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Common fixed point theorems for hybrid pairs of maps in fuzzy metric spaces

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Abstract The purpose of this paper is to introduce the notion of common limit range property (CLR property) for two hybrid pairs of mappings in fuzzy metric spaces, and we prove common fixed point theorems using (CLR) property for these mappings with implicit relation. Our results extend some known results to multi-valued arena. Also, we prove common fixed point theorem in fuzzy metric spaces satisfying an integral type.

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1. Introduction and preliminaries

Fuzzy set [1] is an important concept in topology and analysis. It is a generalization of crisp set. The concept of fuzzy metric spaces has been studied by many authors in several ways. Kramosil and Michalek [2] introduced the concept of KM-fuzzy metric space as a generalization of probabilistic metric space given by Menger [3] and Schweizer and Sklar [4]. George and Veeramani [5] modified this concept to GV-fuzzy metric space and obtained a hausdorff topology for this kind of fuzzy metric spaces. Fuzzy set theory has applications in applied sciences such as mathematical programming, modeling theory, engineering sciences, image processing, control theory, and communication. Many authors have proved fixed and common fixed point theorems in metric and fuzzy metric spaces.

Mishra et al. [6] extended the notion of compatible maps under the name of asymptotically commuting maps in fuzzy metric spaces and prove common fixed point theorems using the continuity of one map and completeness of the involved maps. Singh and Jain [7] introduced the notion of weak and semicompatible maps in fuzzy metric spaces and showed that every pair of compatible maps is weakly compatible but the converse is not true in general. Pant [8] initiated the study of common fixed points of non-compatible maps in metric spaces. For a non-compatible maps, Aamri and El Moutawakil [9] introduced a new property named as (E.A) property, Pant [10] studied the common fixed points for non-compatible maps using (E.A) property in fuzzy metric spaces. Recently, Sintunavarat and Kumam [11] introduced the notion of common limit range property (or (CLR) property) for a pair of maps as a generalization of (E.A) property and prove

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common fixed point theorems in fuzzy metric spaces. The concept of (CLRg) property for hybrid maps is an extending of single maps. There are some similar results in deferent ways such as [12–14].

The aim of this paper is to extend some definitions and prove common fixed point theorems for hybrid maps in fuzzy spaces using the (CLRg) property. Our results are improvement over some relevant results contained in [15–19] besides some other ones.

Now we list some important definitions.

Definition 1.1 [20]. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t-norm if

(I) $*$ is commutative and associative,
(II) $*$ is continuous,
(III) $a * 1 = a$ for all $a \in [0,1]$,
(IV) $a * b \leqslant c * d$ whenever $a \leqslant c$ and $b \leqslant d$ for all $a, b, c, d \in [0,1]$.

Example 1.2 [21]. As classical examples of continuous t-norms we mention the t-norms T_L, T_P, T_M defined through $T_L(a,b) = \max(a+b-1, 0)$, $T_P(a,b) = ab$ and $T_M = \min(a,b)$.

Definition 1.3 [2]. The 3-tuple $(X, M, *)$ is said to be a KM-fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions (for all $x, y, z \in X$ and $t, s > 0$):

(KM_1) $M(x,y,0) = 0$,

(KM_2) $M(x,y,t) = 1\ \forall t > 0$ iff $x = y$,

(KM_3) $M(x,y,t) = M(y,x,t)$,

(KM_4) $M(x,z,t+s) \geqslant M(x,y,t) * M(y,z,s)$,

(KM_5) $M(x,y,.) : [0,\infty) \to [0,1]$ is left continuous.

Remark 1.4 [22]. The function $M(x,y,t)$ is often interpreted as the degree of nearness between x and y with respect to t.

Lemma 1.5 [23]. *For every $x, y \in X$, the mapping $M(x,y,\cdot)$ is nondecreasing on $(0,\infty)$.*

Definition 1.6 [5]. The 3-tuple $(X, M, *)$ is said to be a GV-fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions (for all $x, y, z \in X$ and $t, s > 0$):

$(GV_1) M(x,y,t) > 0$,

$(GV_2) M(x,y,t) = 1$ iff $x = y$,

$(GV_3) M(x,y,t) = M(y,x,t)$,

$(GV_4) M(x,z,t+s) \geqslant M(x,y,t) * M(y,z,s)$,

$(GV_5) M(x,y,.) : (0,\infty) \to [0,1]$ is continuous.

Example 1.7 [5]. Let (X, d) be a metric space wherein $a * b = ab$ for all $a, b \in [0,1]$. Then, one can define a fuzzy metric $M_d(x,y,t)$ by

$$M_d(x,y,t) = \frac{t}{t + d(x,y)},$$

for each $x, y \in X$ and $t > 0$.

Definition 1.7 [24]. Let $CB(X)$ be the set of all nonempty closed bounded subsets of a fuzzy metric space $(X, M, *)$. Then for every $A, B, C \in CB(X)$ and $t > 0$,

$$M(A,B,t) = \min\left\{\min_{a\in A} M(a,B,t),\ \min_{b\in B} M(A,b,t)\right\},$$

where $M(C,y,t) = \max\{M(z,y,t) : z \in C\}$.

Remark 1.8 [16]. Obviously $M(A,B,t) \leqslant M(a,B,t)$ whenever $a \in A$ and $M(A,B,t) = 1$ iff $A = B$. Obviously, $1 = M(A,B,t) \leqslant M(a,B,t)$ for all $a \in A$.

Definition 1.24 [25]. Let $(X, M, *)$ be a fuzzy metric space. We denote by $CP(X)$ the set of nonempty compact subsets of X. We define a function H_M on $CP(X) \times CP(X) \times (0,\infty)$ by

$$H_M(A,B,t) = \min\left\{\inf_{a\in A} M(a,B,t),\ \inf_{b\in B} M(A,b,t)\right\},$$

for all $A, B \in CP(X)$ and $t > 0$, also $(H_M, *)$ is a fuzzy metric on $CP(X)$.

Definition 1.9 [5]. A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be convergent to some $x \in X$ if for all $t > 0$, $\lim_{n\to\infty} M(x_n, x, t) = 1$.

Definition 1.10 [26]. Let $CL(X)$ be the set of all nonempty closed subsets of a metric space (X, d) and $F : Y \subseteq X \to CL(X)$. Then the map $f : Y \to X$ is said to be F-weakly commuting at $x \in X$ if $ffx \in Ffx$ provided that $fx \in Y$ for all $x \in Y$.

Definition 1.12 [27]. Let $(X, M, *)$ be a fuzzy metric space. A map $f : Y \subseteq X \to X$ is said to be coincidentally idempotent w.r.t. a mapping $F : Y \to CL(X)$ if f is idempotent at the coincidence points of (f, F), i.e., $ffx = fx$ for all $x \in Y$ with $fx \in Fx$ provided that $fx \in Y$.

Definition 1.13 [11]. Let (X, d) be metric space. Two mappings $f, g : X \to X$ are said to be satisfy the (CLR_g) property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx,$$

for some $x \in X$.

Motivated from Definition 1.13, we can have

Definition 1.14. Let (X, d) be metric space. Two mappings $f : X \to X$, $F : X \to CL(X)$ are said to be satisfy the property (CLR_f) if there exist sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = fv \in A = \lim_{n\to\infty} Fx_n,$$

for some $v \in X$.

Definition 1.15 [11]. Let $(X, M, *)$ be a fuzzy metric space. Two mappings $f, g : X \to X$ are said to be satisfy the (CLR_g) property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx,$$

for some $x \in X$.

Motivated from Definition 1.15, we can have

Definition 1.16. Let $(X, M, *)$ be a fuzzy metric space. Two mappings $f : X \to X$ and $F : X \to CL(X)$ are said to be satisfy the (CLR_g) property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = u \in A = \lim_{n\to\infty} Fx_n,$$

with $u = fv$, for some $u, v \in X$.

Definition 1.17 [28]. Let $(X, M, *)$ be a fuzzy metric space and $f, g, F, G : X \to X$. The pairs (f, F) and (g, G) are said to satisfy the joint common limit in the range of f and g property (shortly, $(JCLR_{fg})$ property) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gy_n = fu = gu,$$

for some $u \in X$.

Definition 1.18 [29]. Let f, g, S and T be four self-mappings defined on a symmetric space (X, d). Then the pairs (f, S) and (g, T) are said to be have the common limit range property (with respect to S and T) often denoted by $(CLR_{(S,T)})$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = t,$$

with $t = Su = Tw$, for some $t, u, w \in X$.

Motivated from Definition 1.18, we can have

Definition 1.19. Let (X, d) be a symmetric space, $f, g : X \to X$ and $F, G : X \to CL(X)$. Then the pair (f, F) and (g, G) are said to have the property $(CLR_{(f,g)})$ if there exist two sequences $\{x_n\}, \{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A = \lim_{n\to\infty} Fx_n, \lim_{n\to\infty} Gy_n \in CL(X),$$

with $u = fv = gw$, for some $u, v, w \in X$.

Our results involved the following implicit relation.

Definition 1.20 [27]. Let Φ be the family of all lower semicontinuous functions $\phi : [0,1]^6 \to [0,1]$ satisfying the following properties:

(ϕ_1) ϕ is non-increasing in the 3rd, 4th,5th,6th coordinate variables,
(ϕ_{21}) if $\phi(u, 1, 1, u, u, 1) \geqslant 0$ or
(ϕ_{22}) $\phi(u, 1, u, 1, 1, u) \geqslant 0$ $\forall u \in [0, 1]$ implies $u = 1$.

2. Main results

Firstly, we rewrite Definitions 1.18 and 1.19.

Definition 2.1. Let $(X, M, *)$ be a fuzzy metric space, $f, g, F, G : Y \subseteq X \to X$. Then the pairs (f, F) and (g, G) are said to be have the common limit range property (with respect to f and g) often denoted by $(CLR_{(f,g)})$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in Y such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gy_n = t,$$

with $t = fv = gw$, for some $t, v, w \in X$.

Definition 2.2. Let $(X, M, *)$ be a fuzzy metric space, $f, g : Y \subseteq X \to X$ and $F, G : Y \to CL(X)$. Then the hybrid pairs (f, F) and (g, G) are said to have the property $(CLR_{(f,g)})$ if there exist two sequences $\{x_n\}, \{y_n\}$ in Y such that $\lim_{n\to\infty} Gy_n \in CL(X)$ and

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A = \lim_{n\to\infty} Fx_n,$$

with $u = fv = gw$, for some $u, v, w \in X$.

Now, we prove our main theorem as follows.

Theorem 2.3. *Let $f, g : Y \subseteq X \to X$ be two mappings from a subset Y of a fuzzy metric space $(X, M, *)$ into X and $F, G : Y \to CL(X)$ which satisfy the following conditions:*

(a) the hybrid pairs (f, F) and (g, G) are satisfy the property $(CLR_{(f,g)})$,
(b) there exist $\phi \in \Phi$ such that

$$\phi(M(Fx, Gy, t), M(fx, gy, t), M(fx, Fx, t), M(gy, Gy, t), M(fx, Gy, t), M(gy, Fx, t)) \geqslant 0,$$

for all $x, y \in X$. Then

(1) the hybrid pair (f, F) have a coincidence point $v \in Y$,
(2) the hybrid pair (g, G) have a coincidence point $w \in Y$,
(3) the hybrid pair (f, F) have a common fixed point provided that f is F-weakly commuting at $v \in X, ffv = fv$ and $fv \in Y$,
(4) the hybrid pair (g, G) have a common fixed point provided that g is G-weakly commuting at $w \in Y, ggw = gw$ and $gw \in Y$,
(5) f, g, F, G have a common fixed point provided that both (3) and (4) are true.

Proof. Since the hybrid pairs (f, F) and (g, G) are satisfy the property $(CLR_{(f,g)})$, there exist two sequences $\{x_n\}, \{y_n\}$ in Y and $A, B \in CL(X)$ such that $\lim_{n\to\infty} Gy_n = B$ and

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A = \lim_{n\to\infty} Fx_n,$$

with $u = fv = gw$, for some $u, v, w \in X$.
Now, we proceed to show that $A = B$. As

$\phi(M(Fx_n, Gy_n, t), M(fx_n, gy_n, t), M(fx_n, Fx_n, t), M(gy_n, Gy_n, t),$
$M(fx_n, Gy_n, t), M(gy_n, Fx_n, t)) \geqslant 0.$

As $n \to \infty$, we get that

$\phi(M(A, B, t), M(fv, fv, t), M(fv, A, t), M(fv, B, t), M(fv, B, t),$
$M(fv, A, t)) \geqslant 0.$

So that

$$\phi(M(A, B, t), 1, 1, M(A, B, t), M(A, B, t), 1) \geqslant \phi(M(A, B, t), 1, M(fv, A, t), M(fv, B, t), M(fv, B, t), M(fv, A, t)) \geqslant 0.$$

Owing to (ϕ_{21}), we have $M(A, B, t) = 1$ so that $A = B$.

To prove (1), Since $fv \in A$. Now, we show that $A = Fv$. As

$\phi(M(Fv, Gy_n, t), M(fv, gy_n, t), M(fv, Fv, t), M(gy_n, Gy_n, t),$
$M(fv, Gy_n, t), M(gy_n, Fv, t)) \geqslant 0.$

When $n \to \infty$, we have that

$\phi(M(Fv, A, t), M(fv, fv, t), M(fv, Fv, t), M(fv, A, t), M(fv, A, t),$
$M(fv, Fv, t)) \geqslant 0.$

So that

$\phi(M(Fv, A, t), 1, M(A, Fv, t), 1, 1, M(A, Fv, t)) \geqslant \phi(M(Fv, A, t),$
$1, M(fv, Fv, t), M(fv, A, t), M(fv, A, t), M(fv, Fv, t)) \geqslant 0.$

Owing to (ϕ_{22}), this gets us $M(A, Fv, t) = 1$ which implies $A = Fv$. Then $fv \in Fv$ this proves (1). The proof of (2) is similar to that of (1). In order to prove (3), using the conditions given in (3), we have $ffv = fv$ and $ffv \in Ffv$ so that $u = fu \in Fu$. The proof of (4) is similar to that of (3) while (5) follows immediately. This concludes the proof. □

Now, we furnish an example to illustrate Theorem 2.3.

Example 2.4. Let $(X, M, *)$ be a fuzzy metric space, where $X = [0, 1], a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$ and

$$M(x, y, t) = \frac{t}{t + |x - y|},$$

for all $t > 0, x, y \in X$.

Define $\phi : [0, 1]^6 \to [0, 1]$ as

$\phi\{t_1, t_2, t_3, t_4, t_5, t_6\} = t_1 - t_2,$

and define the maps F, G, f, g on X as $Fx = \left[\frac{2x}{3}, 1\right], Gx = [x^2, 1]$ and $fx = \frac{2x}{3}, gx = x^2$ for all $x, y \in X$. Define two sequences $\{x_n\} = \{\frac{1}{n}\}, \{y_n\} = \{\frac{1}{2n}\}, n \in N$ in X. Since,

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = 0 \in [0, 1] = \lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gy_n,$

the hybrid pairs (f, F) and (g, G) are satisfy the property $(CLR_{(f,g)})$ and

$\phi\{M(Fx, Gy, t), M(fx, gy, t), M(fx, Fx, t), M(gy, Gy, t),$
$M(fx, Gy, t).M(gy, Fx, t)\} = 0$

Thus, all the conditions of Theorem 2.3 are satisfied and 0 remains common fixed point for the four involved maps.

Corollary 2.5. *Let $(X, M, *)$ be a fuzzy metric space. If $f : Y \subseteq X \to X$ and $F : Y \to CL(X)$ be a pair of hybrid mappings satisfying the following conditions:*

(*a*) the pair (f, F) satisfy the property (CLR_f),
(*b*) for all $x, y \in Y$,

$$M(fx, Fy, t) \geqslant \min\left\{M(fx, fy, t), \frac{M(fx, Fx, t) + M(fy, Fy, t)}{2}, \frac{M(fx, Fy, t) + M(fy, Fx, t)}{2}\right\}.$$

Then (f, F) have a common fixed point provided that f is F-weakly commuting at $v \in X$ and $ffv = fv$ for $v \in C(f, F)$.

Remark 2.6.

(1) Theorem 2.3 is a extension of Theorem 4.1 [15] for hybrid maps.
(2) Corollary 2.5 is a generalization of Theorem 3.10 [30].

Our next theorem involves a sequence of multi-valued mappings.

Theorem 2.7. *Let $\{F_n\}, n \in N$ be a sequence of multi-valued mappings from a subset Y of a fuzzy metric space $(X, M, *)$ into $CL(X)$ and $f, g : Y \to X$ which satisfy the following conditions:*

(*a*) the pairs (f, F_k) and (g, F_l) are satisfy the property $(CLR_{(f,g)})$ where ($k = 2n - 1$ and $l = 2n$ for all $n \in N$),
(*b*) there exist $\phi \in \Phi$ such that

$\phi(M(F_k x, F_l y, t), M(fx, gy, t), M(fx, F_k x, t), M(gy, F_l y, t),$
$M(fx, F_l y, t), M(gy, F_k x, t)) \geqslant 0,$

for all $x, y \in X$. Then
(1) (f, F_k) have a coincidence point $u_k \in Y$,
(2) (g, F_l) have a coincidence point $u_l \in Y$,
(3) (f, F_k) have a common fixed point provided that f is F_k-weakly commuting at u_k and f is coincidentally idempotent w.r.t. F_k,
(4) (g, F_l) have a common fixed point provided that g is F_l-weakly commuting at u_l and g is coincidentally idempotent w.r.t. F_l.

Proof. Since the hybrid pairs (f, F_k) and (g, F_l) are satisfy the property $(CLR_{(f,g)})$, there exist two sequences $\{x_{kn}\}, \{y_{kn}\}$ in Y and $A_k, B_k \in CL(X)$ such that $\lim_{n\to\infty} F_l y_{kn} = B_k$ and

$$\lim_{n\to\infty} fx_{kn} = \lim_{n\to\infty} gy_{kn} = u_k \in A_k = \lim_{n\to\infty} F_k x_{kn}.$$

with $u_k = fv_k = gw_l$, for some $u_k, v, w \in X$.

Now, we show that $A_k = B_k$. As

$\phi(M(F_k x_{kn}, F_l y_{kn}, t), M(fx_{kn}, gy_{kn}, t), M(fx_{kn}, F_k x_{kn}, t),$
$M(gy_{kn}, F_l y_{kn}, t), M(fx_{kn}, F_l y_{kn}, t), M(gy_{kn}, F_k x_{kn}, t)) \geqslant 0.$

As $n \to \infty$, we find that

$\phi(M(A_k, B_k, t), 1, 1, M(fv_k, B_k, t), M(fv_k, B_k, t), 1) \geqslant 0.$

So that

$$\phi(M(A_k,B_k,t),1,1,M(A_k,B_k,t),M(A_k,B_k,t),1) \geqslant \phi(M(A_k,B_k,t),1,M(fv_k,A_k,t),M(fv_k,B_k,t), M(fv_k,B_k,t),M(fv_k,A_k,t)) \geqslant 0.$$

Owing to (ϕ_{21}), we have $M(A_k,B_k,t)=1$ yielding thereby $A_k=B_k$.

As $u_k\in A_k$, we show that $F_kv_k=A_k$. As

$$\phi(M(F_kv_k,F_ly_{kn},t),M(fv_k,gy_{kn},t),M(fv_k,F_kv_k,t), M(gy_{kn},F_ly_{kn},t),M(fv_k,F_ly_{kn},t), M(gy_{kn},F_kv_k,t)) \geqslant 0,$$

which on making $n\to\infty$ reduces to

$$\phi(M(F_kv_k,A_k,t),1,1,M(u_k,A_k,t),M(u_k,A_k,t),1) \geqslant 0,$$

so that $F_kv_k=A_k$ which proves (1).

The remaining parts are easy to prove. This concludes the proof. □

Remark 2.8. Theorem 2.7 is an extension of Theorem 2 [31] in fuzzy metric space.

The following theorem is an extension of Theorem 5.1 in [15].

Theorem 2.9. *Let $f,g: Y\subseteq X\to X$ be two mappings from a subset Y of a fuzzy metric space $(X,M,*)$ into X and $F,G: Y\to CL(X)$ such that*

(a) the pairs (f,F) and (g,G) are satisfy the (CLR_{fg}) property at $u\in Y$ with respect to F and G,
(b) there exist $\phi\in\Phi$ such that

$$\int_0^{\phi(u,1,1,u,u,1)}\varphi(s)ds \geqslant 0,$$

or

$$\int_0^{\phi(u,1,u,1,1,u)}\varphi(s)ds \geqslant 0,$$

where $\varphi:[0,\infty)\to[0,\infty)$ is a summable non-negative lebesgue integrable function such that for each $\epsilon\in[0,1)$

$$\int_\epsilon^1\varphi(s)ds>0,$$

implies that $u=1$ for all $u,v\in[0,1]$, if

$$\int_0^{\phi(M(Fx,Gy,t),M(fx,gy,t),M(fx,Fx,t),M(gy,Gy,t),M(fx,Gy,t),M(gy,Fx,t))}\varphi(s)ds \geqslant 0,$$

for all $x,y\in X$ and $t>0$. Then

(1) the hybrid pair (f,F) have a coincidence point $v\in Y$,
(2) the hybrid pair (g,G) have a coincidence point $w\in Y$,
(3) the hybrid pair (f,F) have a common fixed point provided that f is F-weakly commuting at $v\in X, ffv=fv$ and $fv\in Y$,
(4) the hybrid pair (g,G) have a common fixed point provided that g is G-weakly commuting at $w\in Y, ggw=gw$ and $gw\in Y$,
(5) f,g,F,G have a common fixed point provided that both(3) and (4) are true.

Proof. Since the hybrid pairs (f,F) and (g,G) satisfy the (CLR_{fg}) property at $u\in Y$ with respect to G then there exist two sequences $\{x_n\},\{y_n\}$ in Y and $A,B\in CL(X)$ such that $\lim_{n\to\infty}Gy_n=B$ and

$$\lim_{n\to\infty}fx_n=\lim_{n\to\infty}gy_n=u\in A=\lim_{n\to\infty}Fx_n,$$

with $u=fv=gw$ for some $u,v,w\in X$.

We show that $A=B$, since

$$\int_0^{\phi(M(Fx_n,Gy_n,t),M(fx_n,gy_n,t),M(fx_n,Fx_n,t),M(gy_n,Gy_n,t),M(fx_n,Gy_n,t),M(gy_n,Fx_n,t))}\varphi(s)ds \geqslant 0.$$

As $n\to\infty$, we have

$$\int_0^{\phi(M(A,B,t),M(u,u,t),M(u,A,t),M(u,B,t),M(u,B,t),M(u,A,t))}\varphi(s)ds \geqslant 0.$$

Then

$$\int_0^{\phi(M(A,B,t),1,M(u,A,t),M(u,B,t),M(u,B,t),M(u,A,t))}\varphi(s)ds \geqslant 0.$$

Or

$$\int_0^{\phi(M(A,B,t),1,1,M(A,B,t),M(A,B,t),1)}\varphi(s)ds \geqslant 0.$$

Then $M_{M,N}(A,B,t)=1$, we have $A=B$.

(1) To prove (1), Since $fv\in A$ we show that $A=Fv$, since

$$\int_0^{\phi(M(Fv,Gy_n,t),M(fv,gy_n,t),M(fv,Fv,t),M(gy_n,Gy_n,t),M(fv,Gy_n,t),M(gy_n,Fv,t))}\varphi(s)ds \geqslant 0.$$

As $n\to\infty$, we have

$$\int_0^{\phi(M(Fv,A,t),M(fv,u,t),M(fv,Fv,t),M(u,A,t),M(fv,A,t),M(u,Fv,t))}\varphi(s)ds \geqslant 0.$$

Then

$$\int_0^{\phi(M(Fv,A,t),1,M(u,Fv,t),M(u,A,t),M(u,A,t),M(u,Fv,t))}\varphi(s)ds \geqslant 0.$$

Or

$$\int_0^{\phi(M(Fv,A,t),1,M(A,Fv,t),1,1,M(A,Fv,t))}\varphi(s)ds \geqslant 0.$$

This show that $M(A,Fv,t)=1$, then $A=Fv$. Then $fv\in Fv$ this proves (1). □

The other results hold immediately.

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