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ORIGINAL ARTICLE

Generalized derivations with power values in rings and Banach algebras

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Abstract Let R be a 2-torsion-free prime ring with center $Z(R)$, F a generalized derivation associated with a nonzero derivation d, L a Lie ideal of R. If $(d(u)^{l_1}F(u)^{l_2}d(u)^{l_3}F(u)^{l_4}\ldots F(u)^{l_k})^n = 0$ for all $u \in L$, where $l_1, l_2, \ldots, l_k$ are fixed non-negative integers not all zero, and n is fixed positive integer, then $L \subseteq Z(R)$. We also examine the case when R is a semiprime ring. Finally, we apply the above result to Banach algebras.

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1. Introduction

In all that follows, unless stated otherwise, R will be an associative ring, $Z(R)$ the center of R, Q its Martindale quotient ring and U its Utumi quotient ring. The center of U, denoted by C, is called the extended centroid of R (we refer the reader to [1] for these objects). By a Banach algebra we shall mean complex normed algebra A whose underlying vector space is a Banach space. The Jacobson radical $rad(A)$ of A is the intersection of all primitive ideals. If the Jacobson radical reduces to the zero element, A is called semisimple. For any $x, y \in R$, the symbol $[x, y]$ denotes the Lie product $xy - yx$. A ring R is called 2-torsion free, if whenever $2x = 0$, with $x \in R$, then $x = 0$. Recall that a ring R is prime if for any $a, b \in R$, $aRb = (0)$ implies $a = 0$ or $b = 0$, and is semiprime if for any $a \in R$, $aRa = (0)$ implies $a = 0$. An additive mapping $d: R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. In particular d is an inner derivation induced by an element $a \in R$, if $d(x) = [a, x]$ for all $x \in R$. The standard identity s_4 in four variables is defined as follows:

$$s_4 = \sum 1)^{\tau} X_{\tau(1)} X_{\tau(2)} X_{\tau(3)} X_{\tau(4)}$$

where $(-1)^{\tau}$ is the sign of a permutation τ of the symmetric group of degree 4.

Let us introduce the background of our investigation. Singer and Werner [2] obtained a fundamental result which stated investigation into the ranges of derivations on Banach algebras. In [2], Singer and Werner proved that any continuous derivation on a commutative Banach algebra has the range in the Jacobson radical of the algebra. In this paper they conjectured that the continuity is not necessary. Thomas [3] verified this conjecture. It is clear that the same result of Singer and Werner does not hold in noncommutative Banach

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algebras because of inner derivations. Hence in this context a very interesting question is how to obtain noncommutative version of Singer-Werner theorem. A first answer to this problem has been obtained by Sinclair in [4]. He proved that every continuous derivation of a Banach algebra leaves primitive ideals of the algebra invariant. In [5], Kim proved that if a noncommutative Banach algebra A admits a continuous linear Jordan derivation d such that $d(x)[d(x),x]d(x)\in rad(A)$ for all $x\in A$ then $d(A)\subseteq rad(A)$. More recently, Park [6] proved that if d is a linear continuous derivation of a noncommutative Banach algebra A such that $[[d(x),x],d(x)]\in rad(A)$ for all $x\in A$ then $d(A)\subseteq rad(A)$. In [7], Filippis extended the Park's result to generalized derivations. In the meanwhile many authors obtained more information about derivations satisfying certain suitable conditions in Banach algebra. For example, in [8] Vukman proved that if d is a linear derivation of a noncommutative semisimple Banach algebra A such that $[d(x),x]d(x)=0$ for all $x\in A$, then $d=0$.

In [9], Brešar introduced the definition of generalized derivation: an additive mapping $F{:}R\to R$ is called a generalized derivation if there exists a derivation $d{:}R\to R$ such that $F(xy)=F(x)y+xd(y)$ holds for all $x,y\in R$, and d is called the associated derivation of F. Hence, the concept of generalized derivations covers both the concepts of a derivation and of a left multiplier (i.e., an additive mapping satisfying $f(xy)=f(x)y$ for all $x,y\in R$). Basic examples are derivations and generalized inner derivations (i.e., mappings of type $x\to ax+xb$ for some $a,b\in R$). We refer to call such mappings generalized inner derivations for the reason they present a generalization of the concept of inner derivations. In [10], Hvala studied generalized derivations in the context of algebras on certain norm spaces. In [11], Lee extended the definition of a generalized derivation as follows: by a generalized derivation we mean an additive mapping $F{:}I\to U$ such that $F(xy)=F(x)y+xd(y)$ holds for all $x,y\in I$, where I is a dense left ideal of R and d is a derivation from I into U. Moreover, Lee also proved that every generalized derivation can be uniquely extended to a generalized derivation of U and thus all generalized derivations of R will be implicitly assumed to be defined on the whole of U. Lee obtained the following: every generalized derivation F on a dense left ideal of R can be uniquely extended to U and assumes the form $F(x)=ax+d(x)$ for some $a\in U$ and a derivation d on U.

On the other hand, a well-known result of Herstein [12] states that if ρ is a right ideal of R such that $u^n=0$ for all $u\in\rho$, where n is a fixed positive integers, then $\rho=0$. In [13], Chang and Lin considered the situation when $d(u)u^n=0$ for all $u\in\rho$, where d is a nonzero derivation of R. In [14], Dhara and De Filippis studied the case when $u^sH(u)u^t=0$ for all $u\in L$, where L a noncommutative Lie ideal of R, H a generalized derivation of R and s,t are fixed nonnegative integers. More precisely, they proved the following: Let R be a prime ring, H a nonzero generalized derivation of R and L a noncommutative Lie ideal of R. Suppose that $u^sH(u)u^t=0$ for all $u\in L$. Then R satisfies s_4, the standard identity in four variables.

The present paper is motivated by the previous results and we here continue this line of investigation by examining what happens a ring R (or an algebra A) satisfying the identity $(d(u)^{l_1}F(u)^{l_2}d(u)^{l_3}F(u)^{l_4}\dots F(u)^{l_k})^n=0$ for all u in some appropriate subset of R (or A).

2. The results

Theorem 2.1. *Let R be a 2-torsion-free prime ring with center $Z(R)$, F a generalized derivation associated with a nonzero derivation d, L a Lie ideal of R. If $(d(u)^{l_1}F(u)^{l_2}d(u)^{l_3}F(u)^{l_4}\dots F(u)^{l_k})^n=0$ for all $u\in L$, where $l_1,l_2,\dots,l_k$ are fixed nonnegative integers not all zero, and n is fixed positive integer, then $L\subseteq Z(R)$.*

Proof. Suppose that $L\not\subseteq Z(R)$. Since R is a prime ring and F is a generalized derivation of R, by Lee [11, Theorem 3], $F(x)=ax+d(x)$ for some $a\in U$. Since Char$(R)\neq 2$ it follows from Herstein [12, pp.4-5], that there exists a nonzero two-sided ideal I of R such that $0\neq[I,R]\subseteq L$. In particular, $[I,I]\subseteq L$, hence without loss of generality we may assume that $L=[I,I]\subseteq L$. By the given hypothesis we have $(d([x,y])^{l_1}F([x,y])^{l_2}d([x,y])^{l_3}F([x,y])^{l_4}\dots F([x,y])^{l_k})^n=0$ for all $x,y\in I$. This implies that $(([d(x),y]+[x,d(y)])^{l_1}(a[x,y]+[d(x),y]+[x,d(y)])^{l_2}\dots(a[x,y]+[d(x),y]+[x,d(y)])^{l_k})^n=0$ for all $x,y\in I$. By Kharchenko [15], we divide the proof into two cases:

Case 1. Let d be an outer derivation of U, then R satisfies the polynomial identity $(([s,y]+[x,t])^{l_1}(a[x,y]+[s,y]+[x,t])^{l_2}\dots(a[x,y]+[s,y]+[x,t])^{l_k})^n=0$ for all $x,y,s,t\in I$. In particular, for $x=0$, we arrive at $[s,y]^p=0$ for all $s,y\in I$, where $p=n(l_1+l_2+\cdots+l_k)$, and by Herstein [16, Theorem 2], R is commutative, a contradiction.

Case 2. Let now d be the inner derivation induced by an element $q\in Q$, that is $d(x)=[q,x]$ for all $x,y\in U$. It follows that $(([[q,x],y]+[x,[q,y]])^{l_1}(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_2}\dots(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_k})^n=0$ for all $x,y\in I$. By Chuang [17, Theorem 2], I and Q satisfy the same generalized polynomial identities (GPIs), we have $(([[q,x],y]+[x,[q,y]])^{l_1}(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_2}\dots(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_k})^n=0$ for all $x,y\in Q$. In case center C of Q is infinite, we have $(([[q,x],y]+[x,[q,y]])^{l_1}(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_2}\dots(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_k})^n=0$ for all $x,y\in Q\otimes_C\overline{C}$, where $\overline{C}$ is the algebraic closure of C. Since both Q and $Q\otimes_C\overline{C}$ are prime and centrally closed [18, Theorem 2.5 and Theorem 3.5], we may replace R by Q or $Q\otimes_C\overline{C}$ according as C is finite or infinite. Thus we may assume that R is centrally closed over C (i.e. $RC=C$) which is either finite or algebraically closed and $(([[q,x],y]+[x,[q,y]])^{l_1}(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_2}\dots(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_k})^n=0$ for all $x,y\in R$. By Martindale [19, Theorem 3], RC (and so R) is a primitive ring which is isomorphic to a dense ring of linear transformations of a vector space V over a division ring D.

Assume that $dim\ V_D\geqslant 3$.

First of all, we want to show that v and qv are linearly D-dependent for all $v\in V$. Since if $qv=0$ then v,qv is D-dependent, suppose that $qv\neq 0$. If v and qv are D-independent, since $dim\ V_D\geqslant 3$, then there exists $w\in V$ such that v,qv,w are also D-independent. By the density of R, there exists $x,y\in R$ such that: $xv=0$, $xqv=w$, $xw=v$; $yv=0$, $yqv=0$, $yw=v$. These imply that $v=(([[q,x],y]+[x,[q,y]])^{l_1}(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_2}\dots(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_k})^n v=0v=0$, which is a contradiction. So we conclude that v and qv are linearly D-dependent for all $v \in V$.

Our next goal is to show that there exists $b \in D$ such that $qv = vb$ for all $v \in V$. In fact, choose $v, w \in V$ linearly independent. Since $dim\ V_D \geqslant 3$, then there exists $u \in V$ such that u, v, w are linearly independent, and so $b_u, b_v, b_w \in D$ such that $qu = ub_u$, $qv = vb_v$, $qw = wb_w$, that is $q(u + v + w) = ub_u + vb_v + wb_w$. Moreover $q(u + v + w) = (u + v + w)b_{u+v+w}$ for a suitable $b_{u+v+w} \in D$. Then $0 = u(b_{u+v+w} - b_u) + v(b_{u+v+w} - b_v) + w(b_{u+v+w} - b_w)$ and because u, v, w are linearly independent, $b_u = b_v = b_w = b_{u+v+w}$, that is b does not depend on the choice of v. Hence now we have $qv = vb$ for all $v \in V$.

Now for $r \in R$, $v \in V$, we have $(rq)v = r(qv) = r(vb) = (rv)b = q(rv)$, that is $[q, R]V = 0$. Since V is a left faithful irreducible R-module, hence $[q, R] = 0$, i.e. $q \in Z(R)$ and so $d = 0$, a contradiction.

Suppose now that $dim\ V_D \leqslant 2$.

In this case R is a simple GPI-ring with 1, and so it is a central simple algebra finite dimensional over its center. By Lanski [20, Lemma 2], it follows that there exists a suitable filed F such that $R \subseteq M_k(F)$, the ring of all $k \times k$ matrices over F, and moreover $M_k(F)$ satisfies the same GPI as R.

Assume $k \geqslant 3$, by the same argument as in the above, we can get a contradiction.

Obviously if $k = 1$, then R is commutative, again a contradiction.

Thus we may assume that $k = 2$, i.e., $R \subseteq M_2(F)$, where $M_2(F)$ satisfies $(([[q,x],y]+[x,[q,y]])^{l_1}(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_2}\dots(a[x,y]+[[q,x],y]+[x,[q,y]])^{l}{}_{k})^n = 0$. Denote e_{ij} the usual matrix unit with 1 in (i, j)-entry and zero elsewhere. Let $[x, y] = [e_{21}, e_{11}] = e_{21}$. It is easy to see that $((qe_{21} - e_{21}q)^{l_1}(ae_{21} + qe_{21} - e_{21}q)^{l_2}\dots(ae_{21} + qe_{21} - e_{21}q)^{l_k})^n = 0$. Right multiplication by e_{21} in the above equation gives that $(-1)^m(e_{21}q)^m e_{21} = ((qe_{21} - e_{21}q)^{l_1}(ae_{21} + qe_{21} - e_{21}q)^{l_2}\dots(ae_{21} + qe_{21} - e_{21}q)^{l_k})^n e_{21} = 0e_{21} = 0$, where $m = n(l_1 + l_2 + \dots + l_k)$. Set $q = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$, then by calculation we find that $(-1)^m\begin{pmatrix} 0 & 0 \\ q_{12}^m & 0 \end{pmatrix} = 0$, which implies that $q_{12} = 0$. Similarly we can see that $q_{21} = 0$. Therefore q is diagonal in $M_2(F)$. Let $f \in Aut(M_2(F))$. Since $(([[f(q),f(x)],f(y)]+[f(x),[f(q),f(y)]])^{l_1}(f(a)[f(x),f(y)]+[[f(q),f(x)],f(y)]+[f(x),[f(q),f(y)]])^{l_2}\dots(f(a)[f(x),f(y)] + [[f(q),f(x)],f(y)] + [f(x),[f(q),f(y)]])^{l}{}_{k})^n = 0$ so $f(q)$ must be a diagonal matrix in $M_2(F)$. In particular, let $f(x) = (1 - e_{ij})x(1 + e_{ij})$ for $i \neq j$, then $f(q) = q + (q_{ii} - q_{jj})e_{ij}$, that is $q_{ii} = q_{jj}$ for $i \neq j$. This implies that q is central in $M_2(F)$, which leads to $d = 0$, a contradiction. This completes the proof of the theorem. □

The following example demonstrates that R to be prime is essential in the hypothesis of Theorem 2.1.

Example 2.2. Let Z be the ring of integers. Set $R = \left\{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} | a, b, c \in Z\right\}$ and $L = \left\{\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} | b \in Z\right\}$. We define the following maps: $F\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} a & 2b \\ 0 & 0 \end{pmatrix}$. $d\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$. Then it is easy to see that L is a Lie ideal and F is a generalized derivation associated with a nonzero derivation d of R. Moreover, it is straightforward to check that F satisfies the property $(d(u)^{l_1}F(u)^{l_2}d(u)^{l_3}F(u)^{l_4}\dots F(u)^{l_k})^n = 0$ for all $u \in L$, where $l_1, l_2, \dots, l_k$ and n are fixed positive integers, however $L \not\subseteq Z(R)$.

Corollary 2.3. *Let R be a 2-torsion-free prime ring, F a generalized derivation associated with a nonzero derivation d. If $(d(r)^{l_1}F(r)^{l_2}d(r)^{l_3}F(r)^{l_4}\dots F(r)^{l_k})^n = 0$ for all $r \in R$, where $l_1, l_2, \dots, l_k$ are fixed non-negative integers not all zero, and n is fixed positive integer, then R is commutative.*

Theorem 2.4. *Let R be a 2-torsion-free semiprime ring, F a generalized derivation associated with a nonzero derivation d. If $(d(r)^{l_1}F(r)^{l_2}d(r)^{l_3}F(r)^{l_4}\dots F(r)^{l_k})^n = 0$ for all $r \in R$, where $l_1, l_2, \dots, l_k$ are fixed non-negative integers not all zero, and n is fixed positive integer, then there exists a central idempotent element e in U such that on the direct sum decomposition $R = eU \oplus (1 - e)U$, d vanishes identically on eU and the ring $(1 - e)U$ is commutative.*

Proof. We are given that $(d(r)^{l_1}F(r)^{l_2}d(r)^{l_3}F(r)^{l_4}\dots F(r)^{l_k})^n = 0$ for all $r \in R$. Since R is semiprime and F is a generalized derivation of R, by Lee [11, Theorem 3], $F(x) = ax + d(x)$ for some $a \in U$. And hence we have $(d(r)^{l_1}(ar + d(r))^{l_2}d(r)^{l_3}(ar + d(r))^{l_4}\dots(ar + d(r))^{l_k})^n = 0$ for all $r \in R$. By Lee [22, Theorem 3], R and U satisfy the same differential identities, then $(d(r)^{l_1}(ar + d(r))^{l_2}d(r)^{l_3}(ar + d(r))^{l_4}\dots(ar + d(r))^{l_k})^n = 0$ for all $r \in U$. Let B be the complete Boolean algebra of idempotents in C and M be any maximal ideal of B. Since U is a B-algebra orthogonal complete [21, p.42], and MU is a prime ideal of U, which is d-invariant. Denote $\overline{U} = U/MU$ and $\bar{d}$ the derivation induced by d on $\overline{U}$, i.e., $\bar{d}(\bar{u}) = \overline{d(u)}$ for all $u \in U$. For all $\bar{r}, \in \overline{U}$, $(\bar{d}(\bar{r})^{l_1}(\bar{a}\bar{r} + \bar{d}(\bar{r})^{l_2}\bar{d}(\bar{r}^{l_3}(\bar{a}\bar{r} + \bar{d}(\bar{r})^{l_4}\dots(\bar{a}\bar{r} + \bar{d}(\bar{r})^{l_k})^n = \bar{0}$. It is obvious that $\overline{U}$ is prime. Therefore by Corollary 2.3, we have either $\overline{U}$ is commutative or $\bar{d} = 0$, that is either $d(U) \subseteq MU$ or $[U, U] \subset MU$. Hence $d(U)[U, U] \subseteq MU$, where MU runs over all prime ideals of U. Since $\cap_M MU = 0$, we obtain $d(U)[U, U] = 0$.

By using the theory of orthogonal completion for semiprime rings (see [1], Chapter 3), it is clear that there exists a central idempotent element e in U such that on the direct sum decomposition $R = eU \oplus (1 - e)U$, d vanishes identically on eU and the ring $(1 - e)U$ is commutative. This completes the proof of the theorem. □

Theorem 2.5. *Let A be a 2-torsion-free non-commutative Banach algebra with Jacobson radical $rad(A)$. Let $F = L_a + d$ be a continuous generalized derivation of R, where L_a denote the left multiplication by some element $a \in A$ and d is the associated derivation of A. If $(d(r)^{l_1}F(r)^{l_2}d(r)^{l_3}F(r)^{l_4}\dots F(r)^{l_k})^n \in rad(A)$ for all $r \in A$, where $l_1, l_2, \dots, l_k$ are fixed non-negative integers not all zero, and n is fixed positive integer, then $d(A) \subseteq rad(A)$.*

Proof. By the hypothesis F is continuous and moreover since it is well-known that L_a also continuous, we get that d is continuous. In [4], Sinclair proved that any continuous derivation of a Banach algebra leaves the primitive ideals invariant. Hence, for any primitive ideal P of A, it is obvious that $F(P) \subseteq aP + d(P) \subseteq P$. It means that the continuous generalized derivation F leaves the primitive ideals invariant. Denote $A/P = \overline{A}$ for any primitive ideals P. Thus we can define the generalized derivation $F_P : \overline{A} \to \overline{A}$ by $F_P(\bar{x}) = F_P(x + P) = F(x) + P = ax + d(x) + P = ax + d_P(\bar{x})$ for all $\bar{x} \in \overline{A}$, where $A/P = \overline{A}$ is a factor Banach algebra. Since P is a primitive ideal, the factor algebra $\overline{A}$ is primitive and so it is prime. The hypothesis $(d(r)^{l_1} F(r)^{l_2} d(r)^{l_3} F(r)^{l_4} \ldots F(r)^{l_k})^n \in rad(A)$ for all $r \in A$, yields that $(d_P(\bar{r})^{l_1} F_P(\bar{r})^{l_2} (d_P(\bar{r})^{l_3} F_P(\bar{r})^{l_4} \ldots F_P(\bar{r})^{l_k})^n = 0$ for all $\bar{r} \in \overline{A}$. From Corollary 2.3, it is immediate that either $\overline{A}$ is commutative or $d = \bar{0}$, that is $[A, A] \subseteq P$ or $d(A) \subseteq P$.

Now we assume that P is a primitive ideal such that $\overline{A}$ is commutative. In [2] Singer and Werner proved that any continuous linear derivation on a commutative Banach algebra maps the algebra into the radical. Furthermore by a result of Jonhson and Sinclair [23], any linear derivation on semisimple Banach algebra is continuous. We know that there are no nonzero linear continuous derivations on commutative semisimple Banach algebras. Therefore $d = \bar{0}$ in $\overline{A}$.

Hence in any case we get $d(A) \subseteq P$ for all primitive ideal P of A. Since $rad(A)$ is the intersection of all primitive ideals, we get $d(A) \subseteq rad(A)$, we get the required conclusion. □

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References

[1] K.I. Beidar, W.S. Martindale, V. Mikhalev, Rings with Generalized Identities, Monographs and Textbooks in Pure and Applied Mathematics, vol. 196, Marcel Dekker, Inc., New York, 1996.

[2] I.M. Singer, J. Werner, Derivations on commutative normed algebras, Math. Ann. 129 (1955) 260–264.

[3] M.P. Thomas, The image of a derivation is contained in the radical, Ann. Math. 128 (3) (1988) 435–460.

[4] A.M. Sinclair, Continuous derivations on Banach algebras, Proc. Amer. Math. Soc. 20 (1969) 166–170.

[5] B.D. Kim, On the derivations of semiprime rings and noncommutative Banach algebras, Acta Math. Sinica 16 (2000) 21–28.

[6] K.H. Park, On derivations in noncommutative semiprime rings and Banach algebras, Bull. Korean Math. Soc. 42 (2005) 671–678.

[7] V.D. Filippis, Generalized derivations in prime rings and noncommutative Banach algebras, Bull. Korean Math. Soc. 45 (2008) 621–629.

[8] J. Vukman, A result concerning derivations in noncommutative Banach algebras, Glas. Math. Ser. III 26 (1-2(46)) (1991) 82–88.

[9] M. Brešar, On the distance of the composition of two derivations to be the generalized derivations, Glasgow Math. J. 33 (1991) 89–93.

[10] B. Hvala, Generalized derivations in prime rings, Comm. Algebra 26 (4) (1998) 1147–1166.

[11] T.K. Lee, Generalized derivations of left faithful rings, Comm. Algebra 27 (8) (1998) 4057–4073.

[12] I.N. Herstein, Topics in ring theory, Univ. of Chicago Press, Chicago, 1969.

[13] C.M. Chang, Y.C. Lin, Derivations on one-sided ideals of prime rings, Tamsui Oxf. J. Math. Sci. 17 (2) (2001) 139–145.

[14] B. Dhara, V. De Filippis, Notes on generalized derivations on Lie ideals in prime rings, Bull. Korean Math. Soc. 46 (3) (2009) 599–605.

[15] V.K. Kharchenko, Differential identities of prime rings, Algebra and Logic 17 (1978) 155–168.

[16] I.N. Herstein, Center-like elements in prime rings, J. Algebra 60 (1979) 567–574.

[17] C.L. Chuang, GPIs having coefficents in Utumi quotient rings, Proc. Amer. Math. Soc. 103 (3) (1988) 723–728.

[18] J.S. Erickson, W.S. Martindale III, J.M. Osborn, Prime nonassociative algebras, Pacific J. Math. 60 (1) (1975) 49–63.

[19] W.S. Martindale III, Prime rings satisfying a generalized polynomial identity, J. Algebra 12 (1969) 176–584.

[20] C. Lanski, An Engel condition with derivation, Proc. Amer. Math. Soc. 118 (3) (1993) 731–734.

[21] C.L. Chuang, Hypercentral derivations, J. Algebra 161 (1994) 37–71.

[22] T.K. Lee, Semiprime rings with differential identities, Bull. Inst. Math. Acad. Sinica 20 (1) (1992) 27–38.

[23] B.E. Johnson, A.M. Sinclair, Continuity of derivations and a problem of Kaplansky, Amer. J. Math. 90 (1968) 1067–1073.