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REVIEW PAPER

On Λ_α-sets and the associated topology T^{Λ_α}

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Abstract In this paper, we introduce the notions of an α-Λ-sets and an α-V-sets in topological space. We study the fundamental properties of α-Λ-sets and α-V-sets. Also, we investigate the topologies defined by these families of sets.

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Contents

1. Introduction

In [1] Maki has introduced the concept of Λ-sets in topological spaces as the sets that coincide with their kernel. The kernel of a set A is the intersection of all open supersets of A.

In this direction we shall introduce the notion of Λ_α-sets, V_α-sets, g.Λ_α-sets and g.V_α-sets. We also investigate properties of these sets and introduce some related new separation axioms.

2. Preliminaries

Njasted [2] introduced a new class of near open sets in a topological space, so called α-open sets. The class of α-open sets is contained in the class of semi-open and preopen sets and contains open sets. In 1986, Maki [1] continued the work of Levine [3] and Dunham [4] on generalized closed sets and exposure operators by introducing the notion of Λ-set in topological

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space. He studied the relationship between the given topology and the topology generated by generalized Λ-sets. Caldas and Dontchev [5] built on Maki's work by introducing Λ_s-sets, V_s-sets, $g\Lambda_s$-sets and gV_s-sets. Ganster et al. [6] introduced the notion of pre-Λ-sets and pre-V-sets and obtained new topologies defined by these families of sets. Also, Caldas et al. [7] introduced and studied the concepts of Λ_p-continuous functions (which includes the class of precontinuous functions). Λ_p-irresolute functions are defined as an analogy of irresolute functions and V_p-closed functions by using Λ_p-sets and V_p-sets. Definitions enable us to obtain conditions under which functions and inverse functions preserve Λ_p-sets and V_p-sets. They introduce a new class of topological spaces called $T^{\vee_p}$-spaces and as an application we show that the image of $T^{\vee_p}$-space under a homeomorphism is a $T^{\vee_p}$-space, and that a $T^{\vee_p}$-space is equivalent to pre-R_0 space. Abd El-Monsef et al. [8] introduced the notion of b-Λ-sets and b-V-sets and obtained new topologies defined by these families of sets. Also, they introduced and studied g.Λ_b-sets and g.V_b-sets and some of its properties. Khalimsky et al. [9] proved that the digital line is a typical example of a $T_{1/2}$ space. The concept of Λ_α-sets and V_α-sets can be applied in rough sets to decrease the boundary region of any subset of an approximation space.

Throughout this paper, (X, τ)(or simply X) represent topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A, respectively.

Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. A subset A of a topological space (X, τ) is called:

(a) semi-open [10] if $A \subseteq \text{cl}(\text{int}(A))$,
(b) preopen [11] if $A \subseteq \text{int}(\text{cl}(A))$,
(c) α-open [2] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$,
(d) β-open [12] [semi-preopen [13]] if $A \subseteq \text{cl}(\text{int}(\text{cl}((A)))$,
(e) b-open [14] [γ-open [15]] if $A \subseteq \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))$.

The class of all semi-open (resp. preopen, α-open, β-open and b-open) denoted by $SO(X, \tau)$(*resp.* $PO(X, \tau)$, $\alpha O(X, \tau)$, $\beta O(X, \tau)$ *and* $BO(X, \tau)$). The complement of these sets called semi-closed (resp. preclosed, α-closed, β-closed and b-closed) and the classes of all these sets and closed sets will be denoted by $SC(X, \tau)$(*resp.*$PC(X, \tau)$, $\alpha C(X, \tau)$, $\beta\, C(X, \tau)$, $BC(X, \tau)$ and $C(X, \tau)$).

Definition 2.2. A subset A of a topological space (X, τ) is called:

(a) Λ-set (resp. V-set) [1] if it is an intersection (resp, union) of open supersets of A (resp. closed sets contained in A).
(b) Λ_s-sets (resp. V_s-sets) [5] if it is an intersection (resp. union) of semi-open supersets of A (resp. semi-closed sets contained in A).
(c) pre-Λ sets (resp. pre-V-set) [6] if it is an intersection (resp. union) of preopen supersets of A (resp. preclosed sets contained in A).
(d) b-Λ-sets (resp. b-V-set) [8] if it is an intersection (resp. union) of b-open supersets of A (resp. b-closed sets contained in A).

Definition 2.3. A subset A of a topological space (X, τ) is called:

(a) generalized Λ-set (resp. generalized V-set) [1] if $\Lambda(A) \subseteq F$ whenever $A \subseteq F$ and $F \in C(X, \tau)$ (resp. $U \subseteq V(A)$ whenever $U \subseteq A$ and $U \in \tau$.
(b) generalized semi-Λ-set (resp. generalized semi-V-set) [5] if $\Lambda_s(A) \subseteq F$ whenever $A \subseteq F$ and $F \in SC(X, \tau)$ (resp. $U \subseteq V_s(A)$ whenever $U \subseteq A$ and $U \in SO(X, \tau)$.
(c) generalized pre-Λ-set (resp. generalized pre-V-set) [6] if $\Lambda_p(A) \subseteq F$ whenever $A \subseteq F$ and $F \in PC(X, \tau)$ (resp. $U \subseteq V_p\,(A)$ whenever $U \subseteq A$ and $U \in PO(X, \tau)$.
(d) generalized b- Λ-set (resp. generalized b-Λ-set) [8] if $\Lambda_b(A) \subseteq F$ whenever $A \subseteq F$ and $F \in BC(X, \tau)$ (resp. $U \subseteq V_b(A)$ whenever $U \subseteq A$ and $U \in BO(X, \tau)$.

Definition 2.4. A topological space (X, τ) is called an $\propto$-T_1-space [16] if to each pair of distinct points x, y of (X, τ) there corresponds an α-open set A containing x but not y and an α-open set B containing y but not x.

3. Λ_α-sets and V_α-sets

In this section we define the notions of an α-Λ-set and an α-V-sets in topological space which is denoted by Λ_α, V_α, and study some of its properties.

Definition 3.1. Let B be a subset of a topological space(X, τ). We define subsets $\Lambda_\alpha(B)$ *and* $V_\alpha(B)$ as follows:

(a) $\Lambda_\alpha(B) = \cap\{G\colon G \supseteq B,\, G \in \alpha O(X, \tau)\}$,
(b) $V_\alpha(B) = \cup\{F\colon F \subseteq B,\, F^C \in \alpha O(X, \tau)\}$.

Example 3.1. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ then $\tau_\alpha = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a.c.d\}\}$ and

$$\bigwedge_\alpha(\{a\}) = \{a\},\ \bigwedge_\alpha(\{b,d\}) = X,\ \bigwedge_\alpha(\{a,d\}) = \{a,c,d\},\ \bigwedge_\alpha(\{a,c,d\}) = \{a,c,d\},$$
$$\vee_\alpha(\{a\}) = \emptyset,\ \vee_\alpha(\{b,d\}) = \{b,d\},\ \vee_\alpha(\{a,d\}) = \{a,d\} \text{ and } \vee_\alpha(\{a,c,d\}) = \{d\}.$$

Proposition 3.1. *Let and* $\{B_\lambda\colon \lambda \in \Omega\}$ *be subsets of a topological space* (X, τ). *Then the following properties are valid:*

(a) $B \subseteq \Lambda_\alpha(B)$.
(b) *If* $A \subseteq B$, *then* $\Lambda_\alpha(A) \subseteq \Lambda_\alpha(B)$.
(c) $\Lambda_\alpha(\Lambda_\alpha(B)) = \Lambda_\alpha(B)$.
(d) *If* $A \in \alpha O(X, \tau)$, *then* $A = \Lambda_\alpha(A)$ *(i.e., A is an* Λ_α*-set)*.
(e) $\Lambda_\alpha\left(\bigcup_{\lambda\in\Omega} B_\lambda\right) = \bigcup_{\lambda\in\Omega}(\Lambda_\alpha(B_\lambda))$.
(f) $\Lambda_\alpha(B^C) = (V_\alpha(B))^C$.
(g) $\Lambda_\alpha\left(\bigcap_{\lambda\in\Omega} B_\lambda\right) \subseteq \bigcap_{\lambda\in\Omega}(\Lambda_\alpha(B_\lambda))$.

Proof.

(a) It is clear by Definition 3.1.
(b) Suppose that $x \notin \Lambda_\alpha(B)$. Then there exists a subset $G \in \alpha O(X, \tau)$ such that $G \supseteq B$ with $x \notin G$ since $B \supseteq A$, then $x \notin \Lambda_\alpha(A)$ and thus $\Lambda_\alpha(A) \subseteq \Lambda_\alpha(B)$.

(c) It follows from (a) and (b) that $\Lambda_\alpha(B) \subseteq \Lambda_\alpha(\Lambda_\alpha(B))$. If $x \notin \Lambda_\alpha(B)$, then there exists $G \in \alpha O(X, \tau)$ such that $B \supseteq G$ *and* $x \notin G$ hence $\Lambda_\alpha(B) \subseteq G$ and so we have $x \notin \Lambda_\alpha(\Lambda_\alpha(B))$. Then $\Lambda_\alpha(\Lambda_\alpha(B)) = \Lambda_\alpha(B)$.

(d) By Definition 3.1 and since $A \in \alpha O(X, \tau)$ we have $\Lambda_\alpha(A) \subseteq A$. By (a) we have that $\Lambda_\alpha(A) = A$.

(e) Suppose that there exists a point x such that $x \notin \Lambda_\alpha(\cup_{\lambda\in\Omega} B_\lambda)$. Then there exists an α- open set G such that $\cup_{\lambda\in\Omega} B_\lambda \subseteq G$ and $x \notin G$. Thus for each $\lambda \in \Omega$ we have $x \notin \Lambda_\alpha(B_\lambda)$. This implies that $x \notin \cup_{\lambda\in\Omega}\Lambda_\alpha(B_\lambda)$. Conversely, suppose that there exists a point $x \in$ X such that$x \notin \cup_{\lambda\in\Omega}\Lambda_\alpha(B_\lambda)$. Then by Definition 3.1 there exists a subsets $G_\lambda \in \alpha O(X, \tau)$ (for all $\lambda \in \Omega$) such that $x \notin G_\lambda$, $B_\lambda \subseteq G_\lambda$. Let $G = \cup_{\lambda\in\Omega} G_\lambda$. Then we have that $x \notin \cup_{\lambda\in\Omega}$-$G_\lambda$, $\cup_{\lambda\in\Omega} B_\lambda \subseteq G$ and $G \in \alpha$ O(X, τ). This implies that $x \notin \Lambda_\alpha$ $(\cup_{\lambda\in\Omega} B_\lambda)$.

(f) $(\mathrm{V}_\alpha(B))^c = \cap\{F^c\colon B^c \subseteq F^c,\ F^c \in \alpha O(\mathrm{X}, \tau)\} = \Lambda_\alpha(B^c)$.

(g) Suppose that there exist a point x such that $x \notin \bigcap_{\lambda\in\Omega}\Lambda_\alpha(B_\lambda)$ then, there exists $\lambda \in \Omega$ such that $x \notin \Lambda_\alpha$ (B_λ) thence there exist $\lambda \in \Omega$ and $G \in \alpha O(\mathrm{X}, \tau)$ such that $G \supseteq B_\lambda$ and $x \notin G$. Thus $x \notin \Lambda_\alpha(\cup_{\lambda\in\Omega} B_\lambda)$. □

By using Proposition 3.1(f) one can easily verify our next result.

Proposition 3.2. *For subsets A, B and $\{B_\lambda\colon \lambda \in \Omega\}$ of a topological space (X, τ) the following properties hold :*

(a) $V_\alpha(B) \subseteq B$.
(b) *If* $A \subseteq B$, *then* $V_\alpha(A) \subseteq V_\alpha(B)$.
(c) $V_\alpha(V_\alpha(B)) = V_\alpha(B)$.
(d) *If B is α-closed in (X, τ) , then* $B = V_\alpha(B)$.
(e) $V_\alpha(\cap_{\lambda\in\Omega} B_\lambda) = \cap_{\lambda\in\Omega}(V_\alpha(B_\lambda))$.
(f) $V_\alpha(\cup_{\lambda\in\Omega} B_\lambda) \supseteq \cup_{\lambda\in\Omega}(V_\alpha(B_\lambda))$.

Proof. (a), (b), (c) and (e) are immediate consequence of Definition 3.1 and Proposition 3.1. To prove (d) let B be α-closed in (X, τ), then $B^c \in \alpha O(\mathrm{X}, \tau)$. By (d) and (f) of Proposition 3.1$B^c = \Lambda_\alpha(B^c) = (\mathrm{V}_\alpha(B))^c$. Hence $B = \mathrm{V}_\alpha(B)$.

To prove (f) by using statement (d) and Proposition 3.1 (f) we have:

$$\mathrm{V}_\alpha\left(\bigcup_{\lambda\in\Omega} B_\lambda\right) = \left[\Lambda_\alpha\left(\bigcup_{\lambda\in\Omega} B_\lambda\right)^c\right]^c = \left[\Lambda_\alpha\left(\bigcap_{\lambda\in\Omega} B_\lambda^c\right)\right]^c [\bigcap_{\lambda\in\Omega}\Lambda_\alpha(B_\lambda^c)]^c$$
$$= \left[\bigcap_{\lambda\in\Omega}(\mathrm{V}_\alpha(B_\lambda))^c\right]^c = \bigcup_{\lambda\in\Omega}(\mathrm{V}_\alpha(B_\lambda)). \qquad \square$$

Remark 3.1. We not in general we have $\Lambda_\alpha(B_1 \cap B_2) \neq \Lambda_\alpha(B_1)$ - $\cap\, \Lambda_\alpha(B_2)$, as the following example shows.

Example 3.2. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. Let $B_1 = \{b\}$ and $B_2 = \{c\}$. Then $\Lambda_\alpha(B_1 \cap B_2) = \phi$, but $\Lambda_\alpha(B_1) \cap \Lambda_\alpha(B_2) = \{a\}$.

Definition 3.2. In a topological space (X, τ), a subset B is called a Λ_α-set (resp. V_α-set) of (X, τ) if $B = \Lambda_\alpha$ (B) (resp. $B = \mathrm{V}_\alpha$-set).

Example 3.3. Let (X, τ) be a topological space as in Example 3.1. Then is Λ_α-set and $B = \{b, d\}$ is V_α-set

Remark 3.2. By Proposition 3.1(d) and Proposition 3.2(d) we have that.

(a) If B is a Λ-set or if $B \in \alpha O(X, \tau)$, then B is a Λ_α-set.
(b) If B is V-set or if B is α-closed set, then B is a V_α-set.

The converse of this remark is not true as shown of the next example.

Example 3.4. Let (R, τ) be a usual topology, a singleton $\{x\}$ is not α-open but Λ_α-set.

Proposition 3.3. *For a topological space (X, τ) the following statements hold:*

(a) *The subsets ϕ and X are Λ_α-sets and V_α-sets.*
(b) *Every union of Λ_α-sets (resp. V_α-sets) is a Λ_α-set(resp. V_α-set).*
(c) *Every intersection Λ_α-sets (resp. V_α-sets) is a Λ_α-set (resp. V_α-set).*
(d) *A subset B is a Λ_α-set if and only if B^c is a V_α-set.*

Proof. We shall only consider the case of Λ_α-sets.

(a) obvious. To prove (b), let $\{B_\lambda\colon \lambda \in \Omega\}$ be a family of Λ_α-sets in (X, τ). If $B = \cup\{B_\lambda\colon \lambda \in \Omega\}$ then by Proposition 3.1 B is Λ_α-sets. To prove (C) , let $\{B_\lambda\colon \lambda \in \Omega\}$ be a family of Λ_α-set in (X, τ). Then by Proposition 3.1 (g) and Definition 3.2 we have $\Lambda_\alpha\left(\bigcap_{\lambda\in\Omega} B_\lambda\right) \subseteq \bigcap_{\lambda\in\Omega}\Lambda_\alpha(B_\lambda) = \bigcap_{\lambda\in\Omega} B_\lambda$, hence by Proposition 3.1(a)

$$\bigcap_{\lambda\in\Omega} B_\lambda = \Lambda_\alpha\left(\bigcap_{\lambda\in\Omega} B_\lambda\right). \qquad \square$$

Remark 3.4. Let τ^{Λ_α} and τ^{V_α} be the set of all Λ_α-sets and V_α-sets from X. Then τ^{Λ_α} and τ^{V_α} is a topology on X containing all α-open (resp. α-closed) sets.

Now we introduce some properties of τ^{Λ_α} and τ^{V_α}.

Proposition 3.4. *For a space (X, τ) the following statements hold:*

(a) $\tau^{\Lambda} \subseteq \tau^{\Lambda_\alpha}$.
(b) $\tau^{\Lambda_\alpha} \subseteq \tau^{\Lambda_s}, \tau^{\Lambda_\alpha} \subseteq \tau^{\Lambda_{\mathrm{P}}}$ and $\tau^{\Lambda_\alpha} \subseteq \tau^{\Lambda_b}$.
(c) $\tau^{\Lambda_s} = (\tau^{\alpha})^{\Lambda_s}$ and $\tau^{\Lambda_s} \subseteq (\tau^{\Lambda_\alpha})^{\overline{\Lambda_s}}$.

Proof.

(a) Let A be a subset of X and A is Λ-set or $(A \in \tau^{\Lambda})$. Then $A = \cap\{U\colon\ A \subseteq U,\ U \in \tau\}$. Since every open set is α-open. Then $A = \cap\{U\colon\ A \subseteq U,\ U$ *is* α-*open*$\}$, so $\tau^{\Lambda} \subseteq \tau^{\Lambda_\alpha}$.
(b) Since $\alpha O(X, \tau) \subset SO(X, \tau)$ [16]. By similar way of (a) then $\tau^{\Lambda_\alpha} \subseteq \tau^{\Lambda s}$. Also, since $\alpha O(X, \tau) \subset PO(X, \tau) \subseteq BO(X, \tau)$, then $\tau^{\Lambda_\alpha} \subseteq \tau^{\Lambda_p} \subseteq \tau^{\Lambda_b}$.

(c) Since $\mathrm{SO}(X, \tau) = \mathrm{SO}(X, \tau^{\alpha})$, then $\tau^{\Lambda_s} = (\tau^{\alpha})^{\Lambda_s}$ and since $\tau^{\alpha} \subseteq \tau^{\Lambda_\alpha}$. So $\tau^{\Lambda_s} \subseteq (\tau^{\Lambda_\alpha})^{\Lambda_s}$. $\square$

Proposition 3.5. *If (X, τ) and (X, σ) are two topological spaces such that $SO(X, \tau) = SO(X, \sigma)$, then $\sigma^{\Lambda} \subset \tau^{\Lambda_\alpha}$.*

Proof. Since for a two topological space (X, τ) and (X, σ) if $SO(X, \tau) = SO(X, \sigma)$ then $\sigma \subseteq \tau^{\alpha}$ and so $(\sigma^{\Lambda}) \subseteq \tau^{\Lambda_\alpha}$. $\square$

Remark 3.5. We note that in the above Proposition if $\tau^{\Lambda_s} = \sigma^{\Lambda_s}$ not necessary lead to $\sigma^{\Lambda} \subset \tau^{\Lambda\alpha}$ the following example show this fact.

Example 3.2. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{x, \phi, \{a\}, \{c\}, \{a, c\}\}$, then $\tau^{\Lambda_s} = \sigma^{\Lambda_s} = PO(X)$ but $\sigma^{\Lambda} = \{X, \phi.\{a\}, \{c\}, \{a, c\}\}$, and $\tau^{\Lambda_\alpha} = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$.

Proposition 3.6. *If $A \in PO(X, \tau)(resp. A \in \tau^{\Lambda_P})$ and $B \in \alpha O(X, \tau)(resp. B \in \tau^{\Lambda_\alpha})$, then $A \cap B \in \tau^{\Lambda_\alpha}(A, \tau_A)$.*

Proof. If A is a preopen set and B is an α-open set then $A \cap B \in \alpha O(A, \tau_A)$. So $A \cap B \in \tau^{\Lambda_\alpha}(A, \tau_A)$. If $A \in \tau^{\Lambda_P}$, then $A \in \cap \{G\colon A \subseteq G, G \in PO(X, \tau)\}$ and if

$B \in \tau^{\alpha}(X, \tau)$, then $B = \cap \{U\colon B \subseteq U, U \in \alpha O(X, \tau)\}$. So

$$A \cap B = \cap\{G \cap U : A \cap B \subseteq G \cap U, G \in PO(X, \tau), U \in \alpha O(X, \tau)\} = \cap\{S : S \in \alpha O(A, \tau), A \cap B \subset S\}, \quad \text{then } A \cap B \in \tau^{\Lambda_\alpha}(A, \tau). \qquad \square$$

The known relationships between some types of sets are summerset in Fig. Figurefigurekk.

Proposition 3.7. *For a space (X, τ) the following are equivalent*

(a) *(X, τ) is $\alpha - T_1$.*
(b) *Every subset A of X is Λ_α-set.*
(c) *Every subset A of X is V_α-set.*

Proof. ($a \to c$) Let $A \subseteq X$. since $A = \cup \{\{x\}\colon x \in A\}$, A is a union of α-*closed* sets, hence A is V_α-set.

($b \iff c$) Clearly by Proposition 3.3.

($c \to a$) Since every subset is V_α-set then it is $\alpha -$ *closed* and so X is α-T_1. $\square$

Proposition 3.8. *For a space (X, τ) the following statements are hold:*

(a) *If (X, τ) is α-T_1, then $(X, \tau^{\Lambda_\alpha})$ and (X, τ^{V_α}) are discrete space.*
(b) *The identity function $f : (X, \tau^{\Lambda_\alpha}) \to (X, \tau)$ is continuous.*

Proof. Obvious. $\square$

Proposition 3.9. *For a space (X, τ) we have $RC(X, \tau^{\Lambda_\alpha}) = RC(X, \tau^{\Lambda})$.*

Proof. Obvious by the result $\mathrm{RC}(X, \tau) = \mathrm{RC}(X, \tau^{\alpha})$. $\square$

4. g.Λ_α-sets and g.V_α-sets

In this section, by using the Λ_α-operator and V_α-operator, we introduce the classes of generalized Λ_α-sets (=g.Λ_α-sets), and generalized V_α-sets (=g.V_α-sets) as an analogy of the sets introduced by Maki [1].

Definition 4.1. In a topological space (X, τ), a subset A is called

(a) g.Λ_α-set of (X, τ) if $\Lambda_\alpha(A) \subseteq F$ whenever $A \subseteq F$ and F is α-*closed*.
(b) g.V_α-set of (X, τ) if $X - A$ is g. Λ_α-set of(X, τ).

Now, $D^{\Lambda_\alpha}(resp. D^{V_\alpha})$ will be denoted by the set of all g.Λ_α-sets (resp. g.V_α-sets) in (X, τ).

Proposition 4.1. *Let (X, τ) be a topological space and I be any index set.*

(a) *Every Λ_α-set is g.Λ_α-set.*
(b) *Every V_α-set is g.V_α-set.*
(c) *If $A_i \in D^{\Lambda_\alpha}$ for $i \in I$, then $\bigcup_{i \in I} A_i \in D^{\Lambda_\alpha}$.*
(d) *If $A_i \in D^{V_\alpha}$ for $i \in I$, then $\bigcup_{i \in I} A_i \in D^{V_\alpha}$.*

Proof. (a) and (b) is proved by Definition 3.2, Proposition 3.1 and Definition 4.1. (c) and (d) is proved by (e) of Proposition 3.1 and Definition 4.1. $\square$

Remark 4.1. The converse of (a) and (b) of Proposition 4.1 is not true in general as shown by the following example.

Example 4.1. Let $X = \{a, b, c,\}$ with $\tau = \{X, \phi, \{a, b\}\}$ then $\tau^{\Lambda_\alpha} = \{X, \phi, \{a, b\}\}$. Then a subset $A = \{a, c\}$ is g.Λ_α-set but it is not Λ_α-set. Also the subset $\{a\}$ is g.V_α-set, but is not V_α-set.

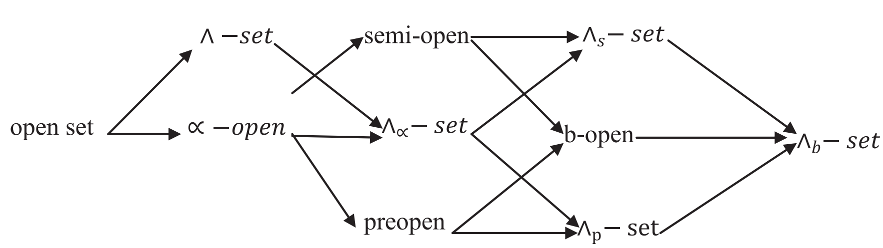

Figure 1 Comparison between sets and types of Λ-sets.

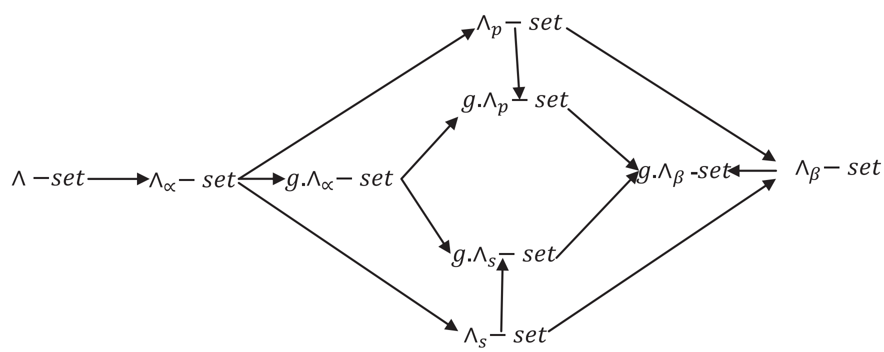

Figure 2 Comparison between types of Λ-sets and types of generalized-Λ-sets.

Remark 4.2. The intersection of two g.Λ_α-sets generally not a g.Λ_α-set. Also the union of two g.V_α-sets is generally not a g.V_α-set as shown in the following example.

Example 4.2. Let (X, τ) be the space in Example 4.1. *If* $A = \{a, c\}$ *and* $B = \{b, c\}$, then A and B are g. Λ_α-sets but $A \cap B = \{c\}$ is not g. Λ_α-set. Also if $A = \{a\}$ and $B = \{b\}$, then A and B are g. V_α-sets but $A \cup B = \{a, b\}$ is not g. V_α-sets.

Proposition 4.2. *Let (X, τ) be a topological space.*

(a) *For each $\{x\} \subset X$ is α-open set or $X - \{x\}$ is g.Λ_α-set of (X, τ).*
(b) *For each $\{x\} \subset X$ is α-open set or $X - \{x\}$ is g.V_α-set of (X, τ).*

Proof.

(a) Let $\{x\}$ be not α-open set then only α-closed set F containing $X - \{x\}$ is X. Thus $(X - \{x\})^{\Lambda_\alpha} \subseteq F = X$ and $X - \{x\}$ is a g.Λ_α-set of (X, τ).
(b) By similar way of (a). □

Theorem 4.1. *Every g.Λ_α-set is g.Λ_p-set (resp. g.Λ_s-set, g.Λ_b-set).*

Proof. Obvious by Theorem 4.1. and Proposition 3.4. □

The known relationships between the types of generalized closed sets are summerset in Fig. Figurefigurekk.

Definition 4.2. A function f: $(X, \tau) \to (Y, \sigma)$ is said to be

(a) Strongly $\propto$-irresolute [17] if for each $x \in X$ and each $\propto$-open set G of Y containing $f(x)$, there is an open set H containing x such that $f(H) \subseteq G$, equivalently, if the inverse image of each $\propto$-open set is open.
(b) $\propto$-irresolute [18] if for each $x \in X$ and each $\propto$-open set G of Y containing $f(x)$, there is an $\propto$-open set H of X containing $\times$ such that $f(H) \subseteq G$, equivalently, if the inverse image of each $\propto$-open set is $\propto$-open.

Theorem 4.2.

(a) *If a function f: $(X, \tau) \to (Y, \sigma)$ is $\propto$-irresolute, then $f: (X, \tau^{\wedge_\propto}) \to (Y, \sigma^{\wedge_\propto})$ is continuous.*
(b) *The identity function $id_x : (X, \tau^{\wedge_\propto}) \to (Y, \sigma)$ is strongly $\propto$-irresolute.*

Proof.

(a) Let G be any Λ_α-set of (Y, σ) i.e. $G \in \sigma^{\wedge_\alpha}$. Then $G = \wedge_\propto(G) = \cap\{W\colon G \subseteq W$ and W is $\propto$-open in $(Y, \sigma)\}$. Since f is $\propto$-irresolute, $f^{-1}(W)$ is $\propto$-open in (X, τ) for each W. Hence we have $f^{-1}(Q) \supseteq \cap\{f^{-1}(W)\colon f^{-1}(G) \subseteq f^{-1}(W)$ and W is $\propto$-open in $(Y, \sigma)\} \supseteq \cap\{H\colon f^{-1}(G) \subseteq H$ and H is $\propto$-open in $(X, \tau)\} = \wedge_\propto(f^{-1}(G))$. On the other hand, by the definition, $f^{-1}(G) \subseteq \wedge_\propto(f^{-1}(G))$. Hence, we obtain $f^{-1}(G) = \wedge_\propto(f^{-1}(G))$. Therefore $f^{-1}(G) \in \tau^{\wedge_\alpha}$ and $f: (X, \tau^{\wedge_\propto}) \to (Y, \sigma^{\wedge_\propto})$ is continuous.
(b) Let G be any $\propto$-open set of (Y, σ). Since G is $\propto$-open, by Proposition 3.1 $(id_x)^{-1}(G) = G \in \tau^{\wedge_\propto}$ and hence id_x is strongly $\propto$-irresolute. □

5. Conclusion

The notions of sets and functions in topological spaces extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences.

By researching generalizations of closed sets, Λ-sets and V-sets, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. The notion of kernel of a set has applications in computer science [19]. This notion is used in most of this paper.

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