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ORIGINAL ARTICLE

Nanofluid flow over a non-linear permeable stretching sheet with partial slip

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KEYWORDS

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Abstract In the present study, the problem of boundary layer flow of a nanofluid over non-linear permeable stretching sheet at prescribed surface temperature in the presence of partial slip is investigated numerically. By means of proper similarity variables, the governing equations are transformed to ordinary differential equations which are solved using symbolic software MATHEMATICA. The similarity solutions that depend on slip parameter, stretching parameter, etc. are elucidated through graphs and tables.

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1. Introduction

The flow over a stretching sheet is relevant to several important engineering applications in the field of metallurgy and chemical engineering processes. These applications involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. The steady two dimensional boundary layer flow of Newtonian fluid over a stretching surface has been studied by Crane [1]. After this pioneering work the flow field over a stretching surface has drawn considerable attention and a good amount of literature has been generated on this problem [2–5]. In this study the fluid velocity is assumed to be zero relative to the solid boundary. But this is not true for fluid flows at the micro- and nanoscale. Investigation shows that slip flow happens when the characteristic size of the flow system is small or the flow pressure is very low. To describe the phenomenon of slip, Navier [6] introduced a boundary condition which states that the component of the fluid velocity tangential to the boundary walls is proportional to tangential stress. Martin and Boyd [7] analyzed Blasius boundary layer problem in the presence of slip boundary condition. The hydrodynamic flow in the presence of partial slip over a stretching sheet with suction has been studied by Wang [8]. Das [9] analyzed the slip effects on heat and mass transfer in MHD micropolar fluid flow. Recently, Das [10] investigated convective heat transfer of nanofluids over a stretching sheet in the presence of partial slip and thermal radiation.

However, all these studies are restricted to linear stretching of the sheet. It is worth mentioning that the stretching is not necessarily linear, as in a polymer extrusion process. The problem of non-linear stretching sheet for different cases of fluid flow has also been analyzed by different researchers. Gupta and Gupta [11] first point out in their study that the stretching

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of the sheet may not necessarily be linear. In view of this, Vajravelu [12] studied flow and heat transfer over a non-linear stretching sheet. Cortell [13] extended the model proposed by [12] considering two different types of thermal boundary conditions on the sheet, constant surface temperature and prescribed surface temperature. Prasad et al. [14] investigated the mixed convection heat transfer over a non-linear stretching surface with variable fluid properties. Recently, Yazdi et al. [15] discussed the slip flow and heat transfer over a non-linear permeable stretching surface.

A nanofluid is a new class of heat transfer fluids that contain a base fluid and nanoparticles. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base liquids. One of the possible mechanisms for anomalous increase in the thermal conductivity of nanofluids is the Brownian motions of the nanoparticles inside the base fluids. It should be noticed that there have been published several recent papers [16,17] on the mathematical and numerical modeling of natural convection heat transfer in nanofluids. A comprehensive survey of convective transport in nanofluids was made by Buongiorno [18] and Kakac and Pramuanjaroenkij [19]. The Buongiorno model [18] has also been used by Khan and Pop [20] to study the boundary layer flow of a nanofluid past a stretching sheet. The boundary layer flow of a nanofluid caused by a stretching surface has drawn the attention of many researchers [21–23]. Very recently Rana and Bhargava [24] investigated the boundary layer flow of a nanofluid flow over a non-linearly stretching sheet.

There have been many theoretical models developed to describe slip flow along the surface. However, to the best of my knowledge, no investigation has been made yet to analyze the slip flow and heat transfer of a nanofluid past a non-linear stretching permeable surface at prescribed surface temperature. The objective of present article was therefore to extend the work of [24] by taking steady boundary layer flow and heat transfer of a nanofluid in the presence of partial slip over a non-linear permeable stretching surface at prescribed surface temperature.

2. Mathematical formulation

Consider the boundary layer flow of nanofluid over a non-linear permeable stretching surface. The flow takes place at $y \geqslant 0$, where y is the coordinate measured normal to the stretching surface. The flow is generated, due to the stretching of the sheet that emerges out of a slit at $x = 0,\ y = 0$. Let us assume that the speed at a point on the plate is proportional to the power of its distance from the slit and the boundary layer approximation are applicable. The sheet is assumed to vary non-linearly with distance x from the leading edge i.e.,

$$u_w = ax^n \tag{1}$$

where a is a positive constant and n is non-linear stretching parameter. The stretching surface is maintained at prescribed surface temperature, T_w as follows:

$$T = T_w (= T_\infty + bx^r) \quad \text{at} \quad y = 0 \tag{2}$$

where b is a positive constant, r is the surface temperature parameter in the prescribed surface temperature boundary condition and T_∞ is the temperature of the fluid far away from the surface. Special case of constant surface temperature is obtained by introducing r equal to zero.

The governing boundary layer equations for this investigation are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + (D_T/T_\infty)\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{5}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + (D_T/T_\infty)\frac{\partial^2 C}{\partial y^2} \tag{6}$$

The associated boundary conditions are

$$\left.\begin{array}{l} u = u_w + u_s,\ v = \pm v_w,\ T = T_w,\ C = C_w \quad \text{at} \quad y = 0 \\ u \to 0,\ T \to T_\infty,\ C \to C_\infty \quad \text{as} \quad y \to \infty \end{array}\right\} \tag{7}$$

where u, v are the velocity components along x and y-axis respectively, ν is the kinematic viscosity, α is the thermal diffusivity, $\tau = (\rho c)_p/(\rho c)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, C is the nanoparticle volumetric fraction, ρ_p is the density of the particles, ρ_f is the density of the base fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, v_w is the suction/injection and u_s is the slip velocity which is assumed to be proportional to the local wall stress as follows:

$$u_s = l\left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{8}$$

where l is the slip length as a proportional constant of the slip velocity.

By using similarity transformations

$$\begin{array}{l} \eta = y\sqrt{\dfrac{a(n+1)}{2\nu}}x^{\frac{n-1}{2}}, \quad u = ax^n f'(\eta), \\ v = -\sqrt{\dfrac{a\nu(n+1)}{2}}x^{\frac{n-1}{2}}\left(f + \left(\dfrac{n-1}{n+1}\right)\eta f'\right) \\ \theta(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \dfrac{C - C_\infty}{C_w - C_\infty} \end{array} \tag{9}$$

the fundamental equations of the boundary layer (3)–(6) are transformed to ordinary differential equations that are locally valid as follows:

$$f''' + ff'' - \left(\frac{2n}{n+1}\right)f'^2 = 0 \tag{10}$$

$$\frac{1}{Pr}\theta'' + f\theta' - \left(\frac{2r}{n+1}\right)f'\theta + Nb\theta'\phi' + Nt\theta'^2 = 0 \tag{11}$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{12}$$

In view of (9), the boundary conditions (7) turn into

$$\left.\begin{array}{l} f = F_w,\ f' = 1 + \zeta_p f'',\ \theta = 1,\ \phi = 1 \quad \text{at} \quad \eta = 0 \\ f' \to 0,\ \theta \to 0,\ \phi \to 0 \quad \text{as} \quad \eta \to \infty \end{array}\right\} \tag{13}$$

Here prime denotes differentiation with respect to η, $Pr = \nu/\alpha$ is the Prandtl number, $Le = \frac{\alpha}{D_B}$ is the Lewis number, $Nb = \frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_f \nu}$ is the Brownian motion parameter, $Nt = \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f \nu T_\infty}$ is the thermophoresis parameter, $F_w = -\frac{v_w}{\sqrt{\frac{a x^{n-1} \nu (n+1)}{2}}}$ is the suction/injection parameter, and $\zeta_p = l\sqrt{\frac{a(n+1)}{2\nu}} x^{n-1}$ is the slip parameter for liquids. It should be noted that n (non-linear stretching parameter) and x (coordinate along the surface) which appear in the F_w and ζ_p tend to break down the similarity solution. Concentrating on the above dimensionless form for F_w and ζ_p can be recognized that n and x are producing in all of them in a special form which we introduce it as the non-linear term, P_{nx} and is given by (see Yazdi et al. [15])

$$P_{nx} = \frac{x^{n-1}(n+1)}{2} \tag{14}$$

This parameter obliges our equations to be solved locally. Redefining F_w and ζ_p based on the non-linear term P_{nx} yields an independent f_w and ζ from x and n as follows:

$$F_w = \frac{f_w}{\sqrt{P_{nx}}}, \quad \zeta_p = \zeta\sqrt{P_{nx}} \tag{15}$$

where $f_w = -\frac{v_w}{\sqrt{a\nu}}$ and $\zeta = l\sqrt{\frac{a}{\nu}}$ are suction/injection and slip parameter based on P_{nx} which are totally independent from x and n. Consequently there is an appropriate possibility by defining these parameters (f_w and ζ) keeping away from difficulties of the dependency of F_w and ζ_p on n and x. Therefore the local similarity solution of the problem for fixed values of the x coordinate, varying n would be obtained properly for the various values of involved parameters of the problem.

3. Method of solution

The non-linear differential Eqs. (10)–(12) with boundary conditions (13) have been solved in the symbolic computation software MATHEMATICA using finite difference code that implements the 3-stage Lobatto IIIa formula for partitioned Runge–Kutta method. We take infinity condition at a large but finite value of η where no considerable variation in velocity, temperature, etc. occur.

To check the validity of the present code, the values of $\theta'(0)$ have been calculated for $\zeta = f_w = r = 0$ and for different values of non-linear stretching parameter n using MATHEMATICA 7.0 in Table 1. From Table 1, it has been observed that the data produced by the present code and those of Cortell [13] and Rana and Bhargava [24] show excellent agreement and the use of the present numerical code is justified.

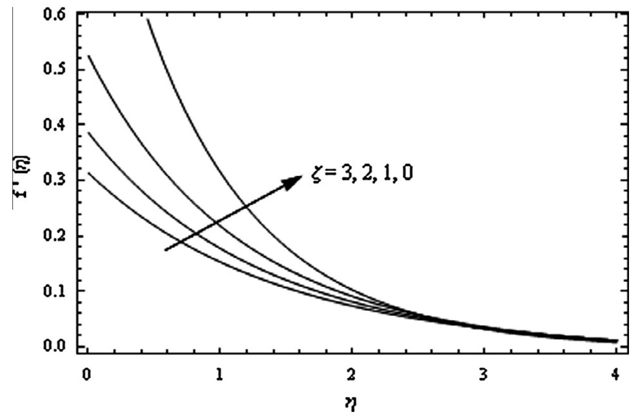

Figure 1 Streamwise velocity for various values of ζ.

4. Numerical results and discussions

In order to get a clear insight into the present problem, the numerical results for velocity, temperature, nanoparticle concentration, etc. have been presented graphically in Figs. 1–10 and in Table 1 for several sets of values of the pertinent parameters such as slip parameter ζ, suction/injection parameter f_w, non-linear stretching parameter n, etc. In the simulation the default values of the parameters are considered as [24] $\zeta = 1.0$, $f_w = 0.2$, $Nb = 0.5$, $Nt = 0.5$, $Le = 5.0$, $Pr = 2.0$ and $n = 2.0$ unless otherwise specified.

The effect of slip parameter ζ can be understood from the variation of the streamwise velocity component $f'(\eta)$ with the similarity independent variable η as illustrated in Fig. 1. As slip parameter increases, the slip at the surface wall increases, and as a result reaches to a smaller amount of penetration due to the stretching surface into the fluid. It is clear from figures that the velocity component at the wall reduces with an increase in the slip parameter ζ for nanofluids and decreases asymptotically to zero at the edge of the hydrodynamic boundary layer. Thus hydrodynamic boundary layer thickness for nanofluids decreases as the slip parameter ζ increases. Fig. 2 shows variation in the temperature profile for various values of slip parameter ζ. Figure indicates that an increase in slip parameter tends to increase temperature in the fluid field. Thus, by escalating ζ, thermal boundary layer thickness enhances. The effect

Table 1 Comparison of the values of $\theta'(0)$ for various values of n.

Pr	n	Cortell [13]	$\zeta = f_w = r = 0$	
			Rana and Bhargava [24]	Present result
1.0	0.2	0.610262	0.6113	0.610571
	0.5	0.595277	0.5967	0.595719
	1.5	0.574537	0.5768	0.574525
5.0	0.2	1.607175	1.5910	1.60713
	0.5	1.586744	1.5839	1.58619
	1.5	1.557463	1.5496	1.55719

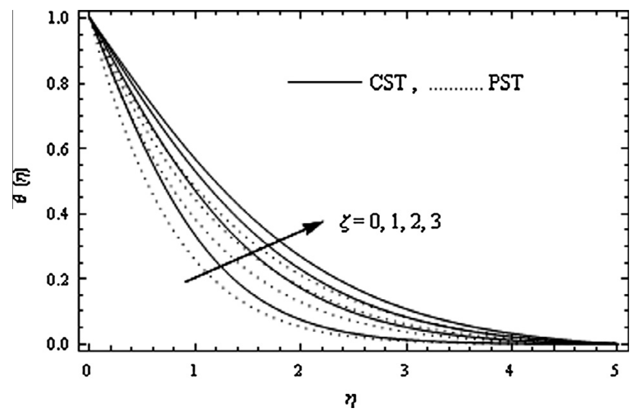

Figure 2 Temperature profiles for various values of ζ.

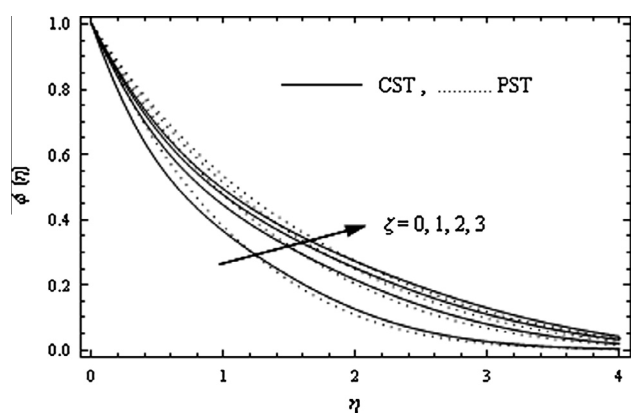

Figure 3 Nanoparticle concentration profiles for various values of ζ.

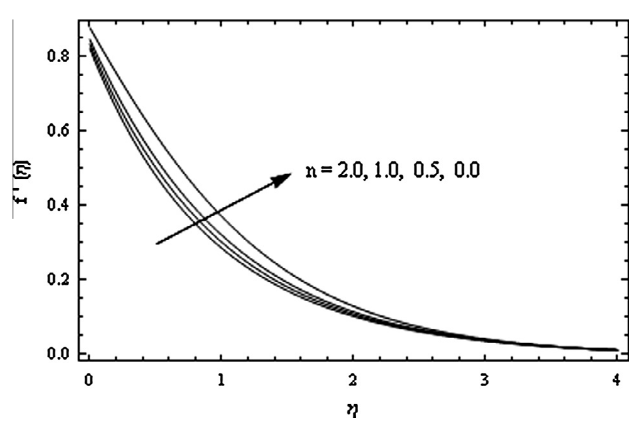

Figure 4 Streamwise velocity for various values of n.

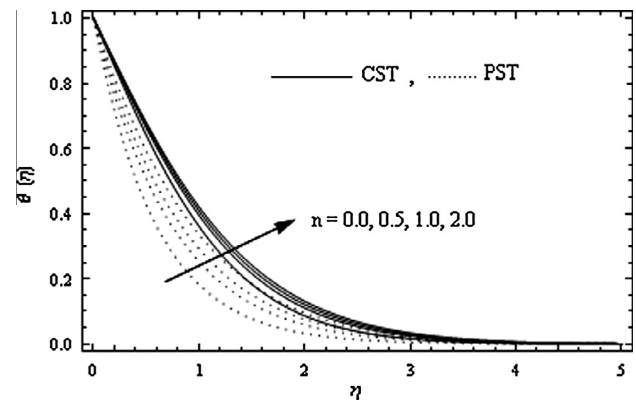

Figure 5 Temperature profiles for various values of n.

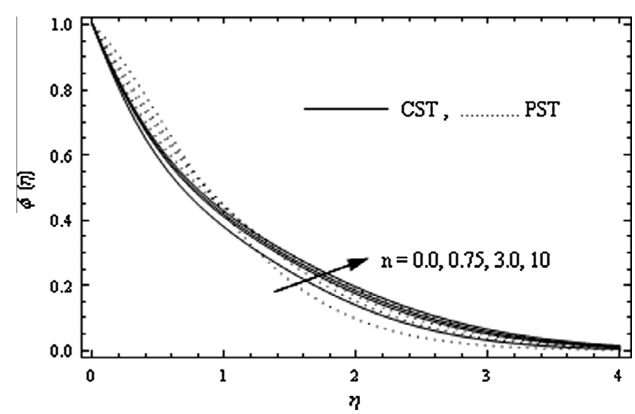

Figure 6 Nanoparticle concentration profiles for various values of n.

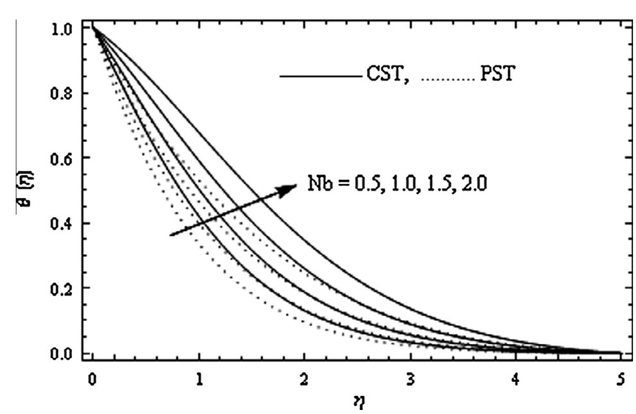

Figure 7 Temperature profiles for various values of Nb.

of slip velocity on nanoparticle concentration is shown in Fig. 3. It can be seen from figure that an increase in the slip parameter ζ leads to increase in the nanoparticle concentration for both constant surface temperature and prescribed surface temperature.

Fig. 4 depicts the variation in the streamwise velocity $f'(\eta)$ with coordinate η for various values of non-linear stretching parameter n. It is observed that an increase in n leads to decrease in $f'(\eta)$. As a result, the momentum boundary layer thickness decreases with increasing non-linear stretching parameter n. Fig. 5 illustrates variation in the temperature versus η for different values of non-linear stretching parameter n. The presence of stretching parameter n leads to an increase in the thickness of the thermal boundary layer profile. Fig. 6 shows variations in the nanoparticle concentration profile as function of η for various values of non-linear stretching

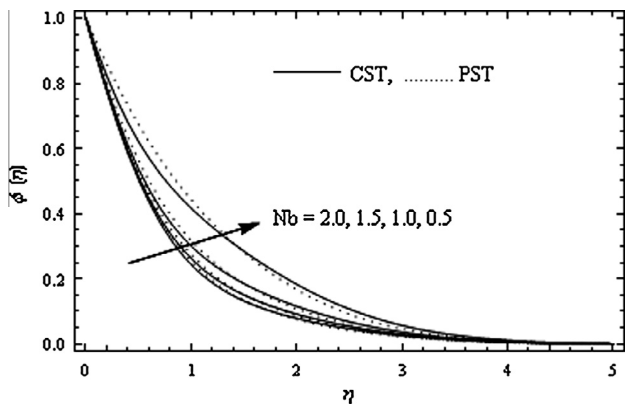

Figure 8 Nanoparticle concentration profiles for various values of *Nb*.

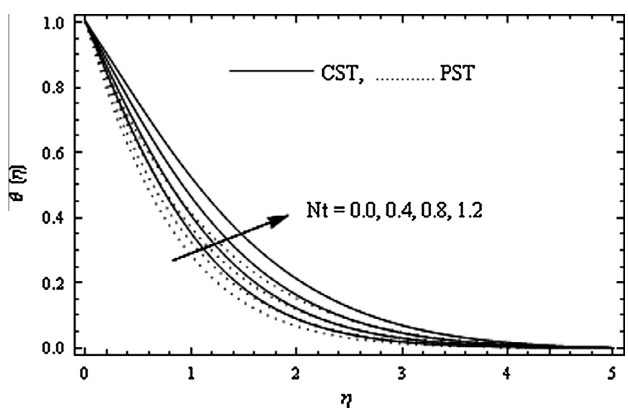

Figure 9 Temperature profiles for various values of *Nt*.

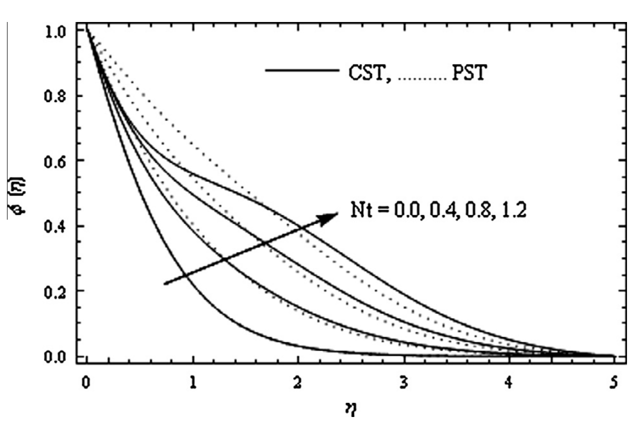

Figure 10 Nanoparticle concentration profiles for various values of *Nt*.

parameter n. One can notice that the concentration increases with increase in non-linear stretching parameter n but the effect is not significant for constant surface temperature.

The influence of Brownian motion parameter Nb on the temperature is shown in Fig. 7. It is found that the temperature increases with Nb across the boundary layer and, as a consequence, thickness of the thermal boundary layer increases by increasing Nb in the flow field for both constant surface temperature and prescribed surface temperature. Fig. 8 represents the dimensionless nanoparticle concentration profiles for different values of Brownian motion parameter Nb. An increase in Brownian motion parameter leads to fall in concentration of the fluid in the boundary layer region.

The impact of thermophoresis parameter Nt on the temperature profiles is presented in Fig. 9. For a non-zero fixed value of η, temperature distribution across the boundary layer increases with the increasing values of Nt for both constant surface temperature and prescribed surface temperature and hence the thickness of thermal boundary layer increases. The variation in concentration profiles for different values of thermophoresis parameter Nt is presented in Fig. 10. It is noticeable that concentration profiles within the boundary layer increase with an increase in thermophoresis parameter.

5. Conclusions

A numerical study is performed for the problem of nanofluid over a non-linear permeable stretching sheet at prescribed surface temperature in the presence of partial slip. A parametric study is performed to explore the effects of various governing parameters on the fluid flow and heat transfer characteristic. Following conclusion can be drawn from the present investigation:

- The streamwise velocity of the nanofluid decrease with increase in slip parameter ζ and non-linear stretching parameter n.
- An increase in the slip parameter and non-linear stretching parameter n leads to increase the thermal boundary layer thickness.
- The nanoparticle concentration is an increasing function of each values of the parameters ζ and n.

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