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ORIGINAL ARTICLE

Finite element approach to study the behavior of fluid distribution in the dermal regions of human body due to thermal stress

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Abstract The human body is a complex structure where the balance of mass and heat transport in all tissues is necessary for its normal functioning. The stabilities of intracellular and extracellular fluids are important physiological factors responsible for homoeostasis. To estimate the effects of thermal stress on the behavior of extracellular fluid concentration in human dermal regions, a mathematical model based on diffusion equation along with appropriate boundary conditions has been formulated. Atmospheric temperature, evaporation rate, moisture concentration and other factors affecting the fluid concentration were taken into account. The variational finite element approach has been employed to solve the model and the results were interpreted graphically.

2010 MATHEMATICAL SUBJECT CLASSIFICATION: 92BXX; 92CXX; 92C35; 92C50; 46N60

1. Introduction

In a normal adult human body 60% of total body weight consists of fluid. The body fluid is classified as intracellular fluid (ICF) and extracellular fluid (ECF). Intracellular fluid contributes two-third to the total body fluid and is located inside the cells. Extracellular fluid which include lymph fluid, interstitial fluid and plasma, contributes one-third to the total body fluid and is located outside the cells, Guyton [1]. The fluid loss in the body occurs mostly from extracellular fluids. The fluid loss may take place due to unmonitored use of diuretics, dehydration, severe vomiting, diarrhea, fever, diaphoresis, bleeding etc. The immediate effects of fluid loss are body weight loss, thirst, concentrated urine, increase heat rate, low blood pressure, decrease blood circulation rate, inadequate tissue perfusion and inefficient transport of substrates to muscle. On the other hand excessive fluid gain is due to failure to excrete fluids from body. The excessive of fluid leads to increase of hydrostatic pressure, dyspnoea, weight gain,

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decreased pulse, increased respirations, cerebral edema etc., as discussed by Black and Hawks [2]. Therefore for normal functioning of the human body the balance between fluid loss and fluid intake is very important. Since the human skin is one of the most important tissue which controls the fluid concentration in human body. Thus it is imperative to study the behavior of fluid concentration in human skin and subcutaneous tissue at adverse environment conditions.

The skin and its accessory structures make up the integumentary system and provide the body with overall protection. The integumentary system mainly consists of three regions; epidermis, dermis and hypodermis. Epidermis is avascular and composed of keratinized and stratified squamous epithelial. It is made up of four or five layers of epithelial cells, depending on its location in human body. In this study, we have divided it into two main layers as stratum corneum and stratum germinativum based on the presence of squamous and keratinized cells in stratum corneum, and melanocytes and basal layer (consisting of cuboidal cells) in stratum germinativum. Dermis is considered as the core of the integumentary system contains lymph, blood vessels, nerves and other structures such as hair follicles and sweat glands. The dermis consists of two layers as papillary layer and reticular layer of connective tissue that compose an interconnected mesh of elastin and collagenous fibers produced by fibroblasts as discussed by Stuhr et al. [3]. Papillary layer is made up of loose, areolar connective tissue, which means the collagen and elastin fibers of this layer form a loose mesh. Within the papillary layer are fibroblasts, a small number of fat cells, and an abundance of small blood vessels. In addition, the papillary layer contains phagocytes, defensive cells that help to fight against bacteria and other infections that breaches the skin. Underlying the papillary layer is the much thicker reticular layer composed of dense and irregular connective tissue. This layer is well vascularized and has a rich sensory and sympathetic nerve supply. The reticular layer appears reticulated due to a tight meshwork of fibers, Stuhr et al. [3]. The inner most layer hypodermis (also called the subcutaneous layer or superficial fascia) is a layer directly below the dermis and serves to connect the skin to the underlying fascia (fibrous tissue) of the bones and muscles. The hypodermis consists of well vascularized, loose, areolar connective tissue and adipose tissue.

In this study we shall estimate the disturbance to the dermal fluid concentration due to thermal stress. Earlier Khanday and Saxena [4] used diffusion equation and have estimated the volumetric pattern of fluid at various skin and subcutaneous layers by making use of Lagrange's interpolation method. Chao et al. [5] studied the heat and water migration in regional skin and subcutaneous tissue. Experimental study was carried out by Wakabayashi et al. [6] on ten Japanese and ten Malaysian males with matched physical characteristics such as height, body weight and peak oxygen consumption, and investigated the effect of hydration differences on body fluid and temperature regulation between tropical and temperate indigenes exercising in the heat. Mcginty et al. [7] proposed a mathematical model of a drug-eluting stent and obtained an analytical solution for the drug concentration both in the target cells and the interstitial region of the tissue in terms of the drug release concentration at the interface between the polymer and the tissue. Taylor [8] and Khanday [9] studied the sweating in extreme environments: heat loss, heat adaptation, body fluid distribution and thermal strain. Nakagawa et al. [10] demonstrated in vivo measurement of the water content in the dermis by confocal Raman spectroscopy.

To estimate the pattern of fluid concentration at various environmental conditions the present study is based on transient mass diffusion equation. The diffusivity and metabolic fluid regulation are taken to be heterogeneous and variational finite element technique will be employed to solve the formulated model. The role of atmospheric temperature T_a, evaporation rate E and moisture concentration C_a is taken into account for reasonable outcomes. The result earlier proved by Khanday and Saxena [4] is based on the steady state diffusion equation but due to radial approach and transient behavior of the model, this study can give insight into the fluid concentration of limbs and other irregular geometric organs dynamically.

2. Mathematical model

The one dimensional transient mass diffusion equation for fluid transport in skin and subcutaneous tissue is given as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(Dr\frac{\partial C}{\partial r}\right)+R=\frac{\partial C}{\partial t} \tag{1}$$

where $C(r,t)$ is the fluid concentration in the tissue, r is the radial distance of layer from the outer skin surface, t denotes the time, D is the mass diffusivity of fluid in the tissue and R is the rate of metabolic fluid generation.

In order to study the process and distribution of fluid in dermal regions of human body by variational finite element method, we consider the domain: skin and subcutaneous tissue consisting of five layers viz. stratum corneum ($l_0 \leqslant r < l_1$), stratum germinativum ($l_1 \leqslant r < l_2$), papillary layer ($l_2 \leqslant r < l_3$), reticular layer ($l_3 \leqslant r < l_4$) and sub-dermal layer ($l_4 \leqslant r < l_5$). The division of skin and subcutaneous tissue into the five segments is based on the properties viz. diffusivity, fluid regulation, density of vessels and blood circulation, as discussed by Khanday and Saxena [4]. The five interfaces joining the sub-domain are: external atmosphere – stratum corneum, stratum corneum – stratum germinativum, stratum germinativum – papillary layer, papillary layer – reticular layer, reticular layer – subdermal layer.

The boundary condition at the interface between external atmosphere and stratum corneum is taken as:

$$D\frac{\partial C}{\partial r}\bigg|_{r=l_0}=h(C-C_a)+LET_a \tag{2}$$

where h is the mass transfer coefficient, C_a is the moisture concentration in the environment, T_a is the atmospheric temperature, L is the latent heat and E is the evaporation rate. Also, the boundary condition at the inner surface of subcutaneous tissue is taken as:

$$C=C_s \tag{3}$$

where C_s is the moisture concentration of the subdermal layer.

2.1. Variational finite element formulation

The numerical solution based on finite element method is considered to be one of the optimal and feasible numerical methods as discussed by Khanday and Saxena [11], Khanday et al. [12,13], Akshara et al. [14]. Moreover this method gives better

numerical approximation for the solution of partial differential equations in a continuous domain.

The variational integral corresponding to differential Eq. (1) together with boundary condition given by Eq. (2) is

$$I=\frac{1}{2}\int_{l_0}^{l_5}\left[D\left(\frac{\partial C}{\partial r}\right)^2-2RC+\frac{\partial C^2}{\partial t}\right]r\,dr + \frac{1}{2}\delta_e\{h(C_0-C_a)^2+2LET_aC_o\} \tag{4}$$

where $\delta_e=0$ for $r>l_1$ and $\delta_e=1$ for $r\leqslant l_1$. So that the variational integrals for the five layers are

$$I_{(i)}=\frac{1}{2}\int_{l_{j-1}}^{l_j}\left[D^{(i)}\left(\frac{\partial C^{(i)}}{\partial r}\right)^2-2R^{(i)}C^{(i)}+\frac{\partial (C^{(i)})^2}{\partial t}\right]r\,dr + \frac{1}{2}\delta_e\{h(C_0-C_a)^2+2LET_aC_o\} \tag{5}$$

where $j=1, 2, 3, 4, 5$ and $i=c, g, p, r, s$ corresponding to the stratum corneum, stratum germinativum, papillary layer, reticular layer and sub-dermal layer.

Assembling these integrals, we have

$$I=\sum_{i=c}^{s}I_{(i)} \tag{6}$$

The fluid concentration at the interfaces of the sub-domains can be computed by optimizing the integral Eq. (6) with respect to the nodal concentrations $C_0,\ C_c,\ C_g,\ C_p$ and C_r.

3. Solution of the model

For the solution process of the above system we assume the solution $C(r,t)$ to be approximated by means of a shape function. Since, the anatomy of dermal layers reveals the fact that the distance between the different layers of integumentary system is of minute length. Therefore, the use of higher degree polynomial functions does not contribute a significant change in the resultant output as described by Saxena and Pardasani [15], Khanday and Saxena [4], Khanday et al. [12] and Aijaz et al. [16]. Thus we shall use linear shape functions for solution process and the element wise linear shape functions are defined as:

$$c^{(i)}(r)=\frac{\lambda_i l_{j+1}C_i-\lambda_{i+1}l_jC_{i+1}}{l_{j+1}-l_j}+\frac{\lambda_{i+1}C_{i+1}-\lambda_iC_i}{l_{j+1}-l_j}r \tag{7}$$

where $j=0, 1, 2, 3, 4$; $i=c, g, p, r, s$; i and $i+1$ denote the adjacent layers. Also λ_i represents the barrier coefficients determining the flux from one region to another. The diffusivity of stratum corneum, stratum germinativum and subdermal layer is $D_c,\ D_g$ and D_s respectively. The diffusivity of papillary layer and reticular layer is same and defined as

$$\frac{l_4D_g-l_2D_s}{l_4-l_2}+\frac{D_s-D_g}{l_4-l_2}r$$

The fluid regulation for stratum corneum, stratum germinativum, papillary layer, reticular layer and subdermal layer is respectively defined as

$$0,\quad \frac{r-l_1}{l_2-l_1}R_p,\quad R_p,\quad \frac{l_4R_p-l_3R_s}{l_4-l_3}+\frac{R_s-R_p}{l_4-l_3}r\quad\text{and}\quad 2R_p$$

The structural configuration of domain and the variation in the parameters viz. fluid concentration, fluid generation and diffusivity of sub-domain is based on the physiological properties as discussed by Ruch and Patton [17], Keener and Sneyd [18], and Guyton [1].

Using these quantitative assumptions in Eq. (5), the assembled integral given by Eq. (6) determines the concentration of fluid in skin and subcutaneous tissue. Now optimizing this resultant equation by differentiating it partially with respect to the nodal concentration $C_0,\ C_c,\ C_g,\ C_p$ and C_r we get the following linear system of equations.

$$\begin{aligned}
&a_1C_0+a_2C_c+a_3\dot{C}_0+a_4\dot{C}_c=q_1\\
&b_1C_0+b_2C_c+b_3C_g+b_4\dot{C}_0+b_5\dot{C}_c+b_6\dot{C}_g=q_2\\
&e_1C_c+e_2C_g+e_3C_p+e_4\dot{C}_c+e_5\dot{C}_g+e_6\dot{C}_p=q_3\\
&f_1C_g+f_2C_p+f_3C_r+f_4\dot{C}_g+f_5\dot{C}_p+f_6\dot{C}_r=q_4\\
&g_1C_p+g_2C_r+g_3\dot{C}_p+g_4\dot{C}_r=q_5
\end{aligned}$$

where dots$(\cdot)$ denote the derivative with respect to t and the expressions for the coefficients are given in Appendix A. In matrix representation we have

$$AC+B\frac{dC}{dt}=Q \tag{8}$$

where

$$A=\begin{bmatrix}a_1&a_2&0&0&0\\b_1&b_2&b_3&0&0\\0&e_1&e_2&e_3&0\\0&0&f_1&f_2&f_3\\0&0&0&g_1&g_2\end{bmatrix},\ C=\begin{bmatrix}C_0\\C_c\\C_g\\C_p\\C_r\end{bmatrix},$$

$$B=\begin{bmatrix}a_3&a_4&0&0&0\\b_4&b_5&b_6&0&0\\0&e_4&e_5&e_6&0\\0&0&f_4&f_5&f_6\\0&0&0&g_3&g_4\end{bmatrix},\ Q=\begin{bmatrix}q_1\\q_2\\q_3\\q_4\\q_5\end{bmatrix}$$

Taking Laplace transform to Eq. (8)

$$\begin{aligned}&AL(C)+BsL(C)=\tfrac{Q}{s}+BC^0\\&or\ (A+sB)L(C)=\tfrac{Q}{s}+BC^0\end{aligned} \tag{9}$$

where $C^0=[C_0(l_0,0),C_g(l_1,0),C_p(l_2,0),C_r(l_3,0),C_s(l_4,0)]'$. Solving Eq. (9) by Gauss elimination method, the value of vector $L(C)=[L(C_0),L(C_c),L(C_g),L(C_p),L(C_r)]'$ can be computed in terms of the parameter s. Inverse Laplace transform is used to eliminate s and obtain the value of $C=[C_0,C_c,C_g,C_p,C_r]'$. Where the prime $'$ denotes the transpose of the vectors.

4. Numerical calculations

To solve Eq. (9), it is desirable to make use of the numerical values of the parameters used in the model. The size of the dermal tissue and other physiological parameters varies from organ to organ and depends on the individual and its geographical location. A suitable numerical approximation of

Table 1 Physiological and numerical value of parameters, Khanday et al. [4,19].

S. No	Parameter	Value	S. No	Parameter	Value
1	D_c	2.02×10^{-3} m^2 min^{-1}	8	l_0	0 cm
2	D_g	2.038×10^{-3} m^2 min^{-1}	9	l_1	0.10 cm
3	D_s	2.045×10^{-3} m^2 min^{-1}	10	l_2	0.20 cm
4	R_p	1.102×10^{-4} l cm^{-3} min^{-1}	11	l_3	0.35 cm
5	R_s	2.196×10^{-4} l cm^{-3} min^{-1}	12	l_4	0.60 cm
6	h	3.6×10^{-1} m min^{-1}	13	l_5	0.75 cm
7	L	2.42 J g^{-1}			

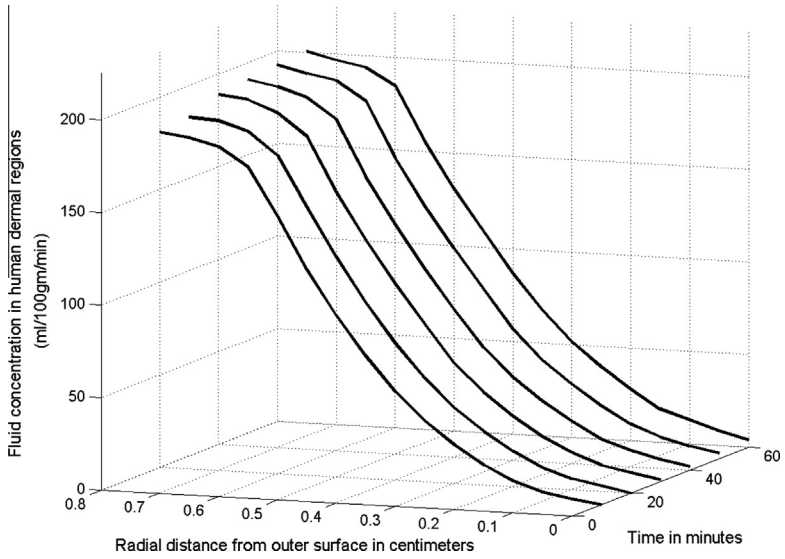

Figure 1 Concentration of fluid in human dermal regions with $T_a = 25\,^{\circ}$C, $C_a = 4.97 \times 10^{-4}\,l\,\text{cm}^{-3}\,\text{min}^{-1}$ and $E = 1.023 \times 10^{-7}\,l\,\text{cm}^{-2}\,\text{min}^{-1}$.

the parameters is given by Table 1 and has been taken from the models used by Khanday et al. [4,19,20], Chao et al. [5], and Shen and Zhang [21]. Due to the minute size and similar anatomy of integumentary system, there is little retardation to diffusion by the barriers of the different layers of skin. But there is a significant retardation to diffusion between atmosphere and stratum corneum due to the change of medium from space to porous. Also there is a moderate retardation to diffusion between the two layers of epidermis due to the presence of dry and dead cells in stratum corneum and the basal cell in stratum germinativum. In dermis both the two components that is papillary layer and reticular layer are made of connective tissue with fibers of collagen extending from one to the other, making the interface between the two somewhat indistinct. Similar situation takes pace with the process of diffusion between the dermis and hypodermis. Therefore the values of barrier coefficients corresponding to these layers are nearly equal to one. The approximate values of the barrier coefficients for stratum corneum, stratum germinativum, papillary layer, reticular layer and subdermal layer are: $\lambda_c = 0.35$, $\lambda_g = 0.85$, $\lambda_r = 0.80$, $\lambda_p = 0.95$ and $\lambda_s = 0.95$ respectively.

5. Discussion and conclusion

For the estimation of effects on the fluid distribution of human integumentary system at various atmospheric temperatures T_a and moisture concentrations C_a, a model based on radial mass diffusion equation along with appropriate boundary conditions has been constructed. The physiological parameters of dermal regions viz. diffusivity, fluid regulation and barrier coefficients have been involved in the model for their significant contribution toward the determination of fluid concentration in the domain. The model has been solved by variational finite element method with linear shape functions as the approximate solutions and the numerical values were obtained using MATLAB software. The Figures 1–3 show the graphs of fluid concentration of dermal regions with respect to radial distance and time. These graphs were drawn at various moisture concentrations, atmospheric temperatures and evaporation rates. It has been observed from these graphs that the temperature significantly effects the fluid concentrations of the various layers of skin and subcutaneous tissue due to the fact that the blood flow and sweat evaporation increase with the increase in

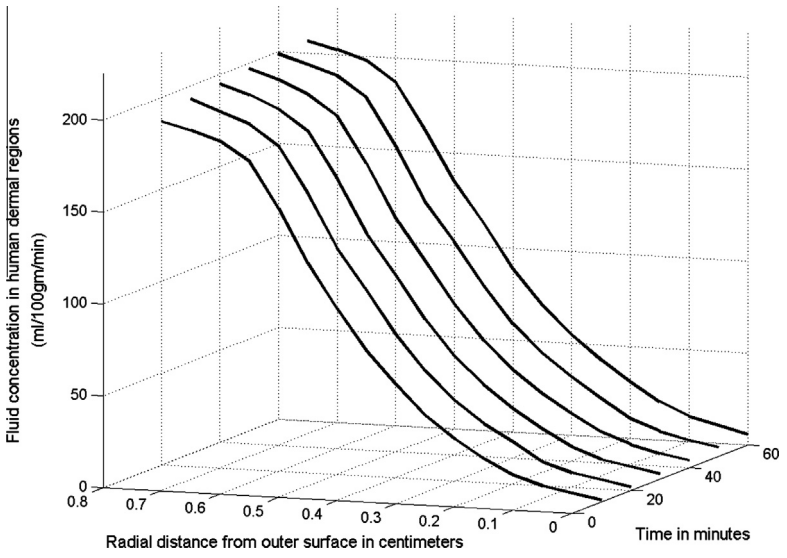

Figure 2 Concentration of fluid in human dermal regions with $T_a = 30\,^{\circ}\mathrm{C}$, $C_a = 5.017 \times 10^{-4}\,\mathrm{l\,cm^{-3}\,min^{-1}}$ and $E = 4.17 \times 10^{-7}\,\mathrm{l\,cm^{-2}\,min^{-1}}$.

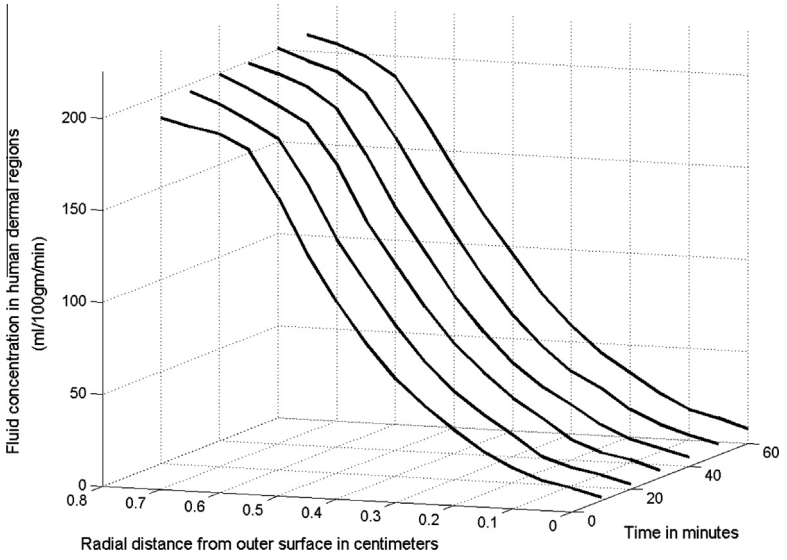

Figure 3 Concentration of fluid in human dermal regions with $T_a = 35\,^{\circ}\mathrm{C}$, $C_a = 5.202 \times 10^{-4}\,\mathrm{l\,cm^{-3}\,min^{-1}}$ and $E = 8.33 \times 10^{-7}\,\mathrm{l\,cm^{-2}\,min^{-1}}$.

atmospheric temperature. Although the overall fluid concentration increases exponentially from outer layer to underlying layers, yet it is evident that with the increase in values of moisture concentration, atmospheric temperature and evaporation rate the fluid concentration increases in outer regions more rapidly than in the inner regions of the dermal system.

The model is significant as a biological point of view due to the fact that any deficit or excess in fluid may lead to various physiological disorders. The model can be used to deal with various biomedical and biophysical problems at different adverse situations. It can also lead to work out the thermal injuries with their behavior at tissue depths depending upon the tissue necrosis. There is a scope of extending this work in two dimensional and three dimensional states to deal with the complex structure of human body. The model can also be modified to study the behavior of fluid concentration due to the immersion of biological tissues in water and other similar situations.

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Appendix A.

$$L_1=\frac{l_1+l_2}{l_2-l_1},\quad L_2=\frac{l_2^2+l_2l_3+l_3^2}{l_3-l_2},\quad L_3=\frac{l_2+l_3}{l_3-l_2},\quad L_4=\frac{l_3+l_4}{l_4-l_3}$$

$$L_5=\frac{l_3^2+l_3l_4+l_4^2}{l_4-l_3},\quad L_6=\frac{l_4+l_5}{l_5-l_4},\quad L_7=\frac{(l_3^3+l_3^2l_4+l_3l_4^2+l_4^3)}{l_4-l_3}$$

$$L_8=\frac{(l_4^2+l_3l_4-2l_3^2)}{(l_4-l_3)},\quad L_9=\frac{(2l_4^2-l_3^2-l_3l_4)}{(l_4-l_3)},$$

$$M_1=\frac{l_4D_s-l_2D_g}{l_4-l_2}$$

$$L_{10}=\frac{(l_3^3+l_3^2l_4+l_3l_4^2-3l_4^2)}{(l_4-l_3)},\ L_{11}=\frac{(l_4^3+l_3^2l_4+l_3l_4^2-3l_3^3)}{(l_4-l_3)},$$

$$a_1=\frac{D_c}{2}+h$$

$$L_1'=\frac{l_1^2+l_1l_2+l_2^2}{l_2-l_1},\ M_2=\frac{D_s-D_g}{l_4-l_2},\quad a_2=\frac{\lambda_cD_c}{2},$$

$$a_3=\frac{l_1^2}{12},\quad a_4=\frac{\lambda_cl_1^2}{12}$$

$$q_1=h_1l_1C_a,\quad b_1=a_2,\quad b_2=\lambda_c^2\left[\frac{D_c}{2}+\frac{D_g(l_1+l_2)}{2(l_2-l_1)}\right],\quad b_4=a_4$$

$$b_3=\lambda_c\lambda_g\frac{D_g(l_1+l_2)}{2(l_2-l_1)},\quad b_5=\lambda_c^2\left[\frac{l_1^2}{4}+\frac{3l_1^3+l_2^3+l_1l_2^2-5l_1^2l_2}{12(l_2-l_1)}\right]$$

$$q_2=\lambda_c\frac{-R_p(l_1^2-l_2^2)}{12},\quad b_6=\frac{\lambda_c\lambda_g(l_1^3+l_2^3-l_1^2l_2-l_1l_2^2)}{12(l_2-l_1)},\quad e_1=b_3$$

$$e_2=\lambda_g^2\left[\frac{D_g(l_1+l_2)}{2(l_2-l_1)}-\frac{M_1L_2}{3}+\frac{M_2L_3}{2}\right],$$

$$e_3=\lambda_g\lambda_p\left[\frac{M_1L_2}{3}-\frac{M_2}{2(l_3-l_2)}\right]$$

$$e_5=\lambda_g^2\left[\frac{l_1^3+3l_2^3+l_1^2l_2-5l_1l_2^2}{12(l_2-l_1)}+\frac{l_2l_3^3-5l_2^2l_3+l_3^3+3l_2^3}{12(l_3-l_2)}\right],\ e_4=b_6$$

$$e_6=\lambda_g\lambda_p\left[\frac{l_2^3+l_3^3-l_2^2l_3-l_2l_3^2}{12(l_3-l_2)}\right],$$

$$f_2=\lambda_p^2\left[\frac{M_2L_3}{2}+\frac{M_1L_2}{3}+\frac{M_2L_4}{2}+\frac{2M_1L_5}{3}\right]$$

$$q_3=\lambda_g\left[\frac{R_pl_1(l_1^2-5l_2^2+l_1l_2)}{12(l_2-l_1)}+\frac{R_p(l_3^2+l_2l_3+l_2^2)}{6}\right],\ f_1=e_3,\ f_4=e_6$$

$$f_3=-\lambda_p\lambda_r\left[\frac{M_2L_4}{2}+\frac{2M_1L_5}{3}\right],\ f_6=\lambda_p\lambda_r\frac{l_3^3+l_4^3-l_3^2l_4-l_3l_4^2}{12(l_4-l_3)}$$

$$f_5=\lambda_p^2\left[\frac{l_2^3+3l_3^3+l_2^2l_3-5l_2l_3^2}{12(l_3-l_2)}+\frac{l_4^3+3l_3^3+l_3l_4^2-5l_3^2l_4}{12(l_4-l_3)}\right]$$

$$q_4=\lambda_p\left[\frac{R_p(2l_3^2-l_2l_3-l_2^2)}{2}+\frac{(l_4R_p-l_3R_s)}{6}L_8+\frac{(R_s-R_p)}{12}L_{11}\right]$$

$$g_1=f_3,\quad g_2=\lambda_r^2\left[\frac{M_2L_4}{2}+\frac{2M_1L_5}{3}+\frac{L_6D_s}{2}\right],\quad g_3=f_6$$

$$g_4=\lambda_r^2\left[\frac{l_3^3+l_3^2l_4+3l_4^3-5l_3l_4^2}{12(l_4-l_3)}+\frac{l_4l_5^2+l_5^3+3l_4^3-5l_4^2l_5}{12(l_5-l_4)}\right]$$

$$q_5=\lambda_r\left[\frac{(l_4R_p-l_3R_s)}{6}L_9-\frac{(R_s-R_p)}{12}L_{10}+\frac{R_p(l_4^2+l_4l_3-2l_3^2)}{3}\right]$$
$$+\lambda_r\lambda_s\frac{L_6D_s}{2}C_s$$

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