-k}[G(gy_0, gy_0, gy_1) + G(gx_0, gx_0, gx_1)],$$

for $m, n \in \mathbb{N}, m < n$, And also

$$G(gy_m, gy_m, gy_n) \leqslant \frac{k^m}{1-k}[G(gy_0, gy_0, gy_1) + G(gx_0, gx_0, gx_1)].$$

Thus $(gx_n)$ and $(gy_n)$ are Cauchy sequences, since $g(x)$ is closed in a complete G-metric space, there exist $(x, y) \in g(X)$ such that

$$\lim_{n\to\infty} gx_n = \lim_{n\to\infty} F(x_n, y_n) = x \text{ and } \lim_{n\to\infty} gy_n = \lim_{n\to\infty} F(y_n, x_n) = y.$$

Condition $(i)$ implies that

$$\lim_{n\to\infty} G(gF(x_n, y_n), gF(x_n, y_n), F(gx_n, gy_n)) = 0,$$

and

$$\lim_{n\to\infty} G(gF(y_n, x_n), gF(y_n, x_n), F(gy_n, gx_n)) = 0.$$

Now suppose that $F$ is continuous. Using triangle inequality we get

$$\begin{aligned} G(gx, gx, F(gx_n, gy_n) &\leqslant G((gx, gx, gF(x_n, y_n)) \\ &\quad + G(gF(x_n, y_n), gF(x_n, y_n), F(gx_n, gy_n)). \end{aligned}$$

Now using continuity of $g$ and $F$, and letting $n \to \infty$ in (3) we get $G(gx, gx, F(x, y)) = 0$, i.e., $gx = F(x, y)$. In a similar way, $gy = F(y, x)$ is obtained. Thus we proved that $(x, y)$ is a coupled coincidence point of $F$ and $g$. Then the set of coupled coincidences is non-empty.

We shall show that if $(x, y)$ and $(w, z)$ are coupled coincidence points, that is, if $gx = F(x, y)$, $g(y) = F(y, x)$, $gw = F(w, z)$ and $gz = F(z, w)$, then

$$gx = gw \text{ and } gy = gz. \tag{3}$$

By assumption, there exists $(u, v) \in X \times X$ such that $(F(u, v), F(v, u))$ is comparable with $(F(x, y), F(y, x))$ and $(F(w, z), F(z, w))$. Put $u_0 = u, v_0 = v$ and choose $u_1, v_1 \in X$ such that $gu_1 = F(u_0, v_0)$ and $gv_1 = F(v_0, u_0)$. Then we can inductively define sequences $(gu_n)$ and $(gv_n)$ in $X$ by $gu_{n+1} = F(u_n, v_n)$ and $gv_{n+1} = F(v_n, u_n)$. Further, set $x_0 = x, y_0 = y$, $w_0 = w$ and $z_0 = z$, in the same way, define the sequences $(gx_n), (gy_n), (gw_n)$ and $(gz_n)$ satisfying $gx_{n+1} = F(x_n, y_n)$, $gy_{n+1} = F(y_n, x_n)$ and $gw_{n+1} = F(w_n, u_n), gz_{n+1} = F(z_n, w_n)$ for $n \in \mathbb{N}$; then we have $x_n = x, y_n = y$ and $w_n = w, z_n = z$, i,e.

$$\begin{aligned} &gx_n = F(x, y), \ gy_n = F(y, x) \text{ and } gw_n = F(w, z), \\ &gz_n = F(z, w) \text{ for } n \in \mathbb{N}. \end{aligned} \tag{4}$$

Since $(F(x, y), F(y, x)) = (gx, gy)$ and $(F(u, v), F(v, u)) = (gu_1, gv_1)$ are comparable, then $gx \asymp gu_1$, $gy \asymp gv_1$ and similarly $gx \asymp gu_n$, $gy \asymp gv_n$. Thus, from (1), we get

$$\begin{aligned} G(gx, gx, gu_{n+1}) &= G(F(x, y), F(x, y), F(u_n, v_n)) \\ &\leqslant \frac{k}{2}(G(gx, gx, gu_n) + G(gy, gy, gv_n)), \end{aligned} \tag{5}$$

and

$$\begin{aligned} G(gy, gy, gv_{n+1}) &= G(F(y, x), F(y, x), F(v_n, u_n)) \\ &\leqslant \frac{k}{2}(G(gy, gy, gv_n) + G(gx, gx, gu_n)). \end{aligned} \tag{6}$$

Adding, we get

$$\begin{aligned} &G(gx, gx, gu_{n+1}) + G(gv_{n+1}, gy, gy) \\ &\quad \leqslant k(G(gx, gx, gu_n) + G(gv_n, gy, gy) \end{aligned}$$

by induction

$$\begin{aligned} &G(gx, gx, gu_{n+1}) + G(gv_{n+1}, gy, gy) \\ &\quad \leqslant k^n(G(gx, gx, gu_1) + G(gv_1, gy, gy) \end{aligned}$$

and by passing to the limit when $n \to \infty$ we get that

$$\lim_{n\to\infty} G(gx, gx, gu_{n+1}) = 0 \text{ and } \lim_{n\to\infty} G(gv_{n+1}, gy, gy) = 0. \tag{7}$$

Similarly, one can show that

$$\lim_{n\to\infty} G(gw, gw, gu_{n+1}) = 0 \text{ and } \lim_{n\to\infty} G(gv_{n+1}, gz, gz) = 0. \tag{8}$$

Therefore, from (7) and (8) we get $gx = gw$ and $gy = gz$. So (4) holds. So, by commutativity of $F$ and $g$, we have

$$g(gx) = g(F(x,y)) = F(gx,gy) \text{ and } g(gy) = g(F(y,x)) = F(gy,gx). \quad (9)$$

Denote now $gx = x^*$ and $gy = y^*$, then by (10), we have

$$gx^* = F(x^*,y^*) \text{ and } gy^* = F(y^*,x^*). \quad (10)$$

Thus, $(x^*,y^*)$ is a coupled coincide point. Then, from (4) with $w = x^*$ and $z = y^*$, we have $gx = gx^*$ and $gy = gy^*$, that is,

$$gx^* = x^* \text{ and } gy^* = y^* \quad (11)$$

then from (10) and (11), we get

$$x^* = gx^* = F(x^*,y^*) \text{ and } y^* = gy^* = F(y^*,x^*).$$

Then, $(x^*,y^*)$ is a coupled common fixed point of $F$ and $g$.

Now, we assume that $(p,q)$ is another coupled common fixed point. Then we have $p = gp = gx^* = x^*$ and $q = gq = gy^* = y^*$. □

Now, we give an example illustrating our main result.

**Example 3.2.** Let $X = [0,+\infty)$ be endowed with the usual metric, and with the usual order in $\mathbb{R}$. Consider the function

$$G : [0,+\infty)^3 \to [0,+\infty), \ G(x,y,z) = \max\{|x-y|,|x-z|,|y-z|\}.$$

It is known from [11] that $(X,G)$ is a complete $G$-metric space. Let $F : X \times X \to X$ and $g : X \to X$ be defined by

$$F(x,y) = \frac{x+y}{30}, \quad g(x) = \frac{x}{10}.$$

It is clear that $F$ is a continuous function and does not satisfy the mixed g-monotone property. In fact, consider $y_1 = \frac{1}{3}$ and $y_2 = \frac{1}{2}$, for $x = 1$ we get $gy_1 = \frac{1}{30} \preceq \frac{1}{20} = gy_2$, but

$$F(x,y_1) = \frac{4}{90} \preceq \frac{1}{20} = F(x,y_2).$$

It is easy to see that, all the required hypotheses of Theorem 3.1 are satisfied. Clearly, $F$ and $g$ have a coupled fixed point, which is (0,0).

In the next theorem, we omit the continuity hypothesis of $F$. We need the following definition.

**Definition 3.3.** Let $(X,\preceq)$ be a partially ordered set and $G$ be a G-metric on $X$. We say that $(X,G,\preceq)$ is regular if the following conditions hold:

(i) if a non-decreasing sequence $(x_n)$ is such that $x_n \to x$, then $x_n \preceq x$ for all $n$,
(ii) if a non-increasing sequence $(y_n)$ is such that $y_n \to y$, then $y \preceq y_n$ for all $n$.

**Theorem 3.3.** *Let $(X,\preceq)$ be a partially ordered set and $G$ be a G-metric on $X$ such that $(X,G,\preceq)$ is regular and let $g : X \to X$ and $F : X \times X \to X$ and $g$ is continuous and $g(X)$ is closed. Suppose that the following hold:*

(i) *$F(X \times X) \subset g(X)$ and $g$ and $F$ are compatible;*
(ii) *if $x,y,u,v \in X$ are such that $gx \asymp F(x,y) = gu$ then $F(x,y) \asymp F(F(x,y),v)$;*
(iii) *there exists $x_0,y_0 \in X$ such that $gx_0 \asymp F(x_0,y_0)$ and $gy_0 \asymp F(y_0,x_0)$;*
(iv) *there exists $k \in [0,1)$ such that for all $x,y,u,v \in$ satisfying $gx \asymp gu$ and $gy \asymp gv$,*

$$G(F(x,y),F(u,v),F(w,z)) \leqslant \frac{k}{2}[G(gx,gu,gw) + G(gy,gv,gz)]. \quad (12)$$

*Then there exist $(x,y) \in X \times X$ such that $gx = F(x,y)$ and $gy = F(y,x)$, that is, $g$ and $F$ have a coupled coincidence point.*

**Proof.** Proceeding exactly as in Theorem 2.1, we have that $(gx_n)$ and $(gy_n)$ are Cauchy sequences in the complete G-metric space $(g(X),G)$. Then, there exist $x,y \in X$ such that $gx_n \to gx$ and $gy_n \to gy$. Since $(gx_n)$ is non-decreasing and $(gy_n)$ is non-decreasing, using the regularity of $(X,G,\preceq)$, we have $gx_n \preceq gx$ and $gy \preceq gy_n$ for all $n \geqslant o$. If $gx_n = gx$ and $gy_n = gy$ for some $n \geqslant o$, then $gx = gx_n \preceq gx_{n+1} \preceq gx = gx_n$ and $gy \preceq gy_{n+1} \preceq gy_n = gy$, which implies that $gx_n = gx_{n+1} = F(x_n,y_n)$ and $gy_n = gy_{n+1} = F(y_n,x_n)$, this is, $(x_n,y_n)$ is a coupled coincidence point of $F$ and $g$. Then, we suppose that $(gx_n,gy_n) \neq (gx,gy)$ for all $n \geqslant 0$. Using the rectangle inequality, we get

$$\begin{aligned} G(F(x,y),gx,gx) &\leqslant G(F(x,y),gx_{n+1},gx_{n+1}) + G(gx_{n+1},gx,gx) \\ &= G(F(x,y),F(x_n,y_n),F(x_n,y_n)) \\ &\quad + G(gx_{n+1},gx,gx) \\ &\leqslant \frac{k}{2}[G(gx,gx_n,gx_n) + G(gy,gy_n,gy_n)] \\ &\quad + G(gx_{n+1},gx,gx). \end{aligned}$$

By letting $n \to +\infty$ in the above inequality, we get

$$G(F(x,y),gx,gx) = 0,$$

which implies that $gx = F(x,y)$. Similarly, we can show that $gy = F(y,x)$. Thus we proved that $(x,y)$ is a coupled coincidence point of $F$ and $g$. □

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