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ORIGINAL ARTICLE

Fully developed natural convective micropolar fluid flow in a vertical channel with slip

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KEYWORDS

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Micropolar fluid;
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Abstract The problem of fully developed natural convective micropolar fluid flow is investigated. The slip boundary conditions for fluid velocity are applied. Non-dimensional variables are introduced. The closed form solutions of the field equations are represented graphically. As expected, it can be seen that the increase in micropolarity parameter results in a decrease in the velocity and an increase in the microrotation. Also, it is observed that the increase in the slip parameter increases the velocity and decreases the microrotation. The no slip case can be recovered as a limiting case of this work when the slip parameter goes to infinity.

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1. Introduction

The theory of micropolar fluids is proposed by Eringen [1] to recover the inadequacy of Navier–Stokes theory to describe the correct behavior of some types of fluids with microstructure such as animal blood, muddy water, colloidal fluids, lubricants and chemical suspensions [1]. In the mathematical theory of micropolar fluids there is, in general, six degrees of freedom, three for translation and three for microrotation of microelements. Extensive reviews of the theory and applications can be found in the review articles [2,3] and in the recent books [1,4].

Buoyancy forces, which results from density differences in a fluid caused by the temperature gradients, are responsible for the fluid motion in natural (or free) convection. Natural convection fluid flows play a significant rule in many practical applications including, for example, cooling of electronic components. Natural convection of micropolar fluid flow has attracted the attention of many researchers because of its wide area of applications such as dilute suspensions of polymer fluids, liquid crystals, and chemical suspensions. Gorla et al studied the natural convection boundary layer flow of a micropolar fluid over a vertical plate with a uniform heat flux [5]. Chamkha et al. [6] considered the problem of fully developed free convection of a micropolar fluid in a vertical channel applying the classical no-slip boundary conditions. The free convective laminar flow in a vertical channel with one region

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filled with micropolar fluid and the other region with a viscous fluid assuming no-slip condition is investigated in [7]. Rohni et al. [8] discussed the problem of mixed convection boundary layer flow near the stagnation point on a heated permeable vertical surface embedded in a saturated porous medium with temperature slip effects.

In the last century, several studies have shown that the classical no-slip condition for velocity might not always hold and that fluid slippage might happen at the solid boundary [9–12]. In the literature, many researchers showed that the no-slip boundary condition may lead to singular or nonrealistic behavior (e.g. [13–15]). A general slip boundary condition that allows the possibility of fluid slip at a solid boundary has been proposed by Navier [16]. It states that the tangential relative velocity of the fluid at a point on the solid boundary is proportional to the tangential stress acting at that point. The constant of proportionality is called the slip coefficient and is assumed to depend only on the nature of the fluid and bounding surface [17]. Recently, the slip condition has been used extensively for viscous fluids [17–23], micropolar fluids [24,25], Maxwell fluids [26] and Ferrofluids [27].

Ahmadi [28] and others restricted gyro-viscosity parameter γ by assuming that $\gamma = (\mu + \kappa/2)j$. One may wonder about the effect of this assumption especially for steady fluid flows [1]. This restriction may be used when studying time dependent fluid flows to allow the field equations to predict the correct behavior of viscous fluid flows in the limiting case when micro-structure effects become negligible and microrotation reduces to the angular velocity [1,25]. This motivated the author to investigate the problem of fully developed free convective micropolar fluid flow in a vertical channel without taking this restriction into consideration. Moreover, the more realistic velocity slip boundary conditions are applied at the two vertical walls and its influence is studied.

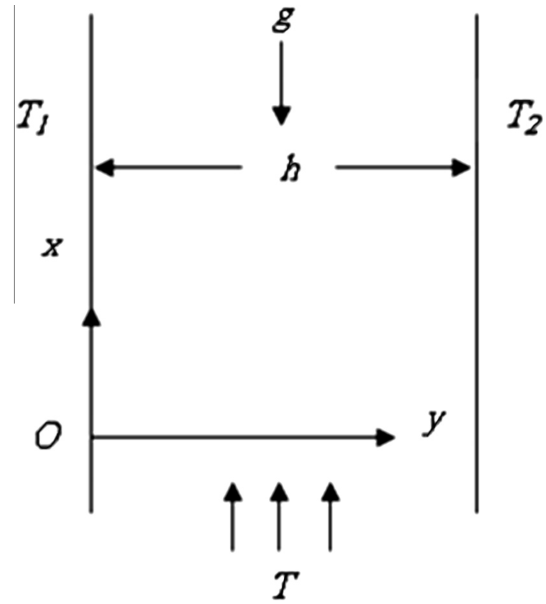

Fig. 1 Geometrical sketch of the problem.

2. Formulation of the problem

Let us consider the laminar free convection flow of an incompressible micropolar fluid between two vertical plates. The motion is assumed to be steady and fully developed and the walls are heated with different uniform temperatures. Working with the Cartesian coordinates (x, y, z), with z axis normal to xy-plane, as shown in Fig. 1, the field equations governing the problem at hand will take the following forms [1].

Balance of momentum:

$$(\mu+\kappa)\frac{d^2u}{dy^2}+\kappa\frac{dv}{dy}+\rho g\beta(T-T_0)=0, \tag{1}$$

Balance of angular momentum:

$$\gamma\frac{d^2v}{dy^2}-\kappa\frac{du}{dy}-2\kappa v=0, \tag{2}$$

Equation of energy:

$$\frac{d^2T}{dy^2}=0, \tag{3}$$

where $u(y)$ is the velocity component along x direction and $v(y)$ is the microrotation about z direction. Also, $T(y)$ represents the fluid temperature. (μ, κ) are the viscousity coefficients and γ is the gyro-viscosity parameter. ρ, β, g and T_0 are, respectively, fluid density, thermal expansion coefficient, gravitational acceleration and temperature of the plane surface.

The following slip and no-spin boundary conditions are proposed

$$\alpha\, u=\tau_{yx},\quad T=T_1,\quad v=0 \quad on\ y=0, \tag{4}$$

$$\alpha\, u=\tau_{yx},\quad T=T_2,\quad v=0 \quad on\ y=h, \tag{5}$$

where T_1 and T_2 are the temperatures of the two walls $y = 0$ and $y = h$, respectively. $0 \leqslant \alpha \leqslant \infty$ is the slip parameter and h is the distance between the two walls. The shear stress τ_{yx} is given by [1]

$$\tau_{yx}=(\mu+\kappa)\frac{du}{dy}+\kappa v. \tag{6}$$

We now introduce the following non-dimensional variables

$$Y=\frac{y}{h},\ U=\frac{u}{U_0},\ N=\frac{h}{U_0}v,\ \theta=\frac{T-T_0}{T_2-T_0},\ T_{yx}=\frac{h}{\mu U_0}\tau_{yx}, \tag{7}$$

where $U_0 = \rho g\beta(T_2 - T_1)h^2/\mu$.

3. Solution of the problem

Substituting the non-dimensional variables (7) into Eqs. (1)–(3), we get

$$(1+K)\frac{d^2U}{dY^2}+K\frac{dN}{dY}+\theta=0, \tag{8}$$

$$\Gamma\frac{d^2N}{dY^2}-K\frac{dU}{dY}-2KN=0, \tag{9}$$

$$\frac{d^2\theta}{dY^2}=0, \tag{10}$$

where $K = \kappa/\mu$ and $\Gamma = \gamma/\mu h^2$.

Note that, if we assumed that $\Gamma = 1 + K/2$, Eq. (9) returns to the classical form assumed by Ahmadi and others who restricted themselves to the relation $\gamma = (\mu + \kappa/2)j$.

Integrating Eq. (10) twice and using boundary conditions (4) and (5) with the aid of non-dimensional variables (7) we arrive at

$$\theta = (1-\theta_1)Y + \theta_1, \tag{11}$$

where $\theta_1 = (T_1 - T_0)/(T_2 - T_0)$.

Substituting for θ from Eq. (11) into Eq. (8) then integrating, we arrive at

$$(1+K)\frac{dU}{dY} + KN + \frac{(1-\theta_1)}{2}Y^2 + \theta_1 Y + C_0 = 0, \tag{12}$$

where C_0 is an arbitrary constant.

Eliminating $\frac{dU}{dY}$ between Eqs. (9) and (12), we get an ordinary differential equation of the unknown N that can be solved to give

$$N = A_1 e^{\eta Y} + B_1 e^{-\eta Y} + \frac{(1-\theta_1)}{2(2+K)}Y^2 + \frac{\theta_1}{(2+K)}Y + C_1, \tag{13}$$

Then substituting the obtained result of N into Eq. (12) and integrating we obtain

$$U = \frac{-K}{\eta(1+K)}\{A_1 e^{\eta Y} - B_1 e^{-\eta Y}\} - \frac{(1-\theta_1)}{3(2+K)}Y^3 - \frac{\theta_1}{(2+K)}Y^2 + \frac{A_2}{(1+K)}Y + \frac{B_2}{(1+K)}, \tag{14}$$

where $\eta^2 = K(2+K)/\Gamma(1+K)$.

Applying the imposed boundary conditions (4) and (5), in non-dimensional form, and then substituting for U and N into Eq. (9), we get a system of five linear algebraic equations that can be solved simultaneously to give

$$A_1 = \frac{1}{\Delta_1}[3M(1-e^\eta)\{2\Gamma\eta(1+K)(\theta_1-1) - K^2(\theta_1+1)\} + \eta K(1+K)\{2M((\theta_1+2)e^\eta + 2(\theta_1+1)) - 3(\theta_1+1)(3K+2)(1-e^\eta)\}], \tag{15}$$

$$B_1 = \frac{e^\eta}{\Delta_1}[3M(1-e^\eta)\{2\Gamma\eta(1+K)(\theta_1-1) + K^2(\theta_1+1)\} - \eta K(1+K)\{2M((2\theta_1+1)e^\eta + \theta_1+2) + 3(\theta_1+1)(3K+2)(1-e^\eta)\}], \tag{16}$$

$$C_1 = \frac{-1}{\Delta_2}[2M\eta(1+K)(1+e^\eta)\{3\Gamma(\theta_1-1) + K(2\theta_1+1)\} + 3K(\theta_1+1)(1-e^\eta)\{\eta(1+K)(3K+2) - MK\}], \tag{17}$$

$$A_2 = \frac{2K(1+K)}{\Delta_2}[\eta(1+K)(1+e^\eta)\{2\mathrm{M}(2\theta_1+1) - 3(\theta_1+1) \times (3K+2)\} + 3M(1-e^\eta)\{2\Gamma(\theta_1-1) - K(\theta_1+1)\}], \tag{18}$$

$$B_2 = \frac{1}{\Delta_1}[2M^2\eta(1+K)(1+e^{2\eta})\{3\Gamma(\theta_1-1) + K(2\theta_1+1)\} + (1-e^{2\eta})\{\eta(1+K)^2(3\eta(\theta_1+1)(3K+2)^2 - 2M[3\Gamma(\theta_1-1)(\eta+2) + \eta(2\theta_1+1)(3K+2)]) - 3M^2K^2(\theta_1+1)\}], \tag{19}$$

where

$$\Delta_1 = -12MK(3K+2)\{\eta(1+K)(1-e^{2\eta}) - K(1-e^\eta)^2\},$$
$$\Delta_2 = 12MK(3K+2)\{\eta(1+K)(1+e^\eta) - K(1-e^\eta)\},$$
$$M = h\alpha/\mu.$$

4. Results and discussion

The system of differential Eqs. (8)–(10) governing the laminar free convection flow of an incompressible micropolar fluid between two vertical plates under the imposed slip and no-spin boundary conditions (4) and (5) have been solved analytically. The closed form solutions given by (11), (13) and, (14) are represented graphically. The non-vanishing components of velocity and microrotation are graphed for different values

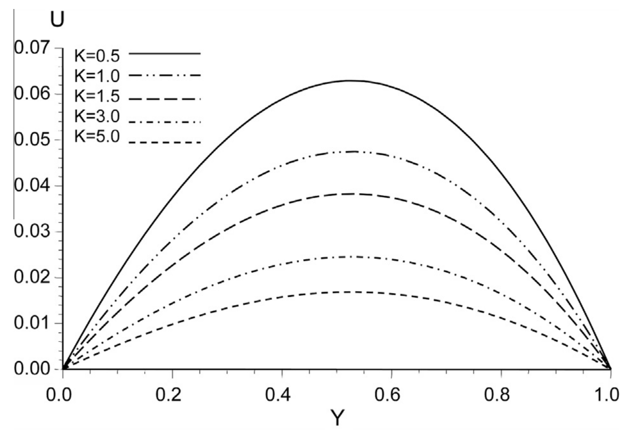

Fig. 2 Velocity profile for $M = \infty$, $\Gamma = 1$ and $\theta_1 = 0.5$.

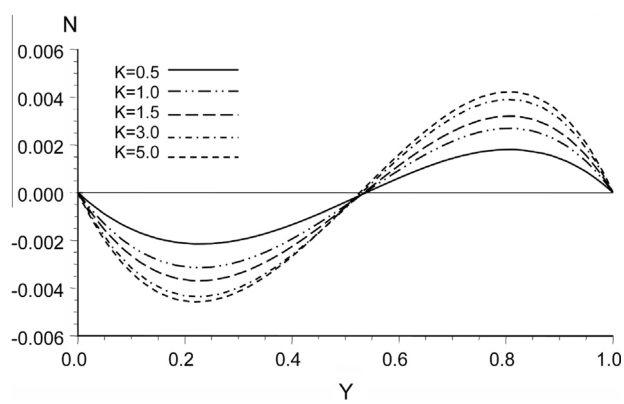

Fig. 3 Microrotation profile for $M = \infty$, $\Gamma = 1$ and $\theta_1 = 0.5$.

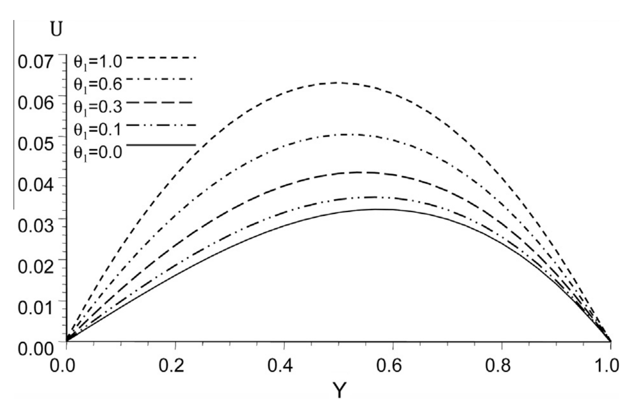

Fig. 4 Velocity Profile for $M = \infty$, $\Gamma = 1$ and $K = 1$.

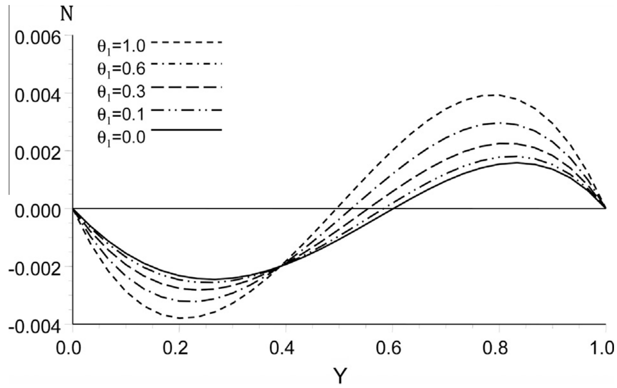

Fig. 5 Microrotation profile for $M = \infty$, $\Gamma = 1$ and $K = 1$.

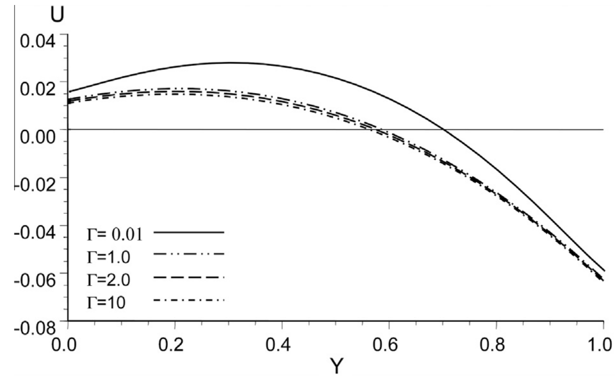

Fig. 8 Velocity profile for $M = 10$, $K = 2$ and $\theta_1 = 0.5$.

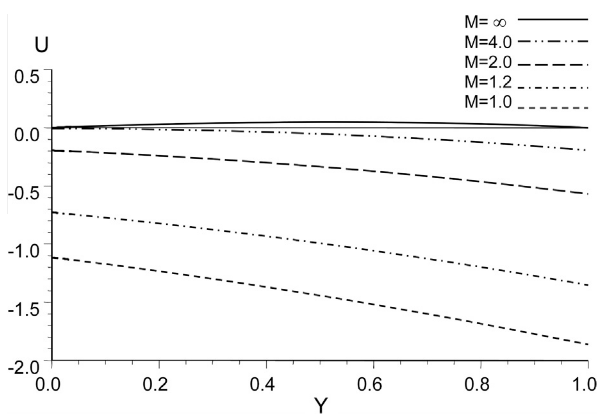

Fig. 6 Velocity profile for $K = 1$, $\Gamma = 2$ and $\theta_1 = 0.5$.

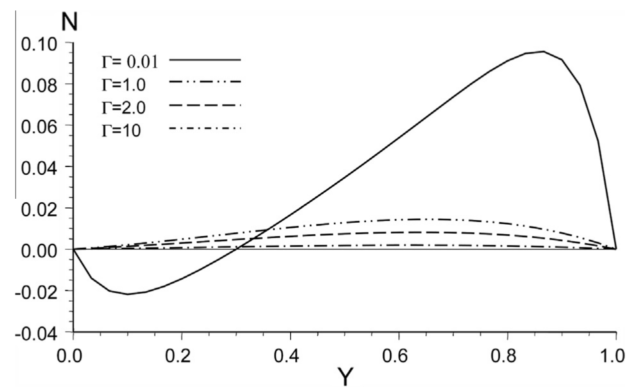

Fig. 9 Microrotation profile for $M = 10$, $K = 2$ and $\theta_1 = 0.5$.

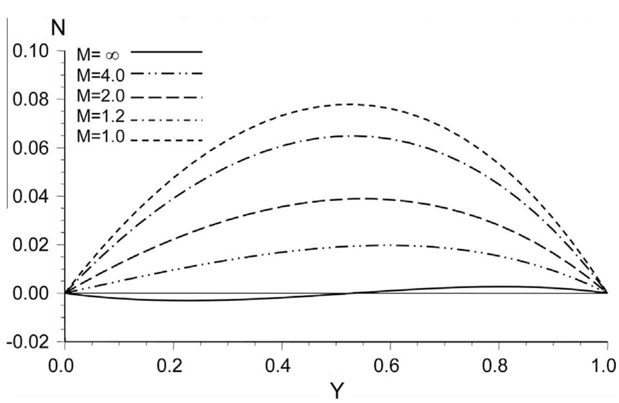

Fig. 7 Microrotation profile for $K = 1$, $\Gamma = 2$ and $\theta_1 = 0.5$.

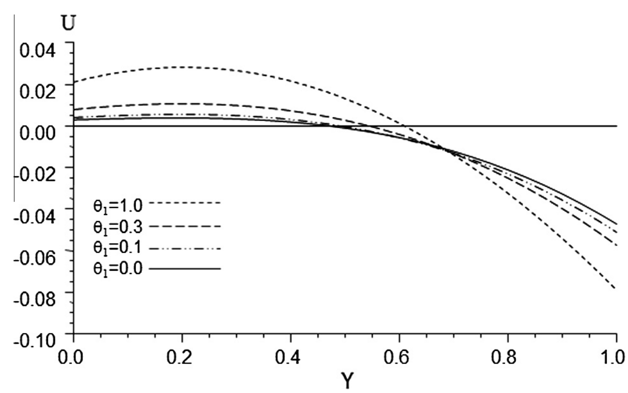

Fig. 10 Velocity profile for $M = 10$, $K = 2$ and $\Gamma = 3$.

of the non-dimensional physical parameters K, θ_1, M and Γ in Figs. 2–10, respectively. Also, the temperature distribution is represented in Fig. 12. From Figs. 2 and 3, it can be observed that the increase in micropolarity parameter K decreases the velocity values but increases the microrotation which means, as expected, that the resistance of the fluid increases with the increase of K. The velocity and microrotation increases monotonically with the increase of θ_1 as seen in Figs. 4 and 5. Figs. 6 and 7 show the velocity and microrotation profiles for different values of the slip parameter M. The classical case of no-slip is recovered when the slip parameter M goes to infinity. It is observed that the increase in slip parameter increases the velocity and decreases the microrotation as shown in the figures. Also, from Figs. 8 and 9 it is seen that the velocity and microrotation values decrease monotonically with the increase in non-dimensional gyro-viscosity coefficient Γ. The increase of θ_1 increases the velocity and microrotation as shown in Figs. 10 and 11. The heat distribution is represented by Fig. 12. The results of Chamkha et al [6] can be recovered as a special case of this work when we let $\Gamma = 1 + K/2$ and M goes to infinity.

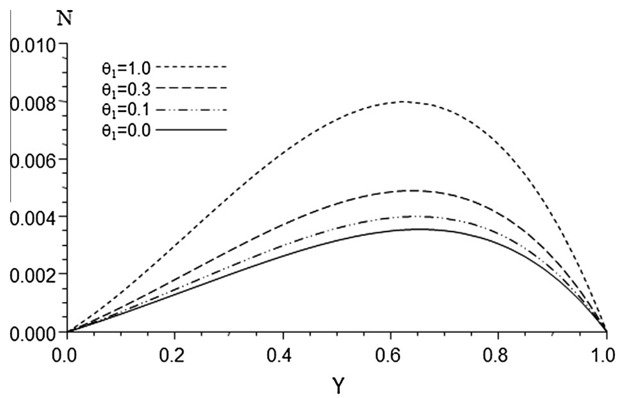

Fig. 11 Microrotation profile for $M = 10$, $K = 2$, $\Gamma = 3$ and $\theta_1 = 0.5$.

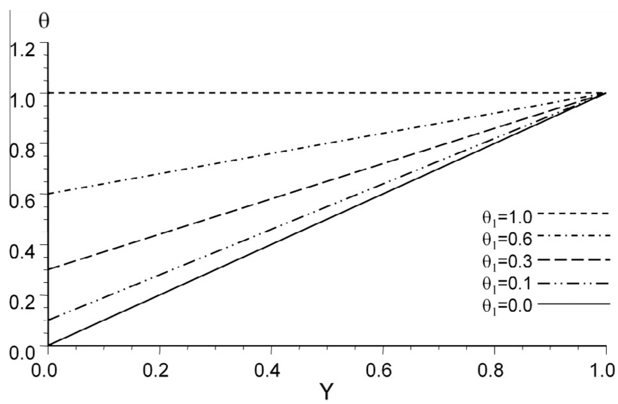

Fig. 12 Temperature distribution.

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References

[1] A.C. Eringen, Microcontinuum Field Theories: II. Fluent Media, Springer-Verlag, New York, 2001.
[2] T. Ariman, M.A. Turk, N.D. Sylvester, Microcontinuum field mechanics – a review, Int. J. Eng. Sci. 11 (1973) 905–929.
[3] T. Ariman, M.A. Turk, N.D. Sylvester, Applications of microcontinuum field mechanics, Int. J. Eng. Sci. 12 (1974) 273–291.
[4] G. Lukaszewicz, Micropolar Fluids, Theory and Application, Birkhäuser, Basel, 1999.
[5] R.S.R. Gorla, A. Slaouti, H.S. Takhar, Free convection in micropolar fluids over a uniformly heated vertical plate, Int. J. Numer. Meth. Heat Fluid Flow 8 (1998) 504–518.
[6] A.J. Chamkha, T. Grosan, I. Pop, Fully developed free convection of a micropolar fluid in a vertical channel, Int. Commun. Heat Mass Transfer 29 (2002) 1119–1127.
[7] J.P. Kumar, J.C. Umavathi, A.J. Chamkha, I. Pop, Fully-developed free-convective flow of micropolar and viscous fluids in a vertical channel, Appl. Math. Model. 34 (2010) 1175–1186.
[8] A.M. Rohni, S. Ahmad, I. Pop, J.H. Merkin, Unsteady mixed convection boundary layer flow with suction and temperature slip effects near the stagnation point on a vertical permeable surface embedded in a porous medium, Transport Porous Med. 92 (2012) 1–14.
[9] J. Happel, H. Brenner, Low Reynolds Number Hydrodynamics, Noordhoff International Publishing, Leyden, Netherlands, 1973.
[10] D.K. Hutchins, M.H. Harper, R.L. Felder, Slip correction measurements for solid spherical particles by modulated dynamic light scattering, Aerosol Sci. Technol. 22 (1995) 202–218.
[11] J. Barrat, L. Bocquet, Large slip effect at a nonwetting fluid-solid interface, Phys. Rev. Lett. 82 (1999) 4671–4674.
[12] R. Pit, H. Hervet, L. Léger, Direct experimental evidence of slip in hexadecane: solid interfaces, Phys. Rev. Lett. 85 (2000) 980–983.
[13] P.A. Thompson, M.O. Robbins, Simulations of contact line motion: slip and the dynamic contact angle, Phys. Rev. Lett. 63 (1989) 766–769.
[14] J. Koplik, J.R. Banavar, Corner flow in the sliding plate problem, Phys. Fluids 7 (1995) 3118–3125.
[15] S. Richardson, On the no-slip boundary condition, J. Fluid Mech. 59 (1973) 707–719.
[16] C.L.M.H. Navier, Memoirs de l'Academie: Royale des Sciences de l'Institut de France, vol. 1, 1823, pp. 414–416.
[17] G. Willmott, Dynamics of a sphere with inhomogeneous slip boundary conditions in Stokes flow, Phys. Rev. E 77 (2008) 055302–055305.
[18] E.A. Ashmawy, Slip at the surface of a general axi-symmetric body rotating in a viscous fluid, Arch. Mech. 63 (2011) 341–361.
[19] F. Yang, Slip boundary condition for viscous flow over solid surfaces, Chem. Eng. Commun. 197 (2010) 544–550.
[20] H.H. Sherief, M.S. Faltas, E.A. Ashmawy, Stokes flow between two confocal rotating spheroids with slip, Arch. Appl. Mech. 82 (2012) 937–948.
[21] E.A. Ashmawy, Unsteady rotational motion of a slip spherical particle in a viscous fluid, ISRN Math. Phys. 2012 (2012) 1–8.
[22] T. Hayat, S.A. Shehzad, M. Qasim, Slip effects on the unsteady stagnation point flow with variable free stream, Walailak J. Sci. Technol. 10 (2013) 385–394.
[23] W.A. Khan, Z.H. Khan, M. Rahi, Fluid flow and heat transfer of carbon nanotubes along a flat plate with Navier slip boundary, Appl. Nanosci. (2013) 1–9.
[24] Y.Y. Lok, I. Pop, D.B. Ingham, Oblique stagnation slip flow of a micropolar fluid, Meccanica 45 (2010) 87–198.
[25] E.A. Ashmawy, Unsteady couette flow of a micropolar fluid with slip, Meccanica 47 (2012) 85–94.
[26] D. Vieru, A. Zafar, Some couette flows of a Maxwell fluid with wall slip condition, Appl. Math. Inf. Sci. 7 (2013) 209–219.
[27] Z.H. Khan, W.A. Khan, I.A. Shah, MHD boundary layer slip flow and heat transfer of ferrofluid along a stretching cylinder with prescribed heat flux, PLoS ONE 9 (2014) 1–6.
[28] G. Ahmadi, Self-similar solution of incompressible micropolar boundary layer flow over a semi-infinite plate, Int. J. Eng. Sci. 14 (1976) 639–646.