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ORIGINAL ARTICLE

Traveling wave solutions for the mKdV equation and the Gardner equations by new approach of the generalized (G'/G)-expansion method

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Abstract In this paper, we demonstrate the effectiveness of the new generalized (G'/G)-expansion method by seeking more exact solutions via the mKdV equation and the Gardner equations. The method is direct, concise and simple to implement compared to other existing methods. The traveling wave solutions obtained by this method are expressed in terms of hyperbolic, trigonometric and rational functions. The method shows a wide application for handling nonlinear wave equations. Moreover, the method reduces the large amount of calculations.

MATHEMATICS SUBJECT CLASSIFICATION: 35C07; 35C08; 35L05

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1. Introduction

A large variety of physical, chemical, and biological phenomena are governed by nonlinear partial differential equations. One of the most exciting advances in the field of nonlinear science and theoretical physics is the development of methods to look for exact solutions of nonlinear partial differential equations. Exact solutions to nonlinear partial differential equations play an important role in nonlinear science, since they can provide much physical information and more insight of the physical aspects of the problem and thus lead to further applications. Wave phenomena in dispersion, dissipation, diffusion, reaction and convection are very much important. In recent years, several powerful methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been established, such as, the ansatz method [1], the Adomian decomposition method [2], the Darboux transformation method [3], the Backlund transformation method [4], the inverse scattering transformation method [5], the Jacobi elliptic function method [6,7], the Exp-function method [8,9], the extended tanh method [10], the Cole-Hopf transformation [11], the (G'/G)-expansion method [12–16], and the modified simple equation method [17,18].

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Recently, Naher and Abdullah [19] established a highly effective extension of the (G'/G)-expansion method, called the new generalized (G'/G)-expansion method to obtain exact traveling wave solutions of NLEEs. The objective of this article is to search for new study relating to the new generalized (G'/G)-expansion method for solving the mKdV equation and the Gardner equations to demonstrate the appropriateness and straightforwardness of the method.

2. Description of the new generalized (G'/G)-expansion method

Let us consider a general nonlinear PDE in the form

$$P(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \ldots) = 0, \tag{1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u(x,t)$ and its derivatives in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives.

Step 1: We combine the real variables x and t by a compound variable Φ,

$$u(x,t) = u(\Phi), \quad \Phi = x \pm Vt, \tag{2}$$

where V is the speed of the traveling wave. The traveling wave transformation (2) converts Eq. (1) into an ordinary differential equation (ODE) for $u = u(\Phi)$:

$$Q(u, u', u'', u''', \ldots) = 0, \tag{3}$$

where Q is a polynomial of u and it derivatives and the superscripts indicate the ordinary derivatives with respect to Φ.

Step 2: Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$u(\Phi) = \sum_{i=0}^{N} a_i (d+H)^i + \sum_{i=1}^{N} b_i (d+H)^{-i}, \tag{4}$$

where either a_N or b_N may be zero, but both a_N and b_N could be zero at a time, a_i $(i = 0, 1, 2, \ldots, N)$ and b_i $(i = 1, 2, \ldots, N)$ and d are arbitrary constants to be determined later and $H(\Phi)$ is given by

$$H(\Phi) = (G'/G) \tag{5}$$

where $G = G(\Phi)$ satisfies the following auxiliary nonlinear ordinary differential equation:

$$AGG'' - BGG' - EG^2 - C(G')^2 = 0 \tag{6}$$

where the prime stands for derivative with respect to Φ; A, B, C and E are real parameters.

Using the general solution of Eq. (6), we have the following solutions of Eq. (5):

Family 1: When $B \neq 0$, $\psi = A - C$ and $\Omega = B^2 + 4E(A - C) > 0$,

$$H(\Phi) = \left(\frac{G'}{G}\right) = \frac{B}{2\psi} + \frac{\sqrt{\Omega}}{2\psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Omega}}{2A}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\Omega}}{2A}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega}}{2A}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\Omega}}{2A}\xi\right)} \tag{7}$$

Family 2: When $B \neq 0$, $\psi = A - C$ and $\Omega = B^2 + 4E(A - C) < 0$,

$$H(\Phi) = \left(\frac{G'}{G}\right) = \frac{B}{2\psi} + \frac{\sqrt{-\Omega}}{2\psi} \frac{-C_1 \sin\left(\frac{\sqrt{-\Omega}}{2A}\xi\right) + C_2 \cos\left(\frac{\sqrt{-\Omega}}{2A}\xi\right)}{C_1 \cos\left(\frac{\sqrt{-\Omega}}{2A}\xi\right) + C_2 \sin\left(\frac{\sqrt{-\Omega}}{2A}\xi\right)} \tag{8}$$

Family 3: When $B \neq 0$, $\psi = A - C$ and $\Omega = B^2 + 4E(A - C) = 0$,

$$H(\Phi) = \left(\frac{G'}{G}\right) = \frac{B}{2\psi} + \frac{C_2}{C_1 + C_2\xi} \tag{9}$$

Family 4: When $B = 0$, $\psi = A - C$ and $\Delta = \psi E > 0$,

$$H(\Phi) = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Delta}}{\psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Delta}}{A}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\Delta}}{A}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\Delta}}{A}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\Delta}}{A}\xi\right)} \tag{10}$$

Family 5: When $B = 0$, $\psi = A - C$ and $\Delta = \psi E < 0$,

$$H(\Phi) = \left(\frac{G'}{G}\right) = \frac{\sqrt{-\Delta}}{\psi} \frac{-C_1 \sin\left(\frac{\sqrt{-\Delta}}{A}\xi\right) + C_2 \cos\left(\frac{\sqrt{-\Delta}}{A}\xi\right)}{C_1 \cos\left(\frac{\sqrt{-\Delta}}{A}\xi\right) + C_2 \sin\left(\frac{\sqrt{-\Delta}}{A}\xi\right)} \tag{11}$$

Step 3: To determine the positive integer N, taking homogeneous balance between highest order nonlinear terms and the derivatives of the highest order appearing in Eq. (3).

Step 4: Substitute Eqs. (4) and (6) including Eq. (5) into Eq. (3) with the value of N obtained in Step 3, we obtain polynomials in $(d + H)^N (N = 0, 1, 2, \ldots)$ and $(d + H)^{-N}$ $(N = 0, 1, 2, \ldots)$. We collect each coefficient of the resulted polynomials to zero yields a set of algebraic equations for a_i $(i = 0, 1, 2, \ldots, N)$ and b_i $(i = 1, 2, \ldots, N)$, d and V. This procedure yields a system of algebraic equations whichever can be solved to find a_i,b_i, d and V. Substituting the values of a_i,b_i, d and V into Eq. (4) along with general solutions of Eq. (6) completes the determination of the solution of Eq. (1).

3. Applications of the method

In this section, we will bring to bear the new generalized (G'/G)-expansion method discussed in Section 2 to the mKdV equation and the Gardner equations which are very important in the field of nonlinear mathematical physics.

3.1. The Gardner equation

In this section, we consider the Gardner equation [20,21] in the following form:

$$u_t = 6uu_x + 6\varepsilon^2 u^2 u_x + u_{xxx}, \quad \delta > 0, \tag{12}$$

This equation known as the combined KdV–mKdV equation is widely studied in various areas of physics that includes plasma physics, fluid dynamics, quantum field theory and solid state physics.

Table 1 Comparison between Taghizade and Neirameh's [13] solutions and the obtained solutions.

Taghizade and Neirameh's [38] solutions	The obtained solutions
i. If $C_1 = 0$, $\lambda = 0$ and $u(\xi) = u_1(\Phi)$, solutions Eq. (18) becomes: $u_1(\Phi) = \frac{i}{2\varepsilon}\sqrt{\lambda^2 - 4\mu}\coth\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right) - \frac{1}{2\varepsilon_2}$	i. If $A = 1$, $C = 0$, $\Omega = \lambda^2 - 4\mu$, $\varepsilon^2 = -\varepsilon_2$ then the solution is $u_1(\Phi) = \frac{i}{2\varepsilon}\sqrt{\lambda^2 - 4\mu}\coth\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right) - \frac{1}{2\varepsilon_2}$
ii. If $C_1 = 0$, $\lambda = 0$ and $u(\xi) = u_3(\Phi)$, solutions Eq. (18) becomes: $u_3(\Phi) = \frac{i}{2\varepsilon}\sqrt{\lambda^2 - 4\mu}\coth\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right) - \frac{1}{2\varepsilon_2}$	ii. If $A = 1$, $C = 0$, $\Omega = \lambda^2 - 4\mu$, $\varepsilon^2 = -\varepsilon_2$ then the solution is $u_3(\Phi) = \frac{i}{2\varepsilon}\sqrt{\lambda^2 - 4\mu}\coth\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right) - \frac{1}{2\varepsilon_2}$
iii. If $\lambda^2 - 4\mu = 0$, $u(\xi) = u_5(\Phi)$, $\lambda = 0$, solution Eq. (18) becomes $u_5(\Phi) = \frac{i}{\varepsilon}\left(\frac{C_2}{C_1+C_2\xi}\right) - \frac{1}{2\varepsilon_2}$	iii. If $A = 1$, $C = 0$, $\varepsilon^2 = \varepsilon_2$ then solutions $u_5(\Phi) = \frac{i}{\varepsilon}\left(\frac{C_2}{C_1+C_2\xi}\right) - \frac{1}{2\varepsilon_2}$

To seek the traveling wave solutions of Eq. (12), we make the transformation $\Phi = x - Vt$. Then (12) reduces to

$$Vu' + 3(u^2)' + 2\varepsilon^2(u^3)' + u''' = 0. \tag{13}$$

where superscripts stand for the derivatives with respect to Φ.

Integrating Eq. (13) once with respect to Φ yields:

$$K + Vu + 3u^2 + 2\varepsilon^2 u^3 + u'' = 0. \tag{14}$$

where K is an integral constant that can be determined later.

Taking homogeneous balance between the highest order nonlinear term u^3 and the highest order derivative u'' in Eq. (14), we obtain $N = 1$. Therefore, the solution of Eq. (14) is of the form

$$u(\Phi) = a_0 + a_1(d + H) + b_1(d + H)^{-1}, \tag{15}$$

where a_0, a_1, b_1 and d are constants to be determined.

Substituting Eq. (15) together with Eqs. (5) and (6) into Eq. (14), and executing the parallel course of algorithm as carried out in Section 2 yields a set of simultaneous algebraic equations (for simplicity which are not presented here) for a_0, a_1, b_1, d and V. Solving these algebraic equations with the help of symbolic computation software, we obtain following:

$$a_0 = -\frac{1}{\pm 2iA\varepsilon^2}(\pm iA - 2d\varepsilon\psi - B\varepsilon), \quad b_1 = 0,$$

$$d = d, \quad a_1 = \frac{\pm i\psi}{\varepsilon A}, \quad V = -\frac{1}{2\varepsilon^2 A^2}(3A^2 + 4E\varepsilon^2\psi + \varepsilon^2 B^2),$$

$$K = \frac{1}{4\varepsilon^4 A^2}(A^2 + 4E\varepsilon^2\psi + B^2\varepsilon^2). \tag{16}$$

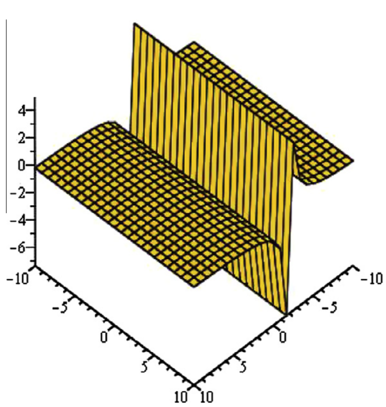

Figure 1 Singular kink of $u_5(\Phi)$ when $C_1 = 2$, $C_2 = 1$, $\delta = 1$, $d = 1$, $A = 1$, $B = 2$, $C = 2$, $E = 1$ and $-10 \leqslant x, t \leqslant 10$.

where $\psi = A - C$, A, B, C and E are free parameters.

Substituting Eq. (16) into Eq. (15), along with Eq. (7) and simplifying yields following traveling wave solutions (if $C_1 = 0$ but $C_2 \neq 0$ and $C_2 = 0$ but $C_1 \neq 0$) respectively:

$$u_1(\Phi) = \pm\frac{1}{2iA\varepsilon^2}\left(\pm iA + \varepsilon\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2A}\Phi\right)\right),$$

$$u_2(\Phi) = \pm\frac{1}{2iA\varepsilon^2}\left(\pm iA + \varepsilon\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2A}\Phi\right)\right),$$

where $\Phi = x + \frac{1}{2\varepsilon^2 A^2}(3A^2 + 4E\varepsilon^2\psi + \varepsilon^2 B^2)t$.

Similarly, substituting Eq. (16) into Eq. (15), along with Eqs. (8)–(11) and simplifying, we obtain respectively the following exact solutions (if $C_1 = 0$ but $C_2 \neq 0; C_2 = 0$ but $C_1 \neq 0$)

$$u_3(\Phi) = \pm\frac{1}{2iA\varepsilon^2}\left(\pm iA + \varepsilon i\sqrt{\Omega}\cot\left(\frac{\sqrt{-\Omega}}{2A}\Phi\right)\right),$$

$$u_4(\Phi) = \pm\frac{1}{2iA\varepsilon^2}\left(\pm iA - \varepsilon i\sqrt{\Omega}\tan\left(\frac{\sqrt{-\Omega}}{2A}\Phi\right)\right),$$

$$u_5(\Phi) = \frac{1}{\pm 2iA\varepsilon^2}\left(\pm iA + 2\varepsilon\psi\left(\frac{C_2}{C_1 + C_2\Phi}\right)\right),$$

$$u_6(\Phi) = \pm\frac{1}{2iA\varepsilon^2}\left(\pm iA - B\varepsilon + 2\varepsilon\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{A}\Phi\right)\right),$$

$$u_7(\Phi) = \pm\frac{1}{2iA\varepsilon^2}\left(\pm iA - B\varepsilon + 2\varepsilon\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{A}\Phi\right)\right),$$

$$u_8(\Phi) = \pm\frac{1}{2iA\varepsilon^2}\left(\pm iA - B\varepsilon + 2i\varepsilon\sqrt{\Delta}\cot\left(\frac{\sqrt{-\Delta}}{A}\Phi\right)\right),$$

$$u_9(\Phi) = \pm\frac{1}{2iA\varepsilon^2}\left(\pm iA - B\varepsilon - 2i\varepsilon\sqrt{\Delta}\tan\left(\frac{\sqrt{-\Delta}}{A}\Phi\right)\right),$$

Beyond the Table 1, we found new exact traveling wave solutions u_2, u_4, u_6, u_7, u_8 and u_9 in this article, which have not been reported in the Ref. [13] (see Table 1).

3.2. The mKdV equation

In this subsection, we consider the mKdV equation in the form

$$u_t - u^2 u_x + \delta u_{xxx} = 0, \quad \delta > 0, \tag{17}$$

We utilize the traveling wave variable $u(\Phi) = u(x, t)$, $\Phi = x - Vt$; Eq. (17) is carried to an ODE

$$-Vu' - u^2u' + \delta u''' = 0. \quad (18)$$

where superscripts stand for the derivatives with respect to Φ.

Integrating Eq. (18) once with respect to Φ yields:

$$K - Vu - \frac{1}{3}u^3 + \delta u''' = 0. \quad (19)$$

where K is an integral constant to be determined later.

Taking homogeneous balance between u^3 and u'' in Eq. (19), we obtain $N = 1$. Therefore, the solution of Eq. (19) is of the form

$$u(\Phi) = a_0 + a_1(d+H) + b_1(d+H)^{-1}, \quad (20)$$

where a_0, a_1, b_1 and d are constants to be determined.

Substituting Eq. (20) together with Eqs. (5) and (6) into Eq. (19), and executing the parallel course of algorithm discussed in Section 2 yields a set of simultaneous algebraic equations (for simplicity which are not presented here) for a_0, a_1, b_1, d and V. Solving these algebraic equations with the help of symbolic computation software, we obtain following:

$$a_0 = -\frac{3\delta(2d\psi + B)}{\pm A\sqrt{6\delta}}, \quad b_1 = 0, \quad d = d,$$

$$a_1 = \frac{\pm\psi\sqrt{6\delta}}{A}, \quad V = -\frac{\delta}{2A^2}(B^2 + 4E\psi), \quad K = 0. \quad (21)$$

where $\psi = A - C$, A, B, C and E are free parameters.

Substituting Eq. (21) into Eq. (20), along with Eq. (7) and simplifying yields following traveling wave solutions (if $C_1 = 0$ but $C_2 \neq 0$ and $C_2 = 0$ but $C_1 \neq 0$) respectively:

$$u_1(\Phi) = \pm\frac{3\delta}{A\sqrt{6\delta}}\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2A}\Phi\right),$$

$$u_2(\Phi) = \pm\frac{3\delta}{A\sqrt{6\delta}}\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2A}\Phi\right),$$

where $\Phi = x + \frac{\delta}{2A^2}(B^2 + 4E\psi)t$.

Similarly, substituting Eq. (21) into Eq. (20), along with Eqs. (8)–(11) and simplifying, we obtain respectively the following exact solutions (if $C_1 = 0$ but $C_2 \neq 0$; $C_2 = 0$ but $C_1 \neq 0$):

$$u_3(\Phi) = \pm\frac{3i\delta}{A\sqrt{6\delta}}\sqrt{\Omega}\cot\left(\frac{\sqrt{-\Omega}}{2A}\Phi\right),$$

$$u_4(\Phi) = \pm\frac{3i\delta}{A\sqrt{6\delta}}\sqrt{\Omega}\tan\left(\frac{\sqrt{-\Omega}}{2A}\Phi\right),$$

$$u_5(\Phi) = \pm\frac{6\delta\psi}{A\sqrt{6\delta}}\left(\frac{C_2}{C_1 + C_2\Phi}\right),$$

$$u_6(\Phi) = \pm\frac{1}{A\sqrt{6\delta}}\left(-3\delta\left(B - 2\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{A}\Phi\right)\right)\right),$$

$$u_7(\Phi) = \pm\frac{1}{A\sqrt{6\delta}}\left(-3\delta\left(B - 2\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{A}\Phi\right)\right)\right),$$

$$u_8(\Phi) = \pm\frac{1}{A\sqrt{6\delta}}\left(-3\delta\left(B - 2i\sqrt{\Delta}\cot\left(\frac{\sqrt{-\Delta}}{A}\Phi\right)\right)\right),$$

$$u_9(\Phi) = \pm\frac{1}{A\sqrt{6\delta}}\left(-3\delta\left(B + 2i\sqrt{\Delta}\tan\left(\frac{\sqrt{-\Delta}}{A}\Phi\right)\right)\right).$$

4. Discussion

From the above solutions we observe that, if we put $A = 1$, $C = 0$ and $\Omega = \lambda^2 - 4\mu$ in our solution $u_1(\Phi)$ then the solution is identical to Wang et al.'s solution $u_{1,2}(\xi)$ obtained if Ref. [12]. Again if we put $A = 1$, $C = 0$ and $\Omega = \lambda^2 - 4\mu$ in our solution $u_3(\Phi)$ then it is identical to Wang et al.'s solution $u_{3,4}(\xi)$ attained in Ref. [12]. Similarly, Wang et al.'s solution $u_{5,6}(\xi)$ is identical to our solution $u_5(\Phi)$. Wang et al. [12] did not find any more solution, but in this article, by using the new generalized (G'/G)-expansion method, we obtain further new exact traveling wave solutions u_2, u_4, u_6, u_7, u_8 and u_9.

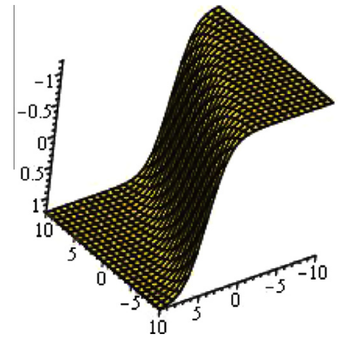

Figure 2 Kink of solution $u_7(\Phi)$ when $\delta = 1$, $d = 1$, $A = 2$, $B = 0$, $C = 1$, $E = 1$ and $-10 \leqslant x, t \leqslant 10$.

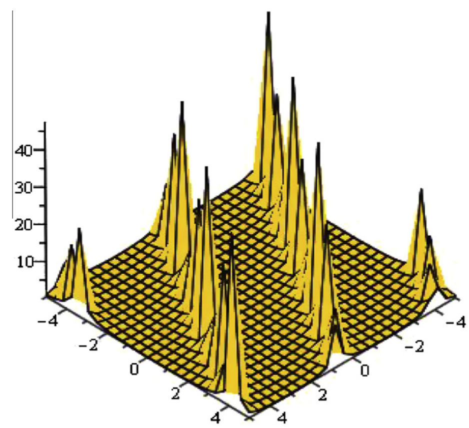

Figure 3 Periodic solutions of $u_4(\Phi)$ when $d = 1$, $\alpha = 1$, $A = 2$, $B = 1$, $C = 4$, $E = 1$ and $-5 \leqslant x, t \leqslant 5$.

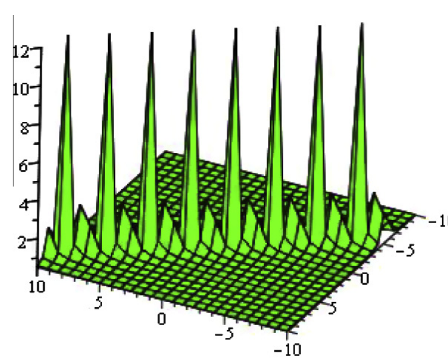

Figure 4 Singular soliton solutions of $u_5(\Phi)$ when $C_1 = 1$, $C_2 = 2$, $A = 1$, $B = 2$, $C = 2$, $E = 1$, $\alpha = 1$, $d = 1$ and $-10 \leqslant x, t \leqslant 10$.

5. Graphical representation of the solutions

The graphical illustrations of the solutions are depicted in the Figs. 1–4 with the aid of symbolic computation software Maple 13.

6. Conclusion

Some new exact traveling wave solutions of the mKdV equation and the Gardner equations have been constructed in this article by using the new generalized (G'/G)-expansion method. The performance of this method is trustworthy and gives many new solutions. Some of our obtained solutions are in good agreement with the existing results which validates our solutions. Therefore, the new generalized (G'/G)-expansion method can be further used to solve many nonlinear evolution equations which frequently arise in various scientific real time application fields.

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