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Generalized Jordan triple derivations associated with Hochschild 2–cocycles of rings

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Abstract
In the present work, we introduce the notion of a generalized Jordan triple derivation associated with a Hochschild 2–cocycle, and we prove results which imply under some conditions that every generalized Jordan triple derivation associated with a Hochschild 2–cocycle of a prime ring with characteristic different from 2 is a generalized derivation associated with a Hochschild 2–cocycle.

Keywords: Prime ring, Derivation, Generalized Jordan triple derivation, Hochschild 2–cocycle

Introduction

Let R denote an associative ring with center $Z(R)$. A ring R is said to have characteristic n if n is the least positive integer such that $nx = 0$ for all $x \in R$, and of characteristic not n if $nx = 0, x \in R$, then $x = 0$. An additive subgroup L of R is called a Lie ideal of R if $[u, r] \in L$ for all $u \in L, r \in R$. A Lie ideal L is said to be a square-closed Lie ideal of R if $u^2 \in L$ for all $u \in L$. An R-bimodule M is a left and right R-module such that $x(my) = (xm)y$ for all $m \in M$ and $x, y \in R$. Recall that a ring R is called prime if $xRy = (0)$ implies that either $x = 0$ or $y = 0$, and R is called semiprime if $xRx = (0)$ implies $x = 0$. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. d is called a Jordan derivation in case $d(x^2) = d(x)x + xd(x)$ for all $x \in R$. Moreover, d is called a Jordan triple derivation if $d(xyx) = d(x)yx + xd(y)x + xyd(x)$ for all $x, y \in R$. It is obvious to see that every derivation is a Jordan derivation and is a Jordan triple derivation but the converse is in general not true. A classical result of Herstein [1] asserts that any Jordan derivation of a prime ring with characteristic different from 2 is a derivation. In [2], Brešar has proved Herstein's result in the case of a semiprime ring. Also, he has shown in [3] that any Jordan triple derivation of a 2-torsion free semiprime ring is a derivation. An additive map f of a ring R is called a generalized derivation if there is a derivation d of R such that for all x, y in R, $f(xy) = f(x)y + xd(y)$ and is called a generalized Jordan derivation if there is a Jordan derivation d such that $f(x^2) = f(x)x + xd(x)$ for all $x \in R$. Furthermore, f is said to be a generalized Jordan triple derivation if there is a Jordan triple derivation d of R such that for all x, y in R, $f(xyx) = f(x)yx + xd(y)x + xyd(x)$. In [4], Jing and Lu have proved in a prime ring R of characteristic not two that every generalized Jordan derivation of R

is a generalized derivation, and also every generalized Jordan triple derivation on R is a generalized derivation.

Let θ and ϕ be endomorphisms of a ring R. f is called a $(\theta,\phi)-$derivation if $f(xy) = f(x)\theta(y) + \phi(x)f(y)$ for all $x,y \in R$. f is called a Jordan $(\theta,\phi)-$derivation if $f(x^2) = f(x)\theta(x) + \phi(x)f(x)$ for all $x \in R$. f is called a Jordan triple $(\theta,\phi)-$derivation if $f(xyx) = f(x)\theta(y)\theta(x) + \phi(x)f(y)\theta(x) + \phi(x)\phi(y)f(x)$ for all $x,y \in R$. In [5], Liu and Shiue have proved that every Jordan triple $(\theta,\phi)-$derivation on a 2-torsion free semiprime ring R is a $(\theta,\phi)-$derivation, where θ and ϕ are automorphisms. An additive mapping $f : R \to R$ is said to be a left (right) centralizer, if $f(xy) = f(x)y$ ($f(xy) = xf(y)$) for all $x,y \in R$. f is called a centralizer, if f is both a left and right centralizer. In [6], Vukman and Kosi-Ulbl have shown that if R is a 2-torsion free semiprime ring and f is an additive mapping of R such that $2f(xyx) = f(x)yx + xyf(x)$ for all $x,y \in R$, then f is a centralizer.

An additive mapping $f : R \to R$ is said to be a left (right) $\theta-$centralizer associated with a function θ of R, if $f(xy) = f(x)\theta(y)$ ($f(xy) = \theta(x)f(y)$) for all $x,y \in R$. f is called a $\theta-$centralizer, if f is both a left and right $\theta-$centralizer. Daif, El-Sayiad, and Muthana in [7] have proved that if R is a $2-$torsion free semiprime ring and f is an additive mapping of R such that $2f(xyx) = f(x)\theta(yx) + \theta(xy)f(x)$ for all $x,y \in R$ with $\theta(Z(R)) = Z(R)$, where θ is a nonzero surjective endomorphism on R, then f is a $\theta-$centralizer.

Now let R be a ring and M be an R-bimodule. A biadditive map $\alpha : R \times R \to M$ is called a Hochschild 2–cocycle, if $x\alpha(y,z) - \alpha(xy,z) + \alpha(x,yz) - \alpha(x,y)z = 0$ for all $x,y,z \in R$, and α is called symmetric if $\alpha(x,y) = \alpha(y,x)$ for all $x,y \in R$. Nakajima [8] has introduced a new type of generalized derivations and generalized Jordan derivations associated with Hochschild 2–cocycles in the following way. An additive map $f : R \to M$ is called a generalized derivation associated with a Hochschild 2–cocycle α if $f(xy) = f(x)y + xf(y) + \alpha(x,y)$ for all $x,y \in R$, and f is called a generalized Jordan derivation associated with α if $f(x^2) = f(x)x + xf(x) + \alpha(x,x)$ for all $x \in R$. If $\alpha = 0$, then f means the usual derivation and Jordan derivation. He has given the following examples:

(1) If f is a generalized derivation associated with a derivation d, then the map $\alpha_1 : R \times R \ni (x,y) \mapsto x(d-f)(y) \in M$ is biadditive and satisfies the 2–cocycle condition. Hence, f is a generalized derivation associated with α_1.

(2) If $f : R \to M$ is a left centralizer, then by $f(xy) = f(x)y + xf(y) + x(-f)(y)$, we have a 2–cocycle $\alpha_2 : R \times R \to M$ defined by, $\alpha_2(x,y) = x(-f)(y)$, and hence, f is a generalized derivation associated with α_2.

(3) Let f be a $(\theta,\phi)-$derivation. Then, the map $\alpha_3 : R \times R \ni (x,y) \mapsto f(x)(\theta(y) - y) + (\phi(x) - x)f(y) \in M$, is biadditive and satisfies the 2–cocycle condition. Since $f(xy) = f(x)y + xf(y) + \alpha_3(x,y)$, then f is a generalized derivation associated with α_3.

(4) In general, he has mentioned the following. Let $f : R \to M$ be an additive map and let $\alpha : R \times R \to M$ be a biadditive map. If $f(xy) = f(x)y + xf(y) + \alpha(x,y)$ holds, then by the associativity $f((xy)z) = f(x(yz))$, α satisfies the 2–cocycle condition. Thus f is a generalized derivation associated with α.

In his work, Nakajima [8] has shown the following result. Let R be a 2-torsion free ring. Then, every generalized Jordan derivation associated with a Hochschild 2–cocycle α is a generalized derivation associated with α in each of the following cases:

(i) R is a noncommutative prime ring.
(ii) There exist $x,y \in R$ such that $[x,y]$ is a nonzero divisor.
(iii) R is commutative and α is symmetric.

Nawzad, et al. [9] have shown the following. Let R be a 2-torsion free ring. Then, every generalized Jordan derivation associated with a Hochschild 2–cocycle α is a generalized derivation associated with α in each of the following cases:

(i) R is a noncommutative semiprime ring and α is symmetric.
(ii) R is commutative.

In [10], Rehman and Hongan have proved the following result. Let R be a 2-torsion free ring and L a square-closed Lie ideal of R. Then, every generalized Jordan derivation associated with a Hochschild 2–cocycle α is a generalized derivation associated with α in each of the following cases.

(i) R is a prime ring and L is noncommutative.
(ii) R is a prime ring, L is commutative and α is symmetric.
(iii) There exist $x, y \in R$ such that $[x, y]$ is a nonzero divisor in L.

In the present article, we introduce the notion of generalized Jordan triple derivations associated with Hochschild 2–cocycles in the following way. Let R be a ring and let M be an R-bimodule. An additive map $f : R \to M$ is called a generalized Jordan triple derivation associated with a Hochschild 2–cocycle α if $f(xyx) = f(x)yx + xf(y)x + \alpha(x, y)x + xyf(x) + \alpha(xy, x)$ for all $x, y \in R$.

Examples (i) If f is a Jordan triple derivation, then the zero map α_1 is biadditive and satisfies the 2–cocycle condition. Therefore f is a generalized Jordan triple derivation associated with α_1.

(ii) If f is a generalized Jordan triple derivation associated with a Jordan triple derivation d, then $\alpha_2(x, y) = x(d-f)(y)$ is biadditive and satisfies the 2–cocycle condition and we can see that $f(xyx) = f(x)yx + xf(y)x + \alpha_2(x, y)x + xyf(x) + \alpha_2(xy, x)$. Hence f is a generalized Jordan triple derivation associated with α_2 .

Our aim in this work is to show that every generalized Jordan triple derivation associated with a Hochschild 2–cocycle α from a prime ring R with characteristic different from 2 to an R-bimodule M is a generalized derivation associated with α.

Preliminary results

The proof of our result is based on the following series of auxiliary lemmas.

Lemma 1 *Let f be a generalized Jordan triple derivation from a ring R to an R- bimodule M associated with a Hochschild 2–cocycle map α from $R \times R$ into M. Then* for all $x, y, z \in R$, $f(xyz+zyx) = f(x)yz+xf(y)z+\alpha(x, y)z+zyf(z)+\alpha(xy, z)+f(z)yx+zf(y)x+\alpha(z, y)x+zyf(x) + \alpha(zy, x)$.

Proof Let $v = f((x + z)y(x + z))$, we have for all $x, y, z \in R$

$$
\begin{aligned}
0 &= v - v \\
&= f(xyx) + f(xyz + zyx) + f(zyz) - \{f(x + z)y(x + z) \\
&\quad + (x + z)f(y)(x + z) + \alpha(x + z, y)(x + z) \\
&\quad + (x + z)yf(x + z) \\
&\quad + \alpha((x + z)y, (x + z))\}.
\end{aligned}
$$

Then,

$$
\begin{aligned}
0 = & f(xyx) + f(xyz + zyx) + f(zyz) - \{f(x)yx \\
& + f(x)yz + f(z)yx \\
& + f(z)yz + xf(y)x + xf(y)z + zf(y)x + zf(y)z \\
& + \alpha(x,y)x + \alpha(x,y)z \\
& + \alpha(z,y)x + \alpha(z,y)z + xyf(x) + xyf(z) + zyf(x) \\
& + zyf(z) + \alpha(xy,x) \\
& + \alpha(xy,z) + \alpha(zy,x) + \alpha(zy,z)\} \text{ for all } x,y,z \in R,
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
f(xyz + zyx) = & f(x)yz + xf(y)z + \alpha(x,y)z + zyf(z) \\
& + \alpha(xy,z) \\
& + f(z)yx + zf(y)x + \alpha(z,y)x + zyf(x) \\
& + \alpha(zy,x) \text{ for all } x,y,z \in R,
\end{aligned}
$$

as required. □

For a generalized Jordan triple derivation f from a ring R to an R-bimodule M associated with a Hochschild 2−cocycle α, we denote by δ, F and β the maps from $R \times R \times R$ into M defined by $\delta(x,y,z) = f(xyz) - f(x)yz - xf(y)z - \alpha(x,y)z - xyf(z) - \alpha(xy,z)$, $F(x,y,z) = f(xyz) - f(x)yz - xf(y)z - xyf(z)$ and $\beta(x,y,z) = xyz - zyx$, respectively. Thus, $\delta(x,y,z) = F(x,y,z) - \alpha(x,y)z - \alpha(xy,z)$.

Lemma 2 *For all x,y,z in a ring R, the following hold:*

(i) $\delta(x,y,z) = -\delta(z,y,x)$, *and*
(ii) $\delta(x,y,z)$ and $\beta(x,y,z)$ *are tri-additive.*

Proof (i) Follows easily from Lemma 1.
(ii) Replace x by $a+b$ in the definition of δ, then (ii) is easily seen. □

Lemma 3 *For any ring R and any $a,b,c,x \in R$,*
$\delta(a,b,c)x\beta(a,b,c) + \beta(a,b,c)x\delta(a,b,c) = 0.$

Proof Let $v = f(abcxcba + cbaxabc)$, then $0 = v - v = f((abc)x(cba) + (cba)x(abc)) - f(a(bcxcb)a + c(baxab)c)$. By the definition of the generalized Jordan triple derivation f associated with a Hochschild 2-cocycle α and by Lemma 1, we get

$$
\begin{aligned}
0 = & f(abc)xcba + abcf(x)cba + \alpha(abc,x)cba + abcxf(cba) \\
& + \alpha(abcx,cba) + f(cba)xabc + cbaf(x)abc + \alpha(cba,x)abc \\
& + cbaxf(abc) + \alpha(cbax,abc) - \{f(a)bcxcba + af(b)cxcba \\
& + abf(c)xcba + abcf(x)cba + ab\alpha(c,x)cba + abcxf(c)ba \\
& + ab\alpha(cx,c)ba + a\alpha(b,cxc)ba + abcxcf(b)a + a\alpha(bcxc,b)a \\
& + \alpha(a,bcxcb)a + abcxcbf(a) + \alpha(abcxcb,a) + f(c)baxabc \\
& + cf(b)axabc + cbf(a)xabc + cbaf(x)abc + cb\alpha(a,x)abc \\
& + cbaxf(a)bc + cb\alpha(ax,a)bc + c\alpha(b,axa)bc + cbaxaf(b)c \\
& + c\alpha(baxa,b)c + \alpha(c,baxab)c + cbaxabf(c) + \alpha(cbaxab,c)\}.
\end{aligned} \tag{1}
$$

Therefore, for all $a, b, c, x \in R$

$$\begin{aligned} 0 &= F(a,b,c)xcba + abcxF(c,b,a) \\ &+ \{\alpha(abc,x) - ab\alpha(c,x)\}cba + \{\alpha(abcx,cba) - \alpha(abcxcb,a)\} \\ &- \{ab\alpha(cx,c)ba + a\alpha(b,cxc)ba + a\alpha(bcxc,b)a + \alpha(a,bcxcb)a\} \\ &+ F(c,b,a)xabc + cbaxF(a,b,c) \\ &+ \{\alpha(cba,x) - cb\alpha(a,x)\}abc + \{\alpha(cbax,abc) - \alpha(cbaxab,c)\} \\ &- \{cb\alpha(ax,a)bc + c\alpha(b,axa)bc + c\alpha(baxa,b)c + \alpha(c,baxab)c\} \end{aligned} \tag{2}$$

Since α is a 2–cocycle map, we obtain the following relations for all $a, b, c, x \in R$:

(i) $\{\alpha((ab)c,x) - (ab)\alpha(c,x)\}cba = \{\alpha(ab,cx) - \alpha(ab,c)x\}cba.$
(ii) $\alpha(abcx,(cb)a) - \alpha((abcx)(cb),a) = \alpha(abcx,cb)a - (abcx)\alpha(cb,a).$
(iii) $\{\alpha((cb)a,x) - (cb)\alpha(a,x)\}abc = \{\alpha(cb,ax) - \alpha(cb,a)x\}abc.$
(iv) $\alpha(cbax,(ab)c) - \alpha((cbax)(ab),c) = \alpha(cbax,ab)c - (cbax)\alpha(ab,c).$

Substituting from (i–iv) in (2), we get for all $a, b, c, x \in R$

$$\begin{aligned} 0 &= F(a,b,c)xcba + abcxF(c,b,a) \\ &+ \{\alpha(ab,cx) - \alpha(ab,c)x\}cba + \{\alpha(abcx,cb)a - abcx\alpha(cb,a)\} \\ &- \{ab\alpha(cx,c)ba + a\alpha(b,cxc)ba + a\alpha(bcxc,b)a + \alpha(a,bcxcb)a\} \\ &+ F(c,b,a)xabc + cbaxF(a,b,c) \\ &+ \{\alpha(cb,ax) - \alpha(cb,a)x\}abc + \{\alpha(cbax,ab)c - cbax\alpha(ab,c)\} \\ &- \{cb\alpha(ax,a)bc + c\alpha(b,axa)bc + c\alpha(baxa,b)c + \alpha(c,baxab)c\} \end{aligned} \tag{3}$$

Since α is a 2–cocycle map, we conclude for all $a, b, c, x \in R$ that
(i) $\alpha(ab,cx) = a\alpha(b,cx) + \alpha(a,b(cx)) - \alpha(a,b)(cx).$
(ii) $\alpha(abcx,cb)a = \{-(abcx)\alpha(c,b) + \alpha((abcx)c,b) + \alpha(abcx,c)b\}a.$
(iii) $\alpha(cb,ax) = c\alpha(b,ax) + \alpha(c,b(ax)) - \alpha(c,b)(ax).$
(iv) $\alpha(cbax,ab)c = \{-(cbax)\alpha(a,b) + \alpha((cbax)a,b) + \alpha(cbax,a)b\}c.$
Substituting from (i–iv) in (3), we obtain

$$\begin{aligned} 0 &= \{F(a,b,c) - \alpha(ab,c) - \alpha(a,b)c\}xcba + abcx\{F(c,b,a) - \alpha(cb,a) \\ &- \alpha(c,b)a\} + \{a\alpha(b,cx)cba - ab\alpha(cx,c)ba - a\alpha(b,cxc)ba\} \\ &+ \{\alpha(abcxc,b)a - a\alpha(bcxc,b)a - \alpha(a,bcxcb)a\} \\ &+ \alpha(a,bcx)cba + \alpha(abcx,c)ba + \{F(c,b,a) - \alpha(cb,a) \\ &- \alpha(c,b)a\}xabc + cbax\{F(a,b,c) - \alpha(ab,c) - \alpha(a,b)c\} \\ &+ \{c\alpha(b,ax)abc - cb\alpha(ax,a)bc - c\alpha(b,axa)bc\} \\ &+ \{\alpha(cbaxa,b)c - c\alpha(baxa,b)c - \alpha(c,baxab)c\} \\ &+ \alpha(c,bax)abc + \alpha(cbax,a)bc, \text{ for all } a,b,c,x \in R. \end{aligned} \tag{4}$$

Again since α is a 2–cocycle map, we have

(i) $a\{\alpha(b,cx)c - b\alpha(cx,c) - \alpha(b,(cx)c)\}ba = -a\alpha(b(cx),c)ba.$
(ii) $\{\alpha(a(bcxc),b) - a\alpha(bcxc,b) - \alpha(a,(bcxc)b)\}a = -\alpha(a,bcxc)ba.$
(iii) $c\{\alpha(b,ax)a - b\alpha(ax,a) - \alpha(b,(ax)a)\}bc = -c\alpha(b(ax),a)bc.$
(iv) $\{\alpha(c(baxa),b) - c\alpha(baxa,b) - \alpha(c,(baxa)b)\}c = -\alpha(c,baxa)bc.$

Replacing (i–iv) into (4), we get, for all $a, b, c, x \in R$

$$\begin{aligned} 0 &= \delta(a,b,c)xcba + abcx\delta(c,b,a) - a\alpha(bcx,c)ba - \alpha(a,bcxc)ba \\ &+ \alpha(a,bcx)cba + \alpha(abcx,c)ba + \delta(c,b,a)xabc + cbax\delta(a,b,c) \\ &- c\alpha(bax,a)bc - \alpha(c,baxa)bc + \alpha(c,bax)abc + \alpha(cbax,a)bc. \end{aligned} \tag{5}$$

Continuing in this manner, we obtain

(i) $\{-a\alpha(bcx,c) - \alpha(a,(bcx)c) + \alpha(a,bcx)c + \alpha(a(bcx),c)\}ba = 0.$
(ii) $\{-c\alpha(bax,a) - \alpha(c,(bax)a) + \alpha(c,bax)a + \alpha(c(bax),a)\}bc = 0.$

By (5), we conclude that $0 = \delta(a,b,c)xcba + abcx\delta(c,b,a) + \delta(c,b,a)xabc + cbax\delta(a,b,c)$ for all $a, b, c, x \in R$. By Lemma 2, we obtain $0 = \delta(a,b,c)xcba - abcx\delta(a,b,c) - \delta(a,b,c)xabc + cbax\delta(a,b,c)$ for all $a, b, c, x \in R$.

Therefore, $\delta(a,b,c)x\beta(a,b,c) + \beta(a,b,c)x\delta(a,b,c) = 0$ for all $a, b, c, x \in R$. This finishes the proof of the lemma. □

Lemma 4 *If R is a prime ring of characteristic not 2, then for all $a, b, c, x \in R, \delta(a,b,c)x\beta(a,b,c) = 0,$.*

Proof By Lemma 3 and Lemma 1.1 of Brešar [3], we get the proof. □

Lemma 5 *If R is a prime ring of characteristic not 2, then $\delta(a_1,b_1,c_1)x\beta(a_2,b_2,c_2) = 0$ for all $a_1, b_1, c_1, a_2, b_2, c_2, x \in R$.*

Proof From Lemma 2(ii), Lemma 4, and Lemma 1.2 of Brešar [3], we get the proof. □

Lemma 6 *Let R be a prime ring. Then, R is commutative iff $\beta(a,b,c) = 0$ for all $a, b, c \in R$.*

Proof If R is commutative, then, by definition of $\beta, \beta(a,b,c) = 0$ for all $a, b, c \in R$. Conversely, assume that $\beta(a,b,c) = 0$ for all $a, b, c \in R$. Let Q be the Martindale right ring of quotients of R defined by Martindale [11]. Then Q is a prime ring with identity that contains the ring R. By Chuang [12], Q satisfies the same generalized polynomial identities as R. In particular $abc - cba = 0$ for all $a, b, c \in Q$. Replacing c by the identity of Q yields the commutativity of Q, and hence R. □

Lemma 7 *Let R be a prime ring of characteristic not 2. Then $\delta(a,b,c) = 0$ for all $a, b, c \in R$, in each of the following cases:*

(i) *R is noncommutative.*
ii *There exist $x, y, z \in R$ such that $\beta(x,y,z)$ is a nonzero divisor in M.*
iii *R is commutative and α is symmetric.*

Proof (i) By Lemmas 5 and 6, we get our requirement.

(ii) By Lemma 5, we have $\delta(a,b,c)r\beta(x,y,z) = 0$ for all $a, b, c, r, x, y, z \in R$. From our assumption $\delta(a,b,c)r = 0$ for all $a, b, c, r \in R$. Thus the primeness of R gives $\delta(a,b,c) = 0$ for all $a, b, c \in R$.

(iii) From Lemma 1 we have $f(abc + cba) = f(a)bc + af(b)c + \alpha(a,b)c + abf(c) + \alpha(ab,c) + f(c)ba + cf(b)a + \alpha(c,b)a + cbf(a) + \alpha(cb,a)$ for all $a, b, c \in R$. Since R is commutative and α is symmetric, we get $0 = 2\{f(abc) - f(a)bc - af(b)c - abf(c)\} -$

$\alpha(a,b)c - \alpha(ab,c) - a\alpha(b,c) - \alpha(a,bc)$ for all $a,b,c \in R$. Since α is 2–cocycle we have $-a\alpha(b,c) - \alpha(a,bc) = -\alpha(a,b)c - \alpha(ab,c)$ for all $a,b,c \in R$. Therefore $0 = 2\{f(abc) - f(a)bc - af(b)c - abf(c) - \alpha(a,b)c - \alpha(ab,c)\}$ for all $a,b,c \in R$. Since R has characteristic not 2, then $\delta(a,b,c) = 0$ for all $a,b,c \in R$, as required. □

Main result

Theorem 1 *Let R be a prime ring of characteristic not 2. Then every generalized Jordan triple derivation associated with a Hochschild 2–cocycle α is a generalized derivation associated with α in each of the following cases.*

(i) R is noncommutative.

(ii) There exist $x,y,z \in R$ such that $\beta(x,y,z)$ is a nonzero divisor in M.

(iii) R is commutative and α is symmetric.

Proof Suppose that f is a generalized Jordan triple derivation associated with a Hochschild 2−cocycle α. We denote by $G(a,b)$ and a^b the elements of M defined by $G(a,b) = f(ab) - f(a)b - af(b)$, and $a^b = f(ab) - f(a)b - af(b) - \alpha(a,b)$, respectively. Thus, $a^b = G(a,b) - \alpha(a,b)$. It is evident that $a^{b+c} = a^b + a^c$, and $(a+b)^c = a^c + b^c$. By Lemma 7, we have $\delta(a,b,c) = 0$ for all $a,b,c \in R$. Thus, for all $a,b,c \in R$

$$f(abc) = f(a)bc + af(b)c + \alpha(a,b)c + abf(c) + \alpha(ab,c). \tag{6}$$

Now let $v = f(abxab)$, then $0 = v - v = f((ab)x(ab)) - f(a(bxa)b)$. By (6), we have for all $a,b,x \in R$

$$\begin{aligned} 0 &= f(ab)xab + abf(x)ab + \alpha(ab,x)ab + abxf(ab) + \alpha(abx,ab) \\ &\quad - f(a)bxab - af(b)xab - abf(x)ab - a\alpha(b,x)ab - abxf(a)b \\ &\quad - a\alpha(bx,a)b - \alpha(a,bxa)b - abxaf(b) - \alpha(abxa,b). \end{aligned}$$

So, for all $a,b,x \in R$

$$\begin{aligned} 0 &= G(a,b)xab + abxG(a,b) + \{\alpha(ab,x) - a\alpha(b,x)\}ab \\ &\quad + \{\alpha(abx,ab) - \alpha(abxa,b)\} - a\alpha(bx,a)b - \alpha(a,bxa)b. \end{aligned} \tag{7}$$

Since α is 2-cocycle we have for all $a,b,x \in R$ that

(i) $\{\alpha(ab,x) - a\alpha(b,x)\}ab = \{\alpha(a,bx) - \alpha(a,b)x\}ab$, and

(ii) $\alpha(abx,ab) - \alpha((abx)a,b) = \alpha(abx,a)b - (abx)\alpha(a,b)$.

Substituting from (i) and (ii) in (7), we get $G(a,b)xab - \alpha(a,b)xab + abxG(a,b) - abx\alpha(a,b) + \alpha(a,bx)ab + \alpha(abx,a)b - a\alpha(bx,a)b - \alpha(a,bxa)b = 0$ for all $a,b,x \in R$. But α is 2–cocycle, hence $\{\alpha(a,bx)a + \alpha(abx,a) - a\alpha(bx,a) - \alpha(a,bxa)\}b = 0$. Therefore $a^bx(ab) + (ab)xa^b = 0$ for all $a,b,x \in R$. By Lemma 1.1 of Brešar [3] , we get

$$a^bx(ab) = (ab)xa^b = 0 \text{ for all } a,b,x \in R. \tag{8}$$

Replacing a by $a+c$ in (8) and using (8), we obtain $a^bxcb = -c^bxab$ for all $a,b,c,x \in R$, and then $(a^bxcb)y(a^bxcb) = -a^bx(cbyc^b)xab = 0$ for all $a,b,c,x,y \in R$. Thus the primeness of R gives

$$a^bxcb = 0 \text{ for all } a,b,c,x \in R. \tag{9}$$

Similarly replacing b by $b+d$ in (9), we get

$$a^bxcd = 0 \text{ for all } a,b,c,d,x \in R. \tag{10}$$

Putting $c = a^b$ and $x = dx$ in (10) we have $a^b dxa^b d = 0$ for all $a, b, d, x \in R$. Again, the primeness of R yields that $a^b d = 0$ for all $a, b, d \in R$, and hence $a^b = 0$ for all $a, b \in R$. Consequently, f is a generalized derivation associated with a Hochschild 2−cocycle α. □

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