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ORIGINAL ARTICLE

Steady mixed convection stagnation point flow of MHD Oldroyd-B fluid over a stretching sheet

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Abstract This study deals with the steady mixed convection stagnation point flow of an incompressible Oldroyd-B fluid over the stretching sheet in the presence of a constant applied magnetic field. It is assumed that the surface temperature varies linearly with the distance from the stagnation point. A coupled system of non-linear differential equations is developed by employing the similarity transformations. To analyze the behavior of the velocity, temperature, skin friction coefficient and rate of heat transfer through the wall, a numerical solution is developed using finite difference scheme. The obtained results are used to discuss the influence of pertinent parameters of interest.

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1. Introduction

The analysis regarding the fluid flow and heat transfer over a stretching sheet is a subject of interest for many researchers working in the area of the two-dimensional boundary layer flows. The reasons lie in the fact that such kinds of investigations have applications in the manufacturing industry. For example, in many manufacturing processes such as glass fiber production, hot rolling, continuous casting, extrusion process, manufacturing of sheets, coating and paper production. The initial work was investigated by Sakiadis [1] for two-dimensional boundary layer flow when the plate is moving with constant velocity. The Sakiadis's problem for heat transfer analysis was studied by Erickson et al. [2]. In the above mention studies the velocity of the sheet is assumed to be constant. This assumption of constant velocity is adequate when we are interested in the analysis of continuous extrusion of polymer sheets. Due to the flexibility of polymer materials, a stretching may occur. Crane [3] found a closed form solution for boundary layer flow by imposing the condition of stretching wall. He assumed a linear variation of stretching velocity with respect to the distance from the origin. The Crane's problem for viscous fluid was extended in different directions in the literatures [4–9], etc.

In above mentioned Refs. [3–9], the fluid flow is induced due to the motion of the surface. But in many real situations the thermal buoyancy is also play part for the occurrence of

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fluid flow. Therefore, the flow behavior is analyzed under the influence of both the mechanism; the motion of solid surface and buoyancy. These buoyancy forces occurs due to the heating and cooling of stretching sheets resulting in producing changes to both flow and temperature fields as discussed by Chen and Strobel [10]. The literature survey indicates that work in this direction is carried out by many workers in the field [11–14] and reference therein. On the other hand, the constitutive relationships for rate type fluids are implicit and elimination of stress components from the equation of motion is not straightforward. This fact is the major cause for the lack of literature on the two-dimensional flow of rate type fluids. The simplest class of rate type fluids is the Maxwell fluid and one can find a number of articles regarding the stretching flow of Maxwell fluid [15–20] and reference there in. However, for an Oldroyd-B fluid there are only two studies regarding the flow over a stretching sheet [21,22].

On the other hand, an extensive literature can be found on the mixed convection flow of Non-Newtonian fluids over continuously moving surfaces. Mixed convection flow of a non-Newtonian micropolar fluid was discussed by Takhar et al. [23]. Mushtaq et al. [24] considered the steady mixed convection flow of a second grade fluid by considering the case of variable surface temperature. The steady mixed Convection flow of a micropolar fluid near the stagnation point on a vertical surface is studied by Lok et al. [25]. In 2008 , Hayat et al. [26] investigated the series solution for the mixed convection flow of a micropolar fluid over a non-linear stretching sheet using homotopy analysis method. Recently, Hsiao [27] studied the heat, mass transfer and mixed convection for MHD flow of a viscoelastic fluid past a continuously moving surface with Ohmic dissipation numerically. Very recently, the mixed convection in the stagnation-point flow of a Maxwell fluid toward a vertical stretching sheet has been investigated by Abbas et al. [28]. They have discussed the results both analytically using homotopy analysis method (HAM) and numerically using finite difference method. To best of our knowledge, no such attention has been given for the mixed convection flow of an Oldroyd-B fluid. Our aim was to discuss the mixed convection in the stagnation-point flow of an Oldroyd-B fluid over a stretching sheet. As a first step the boundary layer equations under these assumptions have been developed and then a numerical solution by employing a finite difference method is presented for the transformed nonlinear coupled ordinary differential equations. The effects of various involving physical parameters on the flow and temperature distributions are discussed through graphs and tables.

2. Formulation of the problem

Consider an incompressible, two-dimensional Oldroyd-B fluid in the region of a stagnation point over a semi-infinite, impermeable stretching sheet at $y = 0$. The assumed stretching velocity and wall temperature of the sheet is $u_w(x) = cx$, with $c > o$ and $T_w(x) = bx$, with $b > 0$, respectively. Far away from the plate the external flow velocity is $u_e(x) = ax$, with $a > 0$ and we assume that body attains a uniform temperature T_∞ outside the boundary layer. A constant magnetic field of strength B_0 is applied in y-direction. Under the usual Boussinesq approximation the governing equations for boundary layer flow and heat transfer are (see [22,30])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=&-\Lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2}+2uv\frac{\partial^2 u}{\partial x\partial y}+v^2\frac{\partial^2 u}{\partial y^2}\right)+a^2x+v\frac{\partial^2 u}{\partial y^2}\\ &+v\Lambda_2\left(u\frac{\partial^3 v}{\partial x\partial^2 y}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right)+\sigma B_0^2\left(ax-u-\Lambda_1 v\frac{\partial u}{\partial y}\right)\\ &+g\beta\left[(T-T_\infty)+\Lambda_1\left(\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}-\frac{\partial u}{\partial x}(T-T_\infty)\right)\right], \end{aligned} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

where u and v are the velocity components in the x and y directions respectively, v is the kinematics viscosity of fluid, p is the pressure, ρ is the density of the fluid, Λ_1 is the relaxation time, Λ_2 is the retardation time, g is the gravitational acceleration, β is the thermal expansion coefficient, c_p is the specific heat, k is the thermal diffusivity and T is the temperature.

The appropriate boundary conditions applicable to the present flow problem are:

$$\begin{aligned} &u=u_w(x)=cx, \quad v=0, \quad T=T_w(x)=T_\infty+bx \text{ at } y=0,\\ &u=\ u_e(x)=ax, \quad \frac{\partial u}{\partial y}\to 0, \quad T=T_\infty \text{ as } y\to\infty. \end{aligned} \tag{4}$$

in which both a and c have the dimension of $(\text{time})^{-1}$ and b is a positive constant. Here, the forth condition in Eq. (4) is the augmented condition discussed by Grag and Rajagopal [29] .

Introducing the standard similarity transformations for a stretching flow

$$u=cxf'(\eta), \quad v=-\sqrt{cv}f(\eta), \quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \quad \eta=\sqrt{\frac{c}{v}}y. \tag{5}$$

Governing flow problem takes the form

$$\begin{aligned} &f'''-f'^2+ff''+\tfrac{a^2}{c^2}+M^2\left(\tfrac{a}{c}-f'-\lambda_1 ff''\right)+A_r(\theta-\lambda_1 f\theta')\\ &\qquad+\lambda_1(2ff'f''-f^2f''')+\lambda_2\left(f''^2-ff^{iv}\right)=0, \end{aligned} \tag{6}$$

$$\theta''+P_r(f\theta'-\theta f')+P_rE_cf''^2=0, \tag{7}$$

$$\begin{aligned} &f=0, \quad f'=1, \quad \theta=1 \text{ at } \eta=0,\\ &f'=\frac{a}{c}, \quad f''=0, \quad \theta=0 \text{ as } \eta\to\infty. \end{aligned} \tag{8}$$

Here prime denotes derivative with respect to η and $\lambda_1 = \Lambda_1 c$ and $\lambda_2 = \Lambda_2 c$ are the dimensionless material parameters of the fluid and $A_r = Gr/Re^2$ is the Archimedes number, $Gr = g\beta b/v^2$ is the Grashof number, $Re = c/v$ is the Reynolds number and $M^2 = \sigma B_0^2/c\rho$ is the Hartmann number.

3. Results and discussion

The numerical procedure explained in detail by Sajid et al. [22] is adopted for the solution of the transformed Eqs. (6)–(8). The effects of the involving parameters for example, the dimensionless relaxation/retardation times (λ_1,λ_2), the Hartmann number M, the ratio of the external flow rate to the stretching rate a/c , the Archimedes number A_r , the Prandtl number Pr and the Eckert number Ec on the velocity $f'(\eta)$ and the temperature $\theta(\eta)$ distributions are presented through Figs. 1–3. The numerical values of $\theta'(0)$ are also given for different physical parameters in Table 1. Fig. 1 elucidates the variations in the velocity component $f'(\eta)$ for various values of a/c and the dimensionless relaxation/retardation time (λ_1, λ_2) , respectively. It is found that in Fig. 1(a) that the velocity $f'(\eta)$ is increased for large values of a/c , furthermore it is also noted that initially when $a/c < 1$ the velocity $f'(\eta)$ decreases by increasing the values of λ_1 but when we take the values of $a/c > 1$, the velocity field $f'(\eta)$ has opposite behavior and velocity increase by increasing the values of λ_1. The boundary layer thickness is decreased as λ_1 increases for both $a/c < 1$ and $a/c > 1$. Moreover, it can be seen from Fig. 1(b) that the retardation time λ_2 has opposite effect on the velocity field $f'(\eta)$ when it compared with the effects of retardation time λ_1 . Fig. 2(a) depicts the influence of a Hartmann number M on the velocity field $f'(\eta)$ when $a/c = 0.1$. As expected both the velocity $f'(\eta)$ and the boundary layer thickness are decreased by increasing the values of M . Physically this fact is due to that the magnetic force acts as a resistance to the flow. Fig. 2(b) presents the variation of the temperature $\theta(\eta)$ for various values of the Eckert number Ec . It is observed that temperature goes to increase as Ec increases. Furthermore, both Pr and Ec have opposite behavior on the temperature as well as on the thermal boundary layer thickness.

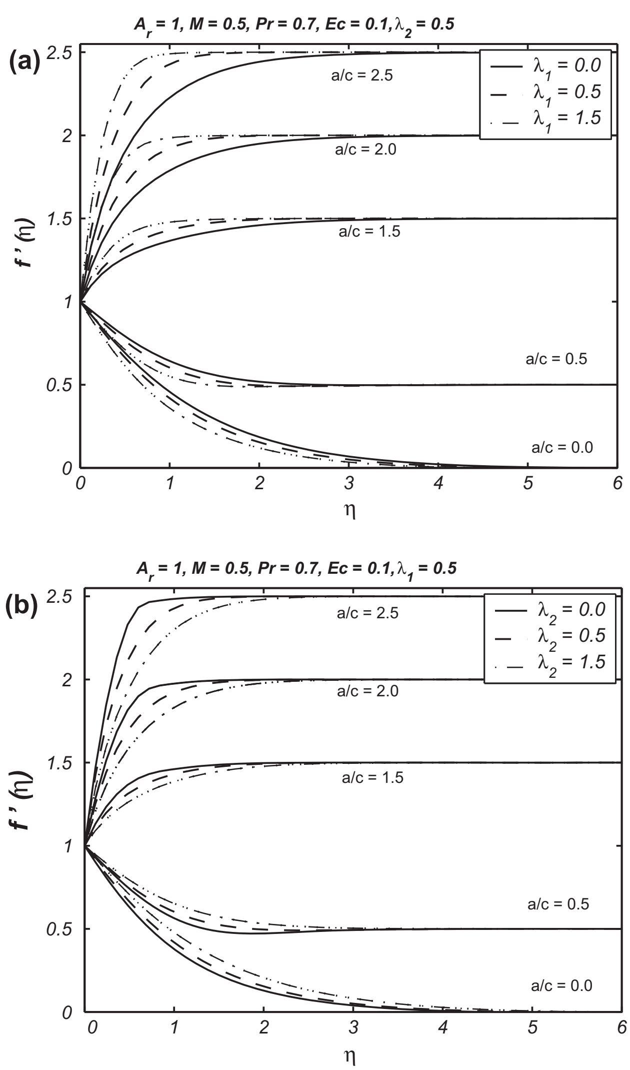

Figure 1 The velocity profile $f'(\eta)$ verses η (a) for different values of a/c and λ_1; and (b) for different values of a/c and λ_2.

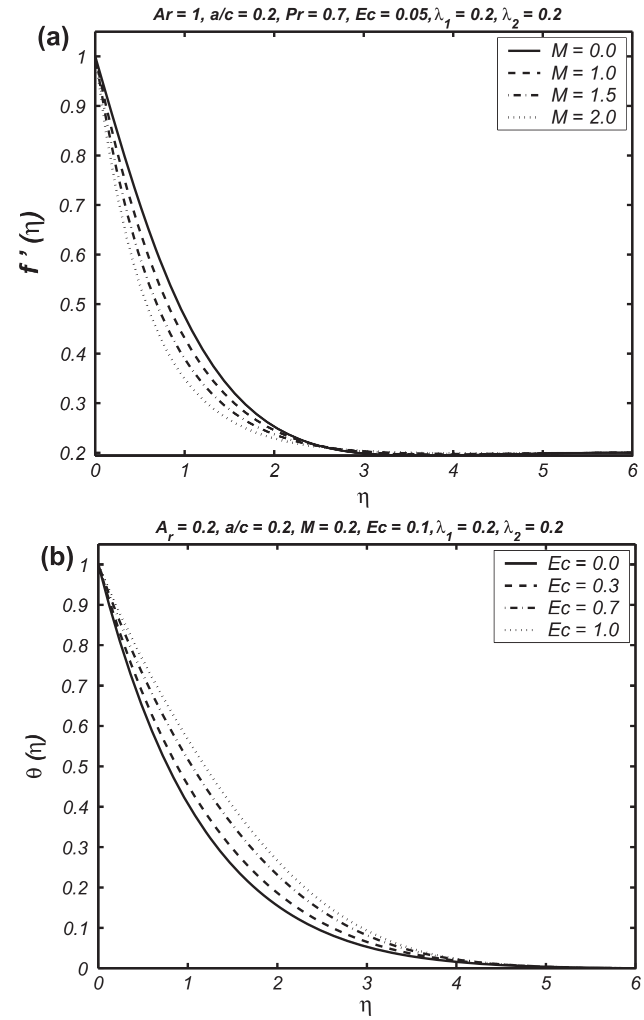

Figure 2 (a) The velocity profile $f'(\eta)$ verses η for different values of M and (b) The temperature profile $\theta(\eta)$ verses η for different values of Ec.

Fig. 3(a) gives the change in the temperature distribution $\theta(\eta)$ for several values of the ratio of the external flow rate to the stretching rate a/c . In this Fig., as we increase the values of a/c the temperature is decreased. It is also noted that the thermal boundary layer thickness decreases for the large values of a/c. Fig. 3(b) is made to see the effects of the Prandtl number Pr on the temperature distribution $\theta(\eta)$. We can say from this Fig. that both the temperature and thermal boundary layer are decreased by increasing the value of Pr.

Table 1 gives the numerical values of the local Nusselt number $-\theta'(0)$ for different values of the Pr, A_r and Ec in case of Oldroyd-B fluid. It is found that the local Nusselt number $-\theta'(0)$ is increased by increasing the values of Prandtl number Pr and the Archimedes number A_r. But the magnitude of $-\theta'(0)$ decreases by increasing the values of Ec.

4. Concluding remarks

The two-dimensional equations incorporating the effects of applied magnetic field under the low magnetic Reynold number assumption and mixed convection effects for an

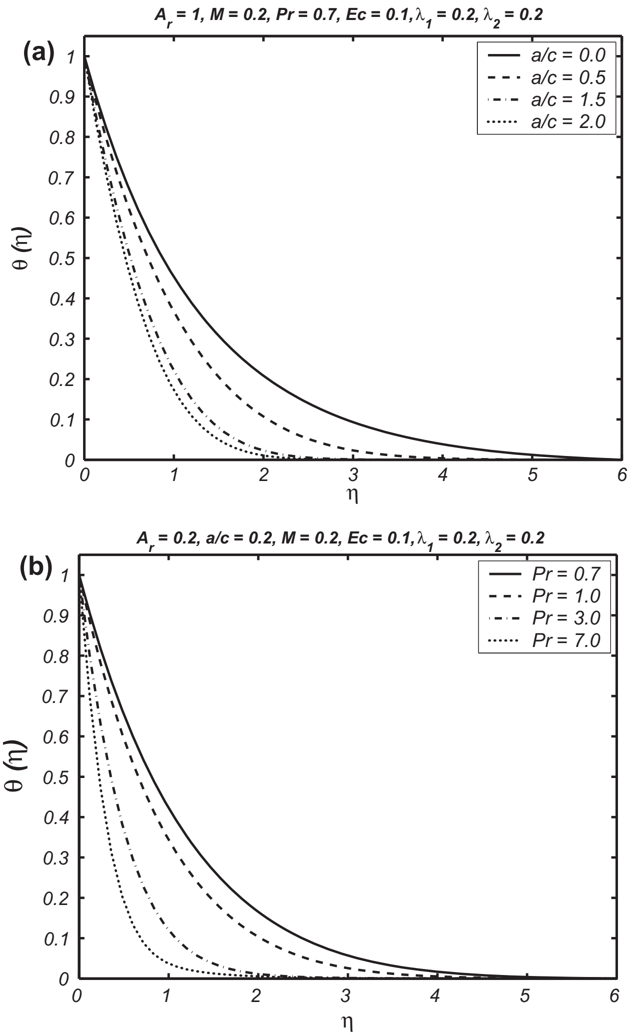

Figure 3 The temperature profile $\theta(\eta)$ verses η (a) for different values of a/c and (b) for different values of Pr.

Table 1 Numerical values of $-\theta'(0)$ for different values of Pr, Ec and A_r .

Pr	A_r	$Ec = 0.1$	$Ec = 0.5$	$Ec = 1.0$
0.70	1.00	0.8283	0.7062	0.5673
1.50	1.00	1.2324	1.0020	0.7495
3.00	1.00	1.7586	1.3680	0.9541
7.00	1.00	2.6772	1.9820	1.2722
10.00	1.00	3.1768	2.3086	1.4351
100.00	1.00	8.5362	5.7227	3.0850
0.50	0.00	0.6075	0.4832	0.3277
0.50	0.20	0.6311	0.5159	0.3740
0.50	0.50	0.6592	0.5549	0.4285
0.50	1.00	0.6952	0.6042	0.4990
0.50	3.00	0.7875	0.7330	0.6841
0.50	5.00	0.8480	0.8183	0.8088

Oldroyd-B fluid are presented in this paper. The obtained equations are then used to discuss the flow and heat transfer analysis for the stagnation point flow over a stretching sheet. The transformed ordinary differential equations are solved numerically by a finite difference method. The quantities of interest are the velocity, temperature and numerical values of $f''(0)$ and the Nusselt number. The results are presented graphically and discussed under the influence of the parameters appearing in the problem. It is found that the magnitude of heat transfer at the wall is increased by an increase in the Archimedes number A_r.

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