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SHORT COMMUNICATION

Entropy generation analysis of magneto hydrodynamic flow of a nanofluid over a stretching sheet

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Abstract An analysis is carried out to study the entropy generation of an incompressible, MHD flow of water based nanofluid over a stretching sheet. The analytical solutions of the governing non-dimensional nonlinear ordinary differential equations are presented in terms of hypergeometric functions and used to compute the entropy generation number. The effects of the physical parameters on velocity and temperature profiles are already studied in our previous work [13]. This work is extended to discuss the effects of magnetic parameter, nanoparticle volume fraction, Hartmann number and the dimensionless group parameter on the entropy generation for Cu, Ag, Al_2O_3 and TiO_2 nanoparticles. The local skin friction coefficient and reduced Nusselt number are tabulated.

MATHEMATICS SUBJECT CLASSIFICATION: 76D10; 76W05; 80A20; 80A99; 82D80

1. Introduction

Entropy of a thermo dynamical system refers to the unavailability of useful work. Physically entropy generation is associated with thermo dynamical irreversibility, which is a common phenomenon in all kinds of heat transfer designs. Greater rate of entropy generation in any thermal system destroys the useful work and greatly reduces the efficiency of the system. Bejan [1,2] presented a method named Entropy Generation Minimization (EGM) to measure and optimizes the disorder or disorganization generated during a process specifically in the fields of refrigeration (cryogenics), heat transfer, storage and solar thermal power conversion. The entropy generation analysis of nanofluids investigated by several authors in different geometries [3–9].

The boundary layer flow over a continuously stretching surface finds many important applications in engineering processes, such as polymer extrusion and drawing of plastic films, and the applied magnetic field may play an important role in controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet. The effect of magnetic field on nanofluids studied by the followers

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Nomenclature

B_0	magnetic field strength	k_f	thermal conductivity of the base fluid
Br	Brinkman number	k_s	thermal conductivity of the nanoparticles
Ha	Hartmann number	σ	electric conductivity
M	Kummer's function	Ω	dimensionless temperature difference
Mn	magnetic parameter	ϕ	the solid volume fraction
N_s	entropy generation number	ρ_{nf}	the effective density of the nanofluid
Pr	Prandtl number	ρ_f	density of the pure fluid
$Re_x^{1/2}C_f$	local skin friction coefficient	ρ_s	density of the nanoparticles
$Re_x^{-1/2}Nu_x$	reduced Nusselt number	μ_{nf}	effective dynamic viscosity of the nanofluid
S_G	local volumetric entropy generation rate	μ_f	dynamic viscosity of the basic fluid
$(S_G)_0$	characteristic entropy generation rate	η	space variable
T	local temperature of the fluid	α_{nf}	thermal diffusivity of the nanofluid
k_{nf}	thermal conductivity of the nanofluid		

[10–12]. Very recently, we investigated the effect of magnetic field on water based nanofluid over a stretching sheet numerically [13] and also we studied the MHD flow of nanofluid with thermal radiation effect both analytically and numerically [14].

The purpose of this attempt is to analyse the entropy generation of magneto hydrodynamic flow of an incompressible viscous nanofluid over a stretching sheet analytically. The analytical solutions of dimensionless governing equations are presented in terms of hypergeometric function. The entropy generation is calculated using the entropy relation by substituting the velocity and temperature fields obtained from the momentum and entropy equations.

2. Formulation of the problem

The entropy analysis for a steady laminar two-dimensional flow of an incompressible viscous nanofluid past a linearly semi-infinite stretching sheet is studied with magnetic field effect. We also consider influence of a constant magnetic field of strength B_0 which is applied normally to the sheet. The temperature at the stretching surface takes the constant value T_w, while the ambient value, attained as y tends to infinity, takes the constant value T_∞. It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field (as the magnetic Reynolds number is small). The fluid is a water based nanofluid containing different types of nanoparticles: Copper (Cu), Aluminium (Al_2O_3), Silver (Ag) and Titanium Oxide (TiO_2). It is also assumed that the base fluid water and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo physical properties of the nanofluid are considered as in [13]. Under the above assumptions, the governing equations can be written in non-dimensional (see [13]) form as

$$F''' + (1-\phi)^{2.5}\{[1-\phi+\phi(\rho_s/\rho_f)](FF''-F'^2) - Mn\,F'\} = 0 \tag{1}$$

$$\theta'' + \frac{Prk_f[1-\phi+\phi(\rho C_p)_s/(\rho C_p)_f]}{k_{nf}}F\theta' = 0 \tag{2}$$

with the corresponding boundary conditions

$$F=0 \quad F'=1 \quad \text{at} \quad \eta=0, \qquad F'\to 0 \quad \text{as} \quad \eta\to\infty \tag{3}$$

$$\theta(0)=1 \qquad \text{and} \qquad \theta(\infty)=0 \tag{4}$$

where ϕ is the solid volume fraction, ρ_f and ρ_s are the densities of the base fluid and nanoparticles, $(\rho C_p)_f$ and $(\rho C_p)_s$ are the specific heat parameters of the base fluid and nanoparticles, k_f is the thermal conductivity of the base fluid, k_{nf} the thermal conductivity of the nanofluid, Pr is the Prandtl number and Mn is the magnetic parameter.

3. Analytical solutions of the flow field and the heat transfer

The exact solution to differential Eq. (1) satisfying the boundary condition (3) is obtained as (see Anjali Devi and Ganga [15])

$$F(\eta) = \frac{1-e^{-m\eta}}{m} \tag{5}$$

where m is the parameter associated with the nanoparticle volume fraction, the magnetic field parameter, the fluid density and the nanoparticle density as follow

$$m = \sqrt{(1-\phi)^{2.5}[Mn+1-\phi+\phi(\rho_s/\rho_f)]} \tag{6}$$

The analytical solution of (2) satisfying (4) interms of η is obtained as

$$\theta(\eta) = e^{-ma_0\eta}\frac{M[a_0, a_0+1, -a_0e^{-m\eta}]}{M[a_0, a_0+1, -a_0]} \tag{7}$$

where M is the Kummer's function ([15]), $\alpha = \frac{k_{nf}}{k_f(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f})}$ and $a_0 = \frac{Pr}{\alpha m^2}$.

The skin friction can be written as

$$Re_x^{1/2}C_f = -\frac{2F''(0)}{(1-\phi)^{2.5}} = -\frac{2}{(1-\phi)^{2.5}}\sqrt{(1-\phi)^{2.5}[Mn+1-\phi+\phi(\rho_s/\rho_f)]} \tag{8}$$

The non-dimensional wall temperature gradient derived from Eq. (7) reads as

$$\theta'(0) = -ma_0 + \frac{ma_0^2}{1+a_0}\,\frac{M[a_0+1, a_0+2, -a_0]}{M[a_0, a_0+1, -a_0]} \tag{9}$$

where Re_x is the local Reynolds number and $Re_x^{1/2}C_f$ is the local skin friction coefficient and $Re_x^{-1/2}Nu_x = -\frac{k_{nf}}{k_f}\theta'(0)$ is the reduced Nusselt number.

4. Entropy generation analysis

The local volumetric rate of entropy generation in the presence of a magnetic field for nanofluids can be expressed as Wood [16]

$$S_G = \frac{k_{nf}}{T_\infty^2}\left[\left(\frac{\partial T}{\partial \bar{x}}\right)^2 + \left(\frac{\partial T}{\partial \bar{y}}\right)^2\right] + \frac{\mu_{nf}}{T_\infty}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{\sigma B_0^2}{T_\infty}\bar{u}^2 \tag{10}$$

The contributions of three sources of entropy generation are considered in Eq. (10). The first term indicates the entropy generation due to heat transfer across a finite temperature difference, the second term the local entropy generation due to viscous dissipation and the third term the local entropy generation due to the effect of the magnetic field. A dimensionless number for entropy generation rate N_S is defined as the ratio of the local volumetric entropy generation rate (S_G) to a characteristic entropy generation rate $(S_G)_0$. For a prescribed boundary condition the characteristic entropy generation rate is

$$(S_G)_0 = \frac{k_{nf}(\Delta T)^2}{\bar{x}^2 T_\infty^2} \tag{11}$$

therefore, the entropy generation number is

$$N_s = \frac{S_G}{(S_G)_0} \tag{12}$$

Using Eqs. (7), (10), (11) and (12) the entropy generation number is given by

$$N_s = \theta'^2(\eta) + \frac{Br}{\Omega}F''^2(\eta) + \frac{Br Ha^2}{\Omega Re_x}F'^2(\eta) \tag{13}$$

where Br is the Brinkman number. Ω and Ha are respectively the dimensionless temperature difference and the Hartmann number. These numbers are given by the following relationships

$$Br = \frac{\mu_{nf}\bar{u}_w^2}{k_{nf}\Delta T} \quad \Omega = \frac{\Delta T}{T_\infty}, \quad Ha = B_0\bar{x}\sqrt{\frac{\sigma}{\mu_{nf}}} \tag{14}$$

5. Results and discussion

In order to get the clear insight of the physical problem the analytical results are discussed with the help of graphical illustrations for Ag–water. The Prandtl number is fixed as 6.2 which is for base fluid water and Re_x is fixed as 1. The effects of nanoparticle volume fraction, magnetic parameter, Hartmann number and the dimensionless group parameter on the entropy generation are discussed for various nanoparticles such as Copper (Cu), Silver (Ag), Aluminium Oxide (Al_2O_3) and Titanium Oxide (TiO_2) when the base fluid is water. In order to validate the present results we have compared our results with those of Wang [17] for reduced Nusslet number $-\theta'(0)$ in the absence of nanoparticle volume fraction and

Table 1 Comparison of results for the reduced Nusselt number.

Pr	Present results with φ = 0 & Mn = 0	Wang [17]
0.07	0.065562	0.0656
0.2	0.169089	0.1691
0.7	0.453916	0.4539
2	0.911358	0.9114
7	1.895400	1.8954
20	3.353900	3.3539
70	6.462200	6.4622

Table 2 Values of $-F''(0)$ for various Mn and φ.

Mn	φ	$-F''(0)$							
		Cu		Ag		Al_2O_3		TiO_2	
		Analytical	Numerical [3]	Analytical	Numerical [13]	Analytical	Numerical [13]	Analytical	Numerical [13]
0	0.05	1.10892	1.1089207	1.13966	1.1396602	1.00538	1.0053797	1.01150	1.0115104
	0.1	1.17475	–	1.22507	–	0.99877	–	1.00952	–
	0.15	1.20886	1.2088625	1.27215	1.2721531	0.98185	0.9818474	0.99603	0.9960410
	0.2	1.21804	1.2180440	1.28979	1.2897881	0.95592	0.9559225	0.97259	0.9726024
0.5	0.05	1.29210	–	1.31858	–	1.20441	—	1.20953	–
	0.1	1.32825	–	1.37296	–	1.17548	–	1.18463	–
	0.15	1.33955	–	1.39694	–	1.13889	–	1.15114	–
	0.2	1.33036	–	1.39634	–	1.09544	–	1.11002	–
1	0.05	1.45236	–	1.47597	–	1.37493	–	1.37941	–
	0.1	1.46576	–	1.50640	–	1.32890	–	1.33700	–
	0.15	1.45858	–	1.51145	–	1.27677	–	1.28771	–
	0.2	1.43390	–	1.49532	–	1.21910	–	1.23222	–
2	0.05	1.72887	1.7288700	1.74875	1.7487500	1.66436	1.6643600	1.66806	1.6680601
	0.1	1.70789	–	1.74289	–	1.59198	–	1.59875	–
	0.15	1.67140	1.6713980	1.71773	1.7177294	1.51534	1.5153352	1.52457	1.5245650
	0.2	1.62126	1.6212641	1.67583	1.6758341	1.43480	1.4347985	1.44596	1.4459580

Table 3 Values of–$\theta'(0)$ for various Mn and φ when Pr = 6.2.

Mn	φ	$-\theta'(0)$							
		Cu		Ag		Al_2O_3		TiO_2	
		Analytical	Numerical [13]	Analytical	Numerical [13]	Analytical	Numerical [13]	Analytical	Numerical [13]
0	0.05	1.55011	1.5500001	1.53274	1.5327023	1.57565	1.5756510	1.59777	1.5977799
	0.1	1.35483	–	1.32448	–	1.39884	–	1.44010	–
	0.15	1.17850	1.1785098	1.13846	1.1384695	1.23631	1.2363100	1.29495	1.2949499
	0.2	1.01615	1.0161489	0.96923	0.9692299	1.08456	1.0845560	1.15984	1.1598065
0.5	0.05	1.50999	–	1.49356	–	1.53206	–	1.55440	–
	0.1	1.32127	–	1.29220	–	1.36014	–	1.40175	–
	0.15	1.15008	–	1.11144	–	1.20194	–	1.26100	–
	0.2	0.99195	–	0.94640	–	1.05408	–	1.12978	–
1	0.05	1.47494	1.4749200	1.45917	1.4591660	1.49474	1.4947500	1.51721	1.5172125
	0.1	1.29134	–	1.26323	–	1.32660	–	1.36842	–
	0.15	1.12438	1.1243800	1.08686	1.0868601	1.17188	1.1718780	1.12312	1.2311705
	0.2	0.96986	0.9698680	0.92561	0.9256201	1.02721	1.0272200	1.10317	1.1031672
2	0.05	1.41485	1.4148091	1.39995	1.3999590	1.43168	1.4316959	1.45429	1.4543474
	0.1	1.23914	–	1.21243	–	1.26949	–	1.31152	–
	0.15	1.07903	1.0790422	1.04325	1.0423810	1.12034	1.1203450	1.17986	1.1798890
	0.2	0.93058	0.9305950	0.88829	0.8883250	0.98092	0.9809399	1.05705	1.0570600

magnetic parameter. The comparison are found to be in good agreement as shown in Table 1.The values of local skin friction coefficient and reduced Nusselt number are calculated and presented in Table 2 and Table 3. The effects of magnetic parameter, solid volume fraction of nanoparticles, Prandtl number on velocity profile, temperature profile, skin friction and reduced Nusselt number are already discussed in the previously published paper (Vishnu Ganesh et al[13]) and a good agreement is observed in analytical and numerical results.

The influence of the magnetic parameter on the entropy generation number is shown in Fig. 1. It is clear that the increasing values of magnetic parameter increase the entropy generation number near the wall and far away from the wall it is not affected by the magnetic parameter. This is due to the increasing of the magnetic parameter causes the resistant forces against the fluid movement and then heat transfer rate in the boundary layer enhances. The presence of the magnetic field creates the entropy in the nanofluid.

Fig. 2 indicates the effect of the nanoparticles volume fraction on the entropy generation number. It is found that the entropy generation number decreases with the increasing of the nanoparticle volume fraction due to the higher dissipation energy resulted from the sharper velocity gradient near the wall and an opposite trend is observed far away from the wall.

The effect of Hartmann number on the entropy generation is depicted in Fig. 3. It can be seen that the entropy generation

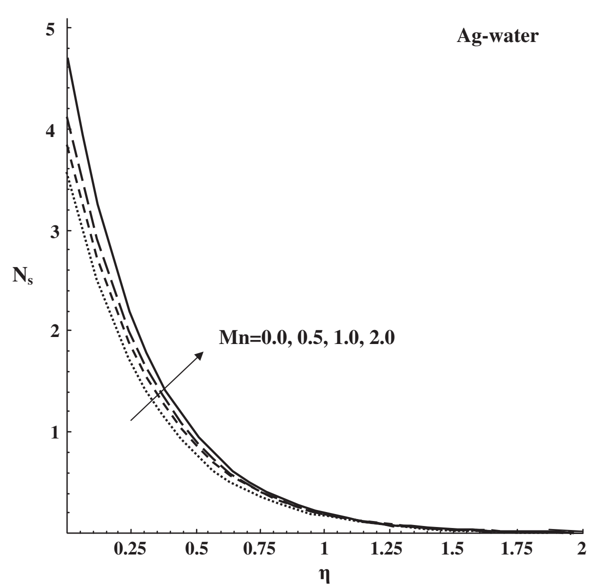

Fig. 1 Dimensionless entropy generation number profiles for different values of magnetic parameter, ϕ = 0.2, Pr = 6.2, $Re_x = 1$, $\mathrm{Br}\Omega^{-1}$ = 1.0, Ha = 1.0.

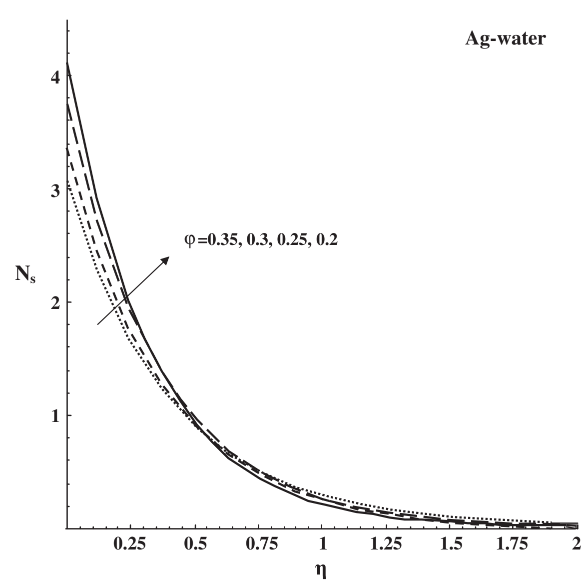

Fig. 2 Dimensionless entropy generation number profiles for different values of nanoparticles volume fraction parameter, Mn = 1.0, Pr = 6.2, $Re_x = 1$, $\mathrm{Br}\Omega^{-1}$ = 1.0, Ha = 1.0.

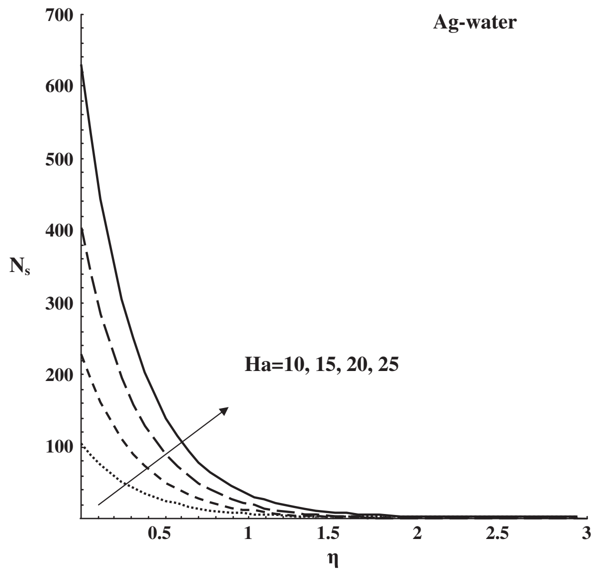

Fig. 3 Dimensionless entropy generation number profiles for different values of Hartman number, Mn = 1.0, Pr = 6.2, $Re_x = 1$, BrΩ^{-1} = 1.0, ϕ = 0.2.

number increases with the increasing of Hartman number. This is due to the fact the increasing of Hartmann number leads to increase the Lorentz forces which strengthen the dissipation energy as a source of irreversibility.

Fig. 4 illustrates the effect of dimensionless group parameter on the entropy generation number. It is observed that the entropy generation number increases with the increasing of dimensionless group parameter. This is because the higher values of the dimensionless group parameter, increase the nanofluid friction.

The effect of different nanoparticles on the entropy generation number is shown in Fig.5. It is clear that the entropy generation number is maximum for Ag–water and minimum

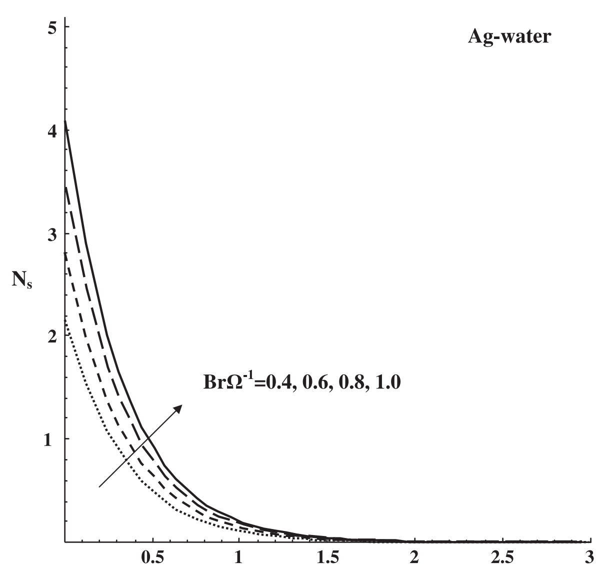

Fig. 4 Dimensionless entropy generation number profiles for different values of dimensionless group parameter, Mn = 1.0, Pr = 6.2, $Re_x = 1$, Ha = 1.0, ϕ = 0.2.

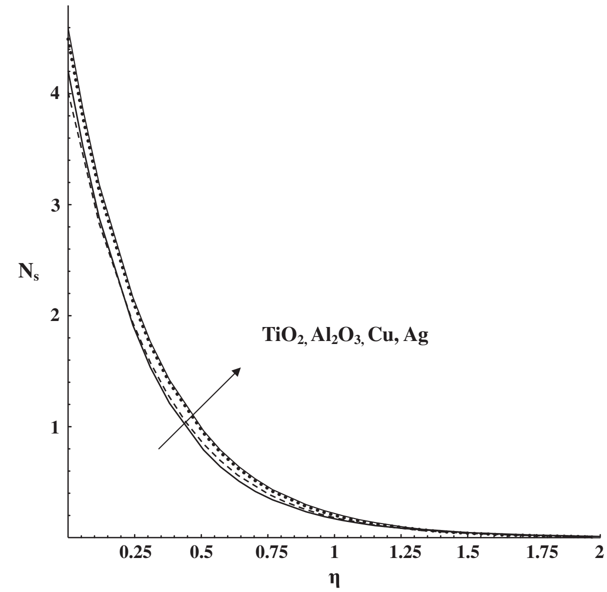

Fig. 5 Entropy generation for different types of nanofluids when Mn = 2.0, Pr = 6.2, $Re_x = 1$, BrΩ^{-1} = 1.0, Ha = 1.0, ϕ = 0.2.

for TiO_2–water. The entropy generation number of Cu–water is greater than Al_2O_3 water. It is also clear that the entropy generation is depend on the thermal conductivity of the nanoparticles which are present in the base fluid. The entropy generation is high for the metallic nanofluids and is low for the non-metallic nanofluids. This is because the metallic nanoparticles have high thermal conductivity and the non-metallic nanoparticles have low thermal conductivity.

6. Conclusion

Entropy generation analysis of hydromagnetic flow of an incompressible viscous nanofluid (Cu–water, *Ag*–water, Al_2O_3–water and TiO_2–water) over a stretching sheet in the presence of transverse magnetic field is investigated analytically. The main conclusions derived from this study are given below:

- The increasing values of magnetic parameter, Hartmann number and dimensionless group parameter leads to increase the generation of entropy in the nanofluid flow field.
- The rising values of nanoparticle volume fraction parameter decrease the entropy generation near the wall and an opposite trend is observed far away from the wall.
- The entropy generation depends on the thermal conductivity of the nanoparticles in the base fluid. The presence of metallic nanoparticles creates the entropy more in the nanofluid flow compared to the non-metallic nanoparticles.

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