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Unified fixed point theorems for mappings in fuzzy metric spaces via implicit relations

Sunny Chauhan [a,*], Mohammad Imdad [b], Wutiphol Sintunavarat [c], Yonghong Shen [d]

[a] Near Nehru Training Centre, H. No. 274, Nai Basti B-14, Bijnor 246 701, Uttar Pradesh, India
[b] Department of Mathematics, Aligarh Muslim University, Aligarh 202 002, India
[c] Department of Mathematics and Statistics, Faculty of Science and Technology, Thammasat University Rangsit Center, Pathumthani 12121, Thailand
[d] School of Mathematics and Statistics, Tianshui Normal University, Tianshui 741001, PR China

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Abstract In this paper, we prove some fixed point theorems for weakly compatible mappings in fuzzy metric spaces employing common limit range property with implicit relations. We also furnish some illustrative examples to support our main results. As an application to our main result, we derive a fixed point theorem for four finite families of self-mappings which can be utilized to derive common fixed point theorems involving any finite number of mappings. Our results improve and extend a host of previously known results including the ones contained in the paper of Gopal et al. (2011).

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1. Introduction

In 1965, Zadeh [1] introduced the well-known concept of a fuzzy set in his seminal paper. In the last two decades there has been a tremendous development and growth in fuzzy mathematics. In 1975, Kramosil and Michálek [2] introduced the concept of fuzzy metric space, which opened an avenue for further development of analysis in such spaces. Further, George and Veeramani [3] modified the concept of fuzzy metric space introduced by Kramosil and Michálek [2] with a view to obtain a Hausdoroff topology on it. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modeling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, and communication.

Mishra et al. [4] extended the notion of compatible mappings to fuzzy metric spaces and proved common fixed point theorems in presence the of continuity of at least one of the mappings, completeness of the underlying space and

* Corresponding author. Tel.: +91 9412452300.
E-mail address: sun.gkv@gmail.com (S. Chauhan).

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containment of the ranges among involved mappings. Further, Singh and Jain [5] weakened the notion of compatibility by using the notion of weakly compatible mappings in fuzzy metric spaces and showed that every pair of compatible mappings is weakly compatible but reverse is not true. Many mathematicians used different conditions on self-mappings and proved several fixed point theorems for contractions in fuzzy metric spaces (see [6–16]). However, the study of common fixed points of non-compatible mappings is also of great interest due to Pant [17]. In 2002, Aamri and Moutawakil [18] defined a property (E.A) for self-mappings which contained the class of non-compatible mappings in metric spaces. In a paper of Ali and Imdad [19], it was pointed out that property (E.A) allows replacing the completeness requirement of the space with a more natural condition of closedness of the range. Afterward, Liu et al. [20] defined a new property which contains the property (E.A) and proved some common fixed point theorems under hybrid contractive conditions. It was observed that the notion of common property (E.A) relatively relaxes the required containment of the range of one mapping into the range of other which is utilized to construct the sequence of joint iterates. Subsequently, there are a number of results proved for contraction mappings satisfying property (E.A) and common property (E.A) in fuzzy metric spaces (see [21–28]). In 2011, Sintunavarat and Kumam [29] coined the idea of "common limit range property" (also see [30–35]) which relaxes the condition of closedness of the underlying subspace. Recently, Imdad et al. [36] extended the notion of common limit range property to two pairs of self-mappings which relaxes the requirement on closedness of the subspaces. Several common fixed point theorems have been proved by many researcher in framework of fuzzy metric spaces via implicit relations (see [5,21,37]).

In fixed point theory, implicit relations are utilized to cover several contraction conditions in one go rather than proving a separate theorem for each contraction condition. In 2005, Singh and Jain [5] proved common fixed point theorems for semi-compatible mappings in fuzzy metric spaces satisfying an implicit function. Recently, Gopal et al. [24] defined two independent classes of implicit functions and obtained some fixed point results for two pairs of weakly compatible mappings satisfying common property (E.A).

In this paper, utilizing the implicit functions of Gopal et al. [24], we prove fixed point theorems for two pairs of weakly compatible mappings employing common limit range property. In process, many known results (especially the ones contained in Gopal et al. [24]) are enriched and improved. Some related results are also derived besides furnishing illustrative examples.

2. Preliminaries

Definition 2.1 [38]. A binary operation $*: [0,1]\times[0,1]\to[0,1]$ is a continuous t-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative,
(2) $*$ is continuous,
(3) $a*1=a$ for every $a\in[0,1]$,
(4) $a*b\leqslant c*d$ if $a\leqslant c$ and $b\leqslant d$ for all $a,b,c,d\in[0,1]$.

Three typical examples of continuous t-norms are minimum t-norm, that is, $a*b=\min\{a,b\}$, product t-norm, that is, $a*b=ab$ and Lukasievicz t-norm, that is, $a*b=\max\{a+b-1,0\}$.

Definition 2.2 [39]. Let X be any set. A fuzzy set in X is a function with domain X and values in $[0,1]$.

Definition 2.3 [3]. A triplet $(X,M,*)$ is a fuzzy metric space whenever X is an arbitrary set, $*$ is a continuous t-norm, and M is a fuzzy set on $X\times X\times(0,+\infty)$ satisfying the following conditions: for every $x,y,z\in X$ and $s,t>0$

(1) $M(x,y,t)>0$,
(2) $M(x,y,t)=1$ if and only if $x=y$,
(3) $M(x,y,t)=M(y,x,t)$,
(4) $M(x,z,t+s)\geqslant M(x,y,t)*M(y,z,s)$,
(5) $M(x,y,\cdot):(0,+\infty)\to(0,1]$ is continuous.

Note that $M(x,y,t)$ can be realized as the measure of nearness between x and y with respect to t. It is known that $M(x,y,\cdot)$ is non-decreasing for all $x,y\in X$. Let $(X,M,*)$ be a fuzzy metric space. For $t>0$, the open ball $\mathcal{B}(x,r,t)$ with center $x\in X$ and radius $0<r<1$ is defined by $\mathcal{B}(x,r,t)=\{y\in X: M(x,y,t)>1-r\}$. Now, the collection $\{\mathcal{B}(x,r,t): x\in X, 0<r<1, t>0\}$ is a neighborhood system for a topology τ on X induced by the fuzzy metric M. This topology is Hausdorff and first countable.

Definition 2.4 [3]. A sequence $\{x_n\}$ in X converges to x if and only if for each $\epsilon>0$ and each $t>0$, there exists $n_0\in\mathbb{N}$ such that $M(x_n,x,t)>1-\epsilon$ for all $n\geqslant n_0$.

In the following example, we know that every metric induces a fuzzy metric:

Example 2.1 [3]. Let (X,d) be a metric space. We define $a*b=ab$ for all $a,b\in[0,1]$ and let M_d be a fuzzy set on $X^2\times(0,+\infty)$ defined as follows:

$$M_d(x,y,t)=\frac{t}{t+d(x,y)}.$$

Then $(X,M_d,*)$ is a fuzzy metric space and the fuzzy metric M induced by the metric d is often referred to as the standard fuzzy metric. The fuzzy metric space $(X,M_d,*)$ is complete if and only if the metric space (X,d) is complete.

Definition 2.5 [4]. A pair (A,S) of self-mappings of a fuzzy metric space $(X,M,*)$ is said to be compatible if and only if $M(ASx_n,SAx_n,t)\to1$ for all $t>0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n,Sx_n\to z$ for some $z\in X$ as $n\to\infty$.

Definition 2.6 [40]. A pair (A,S) of self-mappings of a nonempty set X is said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, that is, if $Az=Sz$ some $z\in X$, then $ASz=SAz$.

Remark 2.1 [40]. Two compatible self-mappings are weakly compatible, but the converse is not true. Therefore the concept of weak compatibility is more general than that of compatibility.

Definition 2.7 [22]. A pair (A, S) of self-mappings of a fuzzy metric space $(X, M, *)$ is said to satisfy the property (E.A), if there exists a sequence $\{x_n\}$ in X for some $z \in X$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

for some $z \in X$.

Definition 2.8 [22]. A pair (A, S) of self-mappings of a fuzzy metric space $(X, M, *)$ is said to be non-compatible if and only if there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$, but for some $t > 0, \lim_{n\to\infty} M(ASx_n, SAx_n, t)$ is either less than 1 or non-existent.

Definition 2.9 [22]. Two pairs (A, S) and (B, T) of self-mappings of a fuzzy metric space $(X, M, *)$ are said to satisfy the common property (E.A), if there exist two sequences $\{x_n\}, \{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

for some $z \in X$.

Definition 2.10 [29]. A pair (A, S) of self-mappings of a fuzzy metric space $(X, M, *)$ is said to satisfy the common limit range property with respect to mapping S (briefly, (CLR_S) property), if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

where $z \in S(X)$.

Definition 2.11 [30]. Two pairs (A, S) and (B, T) of self-mappings of a fuzzy metric space $(X, M, *)$ are said to satisfy the common limit range property with respect to mappings S and T (briefly, (CLR_{ST}) property), if there exist two sequences $\{x_n\}, \{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

where $z \in S(X) \cap T(X)$.

Definition 2.12 [41]. The pair $(A_1A_2 \dots A_m, S_1S_2 \dots S_n)$ of two families of self-mappings $\{A_i\}_{i=1}^m$ and $\{S_k\}_{k=1}^n$ are said to be pairwise commuting if

(1) $A_iA_j = A_jA_i$ for all $i, j \in \{1, 2, \dots, m\}$,
(2) $S_kS_l = S_lS_k$ for all $k, l \in \{1, 2, \dots, n\}$,
(3) $A_iS_k = S_kA_i$ for all $i \in \{1, 2, \dots, m\}$ and $k \in \{1, 2, \dots, n\}$.

Lemma 2.1 [4]. *Let $(X, M, *)$ be a fuzzy metric space with $t * t \geqslant t$ for all $t \in [0, 1]$. If there exists a constant $k \in (0, 1)$ such that*

$$M(x, y, kt) \geqslant M(x, y, t),$$

for all $x, y \in X$, then $x = y$.

3. Implicit relations

In 2005, Singh and Jain [5] defined the following class of a implicit function.

Let Φ be the set of all real continuous functions $\phi : [0, 1]^4 \to \mathbb{R}$, non-decreasing in first argument and satisfy:

(ϕ_1) for $u, v \geqslant 0, \phi(u, v, u, v) \geqslant 0$, or $\phi(u, v, v, u) \geqslant 0$ implies that $u \geqslant v$,
(ϕ_2) $\phi(u, u, 1, 1) \geqslant 0$ implies that $u \geqslant 1$.

Example 3.1. Define $\phi : [0, 1]^4 \to \mathbb{R}$ as $\phi(t_1, t_2, t_3, t_4) = 15t_1 - 13t_2 + 5t_3 - 7t_4$. Then, $\phi \in \Phi$.

Since then, Imdad and Ali [37] introduced a new class of implicit function.

Let Ψ denotes the family of all continuous functions $\psi : [0, 1]^4 \to \mathbb{R}$ satisfying the following conditions:

(ψ_1) for every $u > 0, v \geqslant 0$ with $\psi(u, v, u, v) \geqslant 0$ or $\psi(u, v, v, u) \geqslant 0$, we have $u > v$,
(ψ_2) $\psi(u, u, 1, 1) < 0$, for each $0 < u < 1$.

Example 3.2 [37]. Define $\psi : [0, 1]^4 \to \mathbb{R}$ as $\psi(t_1, t_2, t_3, t_4) = t_1 - \varphi(\min\{t_2, t_3, t_4\})$, where $\varphi : [0, 1] \to [0, 1]$ is a continuous function such that $\varphi(s) > s$ for $0 < s < 1$. Then, $\psi \in \Psi$.

Example 3.3 [37]. Define $\psi : [0, 1]^4 \to \mathbb{R}$ as $\psi(t_1, t_2, t_3, t_4) = t_1 - k \min\{t_2, t_3, t_4\}$, where $k > 1$. Then, $\psi \in \Psi$.

Example 3.4 [37]. Define $\psi : [0, 1]^4 \to \mathbb{R}$ as $\psi(t_1, t_2, t_3, t_4) = t_1 - kt_2 - \min\{t_3, t_4\}$, where $k > 0$. Then, $\psi \in \Psi$.

Example 3.5 [37]. Define $\psi : [0, 1]^4 \to \mathbb{R}$ as $\psi(t_1, t_2, t_3, t_4) = t_1 - at_2 - bt_3 - ct_4$, where $a > 1$ and $b, c \geqslant 0, (b, c \neq 1)$. Then, $\psi \in \Psi$.

Example 3.6 [37]. Define $\psi : [0, 1]^4 \to \mathbb{R}$ as $\psi(t_1, t_2, t_3, t_4) = t_1 - at_2 - b(t_3 + t_4)$, where $a > 1$ and $0 \leqslant b < 1$. Then, $\psi \in \Psi$.

Example 3.7 [37]. Define $\psi : [0, 1]^4 \to \mathbb{R}$ as $\psi(t_1, t_2, t_3, t_4) = t_1^3 - kt_2t_3t_4$, where $k > 1$. Then, $\psi \in \Psi$.

In [37], it is also showed that the above mentioned classes of functions ϕ and ψ are independent.

4. Main results

We begin with the following observation.

Lemma 4.1. *Let A, B, S and T be four self-mappings of a fuzzy metric space $(X, M, *)$. Suppose that*

(1) *The pair (A, S) satisfies the (CLR_S) property (or (B, T) satisfies the (CLR_T) property),*
(2) $A(X) \subset T(X)$ (or $B(X) \subset S(X)$),
(3) $T(X)$ (or $S(X)$) *is a closed subset of X,*

(4) *$B(y_n)$ converges for every sequence $\{y_n\}$ in X whenever $T(y_n)$ converges (or $A(x_n)$ converges for every sequence $\{x_n\}$ in X whenever $S(x_n)$ converges),*
(5) *there exists $\psi \in \Psi$ such that*

$$\psi(M(Ax,By,t),M(Sx,Ty,t),M(Ax,Sx,t),M(By,Ty,t)) \geqslant 0 \tag{4.1}$$

for all $x, y \in X$ and $t > 0$.

Then the pairs (A,S) and (B,T) enjoy the (CLR_{ST}) property.

Proof. Suppose that the pair (A,S) enjoys the (CLR_S) property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

where $z \in S(X)$. Since $A(X) \subset T(X)$, for each sequence $\{x_n\}$ there exists a sequence $\{y_n\}$ in X such that $Ax_n = Ty_n$. Therefore, due to the closedness of $T(X)$,

$$\lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Ax_n = z,$$

where $z \in S(X) \cap T(X)$. Thus, we have $Ax_n \to z, Sx_n \to z$ and $Ty_n \to z$ as $n \to \infty$. By (4), the sequence $\{By_n\}$ converges and in all we need to show that $By_n \to z$ as $n \to \infty$. On the contrary, $By_n \to l(\neq z)$ as $n \to \infty$. On using inequality (4.1) with $x = x_n, y = y_n$, we have (for $t > 0$)

$$\psi(M(Ax_n,By_n,t),M(Sx_n,Ty_n,t),M(Ax_n,Sx_n,t),$$
$$M(By_n,Ty_n,t)) \geqslant 0.$$

Taking the limit as $n \to \infty$, we have

$$\psi(M(z,l,t),M(z,z,t),M(z,z,t),M(l,z,t)) \geqslant 0,$$

or, equivalently,

$$\psi(M(z,l,t),1,1,M(l,z,t)) \geqslant 0,$$

yielding thereby, $M(z,l,t) > 1$, a contradiction (due to (ψ_1)). Then we get, $z = l$. Therefore, the pairs (A,S) and (B,T) enjoy the (CLR_{ST}) property.

In case of (B,T) satisfies the (CLR_T) property is similar to previous case. Then, in order to avoid repetition, the details are omitted. □

Remark 4.1. The converse of Lemma 4.1 is not true in general. For a counter example, we refer to Example 4.1.

Theorem 4.1. *Let A, B, S and T be four self-mappings of a fuzzy metric space $(X, M, *)$ satisfying inequality (4.1). Suppose that the pairs (A,S) and (B,T) satisfy the (CLR_{ST}) property, then the pairs (A,S) and (B,T) have a coincidence point each. Moreover, A, B, S and T have a unique common fixed point provided that both the pairs (A,S) and (B,T) are weakly compatible.*

Proof. If the pairs (A,S) and (B,T) satisfy the (CLR_{ST}) property, then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to+\infty} Ax_n = \lim_{n\to+\infty} Sx_n = \lim_{n\to+\infty} By_n = \lim_{n\to+\infty} Ty_n = z,$$

where $z \in S(X) \cap T(X)$. Since $z \in S(X)$, there exists a point $u \in X$ such that $Su = z$. We show that $Au = Su$. Suppose not, then putting $x = u$ and $y = y_n$ in (4.1), we get

$$\psi(M(Au,By_n,t),M(Su,Ty_n,t),M(Au,Su,t),$$
$$M(By_n,Ty_n,t)) \geqslant 0$$

which on making $n \to \infty$, reduces to

$$\psi(M(Au,z,t),M(z,z,t),M(Au,z,t),M(z,z,t)) \geqslant 0,$$

and so,

$$\psi(M(Au,z,t),1,M(Au,z,t),1) \geqslant 0.$$

In view of (ψ_1), we get $M(Au,z,t) > 1$, a contradiction. Hence, $Au = z = Su$ which shows that u is a coincidence point of the pair (A,S).

Also $z \in T(X)$, there exists a point $v \in X$ such that $Tv = z$. Next, we show that $Bv = Tv$. If not, then using (4.1) with $x = u, y = v$, we have (for $t > 0$)

$$\psi(M(Au,Bv,t),M(Su,Tv,t),M(Au,Su,t),M(Bv,Tv,t)) \geqslant 0,$$

or, equivalently,

$$\psi(M(z,Bv,t),M(z,z,t),M(z,z,t),M(Bv,z,t)) \geqslant 0,$$

and so

$$\psi(M(z,Bv,t),1,1,M(Bv,z,t)) \geqslant 0,$$

implying thereby, $M(z,Bv,t) > 1$, a contradiction (due to (ψ_1)). Hence $Bv = Tv$ which shows that v is a coincidence point of the pair (B,T).

Since the pair (A,S) is weakly compatible and $Au = Su$, hence $Az = ASu = SAu = Sz$. We assert that $Az = z$. Suppose that $Az \neq z$, then putting $x = z$ and $y = v$ in (4.1), we get (for $t > 0$)

$$\psi(M(Az,Bv,t),M(Sz,Tv,t),M(Az,Sz,t),M(Bv,Tv,t)) \geqslant 0,$$

or, equivalently,

$$\psi(M(Az,z,t),M(Az,z,t),1,1) \geqslant 0,$$

which contradicts (ψ_2). Then we have $Az = z = Sz$ which shows that z is a common fixed point of the pair (A,S).

Also the pair (B,T) is weakly compatible and $Bv = Tv$, then $Bz = BTv = TBv = Tz$. Now we show that $Bz = z$. If not, then using (4.1) with $x = u, y = z$, we have

$$\psi(M(Au,Bz,t),M(Su,Tz,t),M(Au,Su,t),M(Bz,Tz,t)) \geqslant 0,$$

and so

$$\psi(M(z,Bz,t),M(z,Bz,t),1,1) \geqslant 0,$$

a contradiction (due to (ψ_2)). Hence, $Bz = z = Tz$ which shows that z is a common fixed point of the pair (B,T). Therefore, z is a common fixed point of the mappings A, B, S and T. The uniqueness is a direct consequence of the inequality (4.1). This concludes the proof. □

Remark 4.2. Theorem 4.1 improves the corresponding results contained in Gopal et al. [24, Theorem 3.9]as closedness of the underlying subspaces is not required.

Theorem 4.2. *Let A, B, S and T be four self-mappings of a fuzzy metric space $(X, M, *)$ satisfying all the hypotheses of Lemma 4.1. Then the pairs (A, S) and (B, T) have a coincidence point each. Moreover, A, B, S and T have a unique common fixed point provided that both the pairs (A, S) and (B, T) are weakly compatible.*

Proof. In view of Lemma 4.1, the pairs (A, S) and (B, T) enjoy the (CLR_{ST}) property, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

where $z \in S(X) \cap T(X)$. The rest of the proof runs on the lines of the proof of Theorem 4.1. □

Example 4.1. Let $(X, M, *)$ be a fuzzy metric space, where $X = [2, 19]$, with continuous t-norm $*$ is defined by $a * b = ab$ for all $a, b \in [0, 1]$ and

$$M(x, y, t) = \left(\frac{t}{t+1}\right)^{|x-y|}$$

for all $x, y \in X$. Define the self-mappings A, B, S and T by

$$A(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3, 19]; \\ 3, & \text{if } x \in (2, 3]. \end{cases}$$

$$B(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3, 19]; \\ 2.5, & \text{if } x \in (2, 3]. \end{cases}$$

$$S(x) = \begin{cases} 2, & \text{if } x = 2; \\ 10, & \text{if } x \in (2, 3]; \\ \frac{x+77}{40}, & \text{if } x \in (3, 19]. \end{cases}$$

$$T(x) = \begin{cases} 2, & \text{if } x = 2; \\ 13, & \text{if } x \in (2, 3); \\ 14, & \text{if } x = 3; \\ \frac{x+77}{40}, & \text{if } x \in (3, 19]. \end{cases}$$

Also, define implicit function $\psi : [0, 1]^4 \to \mathbb{R}$ as $\psi(t_1, t_2, t_3, t_4) = t_1 - \varphi(\min\{t_2, t_3, t_4\})$, where $\varphi(s) = \sqrt{s}$.

It is noted that

$$A(X) = \{2, 3\} \not\subset [2, 2.4] \cup \{13, 14\} = T(X)$$

and

$$B(X) = \{2, 2.5\} \not\subset [2, 2.4] \cup \{10\} = S(X).$$

Theorem 4.2 is not applicable to this example as $A(X) \not\subset T(X)$ and $B(X) \not\subset S(X)$.

Taking $\{x_n\} = \{3 + \frac{1}{n}\}, \{y_n\} = \{2\}$ or $\{x_n\} = \{2\}, \{y_n\} = \{3 + \frac{1}{n}\}$. It obtain that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 2 \in S(X) \cap T(X).$$

which shows that the pairs (A, S) and (B, T) enjoy the (CLR_{ST}) property.

Next, we show that the following inequality holds:

$$M(Ax, By, t) \geqslant \sqrt{\min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)\}} \tag{4.2}$$

for all $x, y \in X$ and $t > 0$. Define

$$\begin{cases} (I): & x = 2 \\ (II): & x \in (2, 3) \\ (III): & x = 3 \\ (IV): & x \in (3, 19] \end{cases} \quad \text{and} \quad \begin{cases} (i): & y = 2 \\ (ii): & y \in (2, 3) \\ (iii): & y = 3 \\ (iv): & y \in (3, 19] \end{cases}$$

There are 16 possibilities which are $(I, i), (I, ii), (I, iii), (I, iv), (II, i), (II, ii), (II, iii), (II, iv), (III, i), (III, ii), (III, iii), (III, iv), (IV, i), (IV, ii), (IV, iii)$ and (IV, iv).

Case (1) If (I, i) holds, we have the inequality (4.2) holds.
Case (2) If (I, ii) holds, we have

$$\begin{aligned} M(Ax, By, t) &= \left(\frac{t}{t+1}\right)^{0.5} \\ &\geqslant \left(\frac{t}{t+1}\right)^{5.5} \\ &= \sqrt{\left(\frac{t}{t+1}\right)^{11}} \\ &= \sqrt{M(Sx, Ty, t)} \\ &\geqslant \sqrt{\min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)\}}. \end{aligned}$$

Case (3) If (I, iii) holds, we have

$$\begin{aligned} M(Ax, By, t) &= \left(\frac{t}{t+1}\right)^{0.5} \\ &\geqslant \left(\frac{t}{t+1}\right)^{6} \\ &= \sqrt{\left(\frac{t}{t+1}\right)^{12}} \\ &= \sqrt{M(Sx, Ty, t)} \\ &\geqslant \sqrt{\min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)\}}. \end{aligned}$$

Case (4) If (I, iv) holds, we have the inequality (4.2) holds.
Case (5) If (II, i) holds, we have

$$\begin{aligned} M(Ax, By, t) &= \left(\frac{t}{t+1}\right)^{1} \\ &\geqslant \left(\frac{t}{t+1}\right)^{4} \\ &= \sqrt{\left(\frac{t}{t+1}\right)^{8}} \\ &= \sqrt{M(Sx, Ty, t)} \\ &\geqslant \sqrt{\min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)\}}. \end{aligned}$$

Case (6) If (II, ii) holds, we have

$$\begin{aligned} M(Ax, By, t) &= \left(\frac{t}{t+1}\right)^{0.5} \\ &\geqslant \left(\frac{t}{t+1}\right)^{1.5} \\ &= \sqrt{\left(\frac{t}{t+1}\right)^{3}} \\ &= \sqrt{M(Sx, Ty, t)} \\ &\geqslant \sqrt{\min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)\}}. \end{aligned}$$

Case (7) If (II, iii) holds, we have

$$\begin{aligned} M(Ax,By,t) &= \left(\frac{t}{t+1}\right)^{0.5} \\ &\geqslant \left(\frac{t}{t+1}\right)^{2} \\ &= \sqrt{\left(\frac{t}{t+1}\right)^{4}} \\ &= \sqrt{M(Sx,Ty,t)} \\ &\geqslant \sqrt{\min\{M(Sx,Ty,t),M(Ax,Sx,t),M(By,Ty,t)\}}. \end{aligned}$$

Case (8) If (II, iv) holds, we have

$$\begin{aligned} M(Ax,By,t) &= \left(\frac{t}{t+1}\right)^{1} \\ &\geqslant \left(\frac{t}{t+1}\right)^{3.5} \\ &= \sqrt{\left(\frac{t}{t+1}\right)^{7}} \\ &= \sqrt{M(Ax,Sx,t)} \\ &\geqslant \sqrt{\min\{M(Sx,Ty,t),M(Ax,Sx,t),M(By,Ty,t)\}}. \end{aligned}$$

Case (9) This case corresponding to (III, i) is as to Case 5.
Case (10) This case corresponding to (III, ii) is as to Case 6.
Case (11) This case corresponding to (III, iii) is as to Case 7.
Case (12) If (III, iv) holds, we have

$$\begin{aligned} M(Ax,By,t) &= \left(\frac{t}{t+1}\right)^{0.5} \\ &\geqslant \left(\frac{t}{t+1}\right)^{3.5} \\ &= \sqrt{\left(\frac{t}{t+1}\right)^{7}} \\ &= \sqrt{M(Ax,Sx,t)} \\ &\geqslant \sqrt{\min\{M(Sx,Ty,t),M(Ax,Sx,t),M(By,Ty,t)\}}. \end{aligned}$$

Case (13) If (IV, i) holds, we have the inequality (4.2) holds.
Case (14) If (IV, ii) holds, we have

$$\begin{aligned} M(Ax,By,t) &= \left(\frac{t}{t+1}\right)^{0.5} \\ &\geqslant \left(\frac{t}{t+1}\right)^{5.25} \\ &= \sqrt{\left(\frac{t}{t+1}\right)^{10.5}} \\ &= \sqrt{M(By,Ty,t)} \\ &\geqslant \sqrt{\min\{M(Sx,Ty,t),M(Ax,Sx,t),M(By,Ty,t)\}}. \end{aligned}$$

Case (15) If (IV, iii) holds, we have

$$\begin{aligned} M(Ax,By,t) &= \left(\frac{t}{t+1}\right)^{0.5} \\ &\geqslant \left(\frac{t}{t+1}\right)^{5.75} \\ &= \sqrt{\left(\frac{t}{t+1}\right)^{11.5}} \\ &= \sqrt{M(By,Ty,t)} \\ &\geqslant \sqrt{\min\{M(Sx,Ty,t),M(Ax,Sx,t),M(By,Ty,t)\}}. \end{aligned}$$

Case (16) If (IV, iv) holds, we have the inequality (4.2) holds.

Therefore, inequality (4.2) holds for all $x, y \in X$.

Also, the pairs (A, S) and (B, T) commute at 2 which is their common coincidence point. Thus all the conditions of Theorem 4.1 are satisfied and 2 is the unique common fixed point of the pairs (A, S) and (B, T) which also remains a point of coincidence as well. Also, notice that some mappings in this example are even discontinuous at their unique common fixed point 2.

Example 4.2. In the setting of Example 4.1, replace the self-mappings A, B, S and T by the following besides retaining the rest:

$$A(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3, 19]; \\ 3, & \text{if } x \in (2, 3]. \end{cases}$$

$$B(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3, 19]; \\ 4, & \text{if } x \in (2, 3]. \end{cases}$$

$$S(x) = \begin{cases} 2, & \text{if } x = 2; \\ 14, & \text{if } x \in (2, 3]; \\ \frac{x+1}{2}, & \text{if } x \in (3, 19]. \end{cases}$$

$$T(x) = \begin{cases} 2, & \text{if } x = 2; \\ 11 + x, & \text{if } x \in (2, 3]; \\ \frac{x+1}{2}, & \text{if } x \in (3, 19]. \end{cases}$$

Also, define implicit function $\psi : [0,1]^4 \to \mathbb{R}$ as $\psi(t_1, t_2, t_3, t_4) = t_1 - \varphi(\min\{t_2, t_3, t_4\})$, where $\varphi(s) = \sqrt{s}$.

It is noted that

$$A(X) = \{2, 3\} \subset [2, 10] \cup (13, 14] = T(X)$$

and

$$B(X) = \{2, 4\} \subset [2, 10] \cup \{14\} = S(X).$$

Taking $\{x_n\} = \{3 + \frac{1}{n}\}, \{y_n\} = \{2\}$ or $\{x_n\} = \{2\}, \{y_n\} = \{3 + \frac{1}{n}\}$. It obtain that

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 2 \in T(X).$$

which shows that the pair (B, T) enjoy the (CLR_T) property.

Similar to Example 4.1, one can easily verify the inequality (4.1). Also, the pairs (A, S) and (B, T) commute at 2 which is their common coincidence point. Thus all the conditions of Theorem 4.2 are satisfied and 2 is the unique common fixed point of the pairs (A, S) and (B, T) which also remains a point of coincidence as well. Also, notice that some mappings in this example are even discontinuous at their unique common fixed point 2.

Corollary 4.1. *The conclusions of Lemma 4.1, Theorems 4.1 and 4.2 remain true if inequality (4.1) is replaced by one of the following contraction conditions: For all $x, y \in X$ and $t > 0$*

$$M(Ax, By, t) \geqslant \varphi(\min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)\}), \tag{4.3}$$

where $\varphi : [0, 1] \to [0, 1]$ is a continuous function such that $\varphi(s) > s$ for $0 < s < 1$.

$$M(Ax,By,t) \geqslant k(\min\{M(Sx,Ty,t),M(Ax,Sx,t),M(By,Ty,t)\}), \quad (4.4)$$

where $k > 1$.

$$M(Ax,By,t) \geqslant kM(Sx,Ty,t) + \min\{M(Ax,Sx,t),M(By,Ty,t)\}, \quad (4.5)$$

where $k > 0$.

$$M(Ax,By,t) \geqslant aM(Sx,Ty,t) + bM(Ax,Sx,t) + cM(By,Ty,t), \quad (4.6)$$

where $a > 1$ *and* $b,c \geqslant 0 (b,c \neq 1)$.

$$M(Ax,By,t) \geqslant aM(Sx,Ty,t) + b[M(Ax,Sx,t) + M(By,Ty,t)], \quad (4.7)$$

where $a > 1$ *and* $0 \leqslant b < 1$.

$$M(Ax,By,t) \geqslant kM(Sx,Ty,t)M(Ax,Sx,t)M(By,Ty,t), \quad (4.8)$$

where $k > 1$.

Proof. The proof of each inequality (4.3)–(4.8) easily follows from Theorem 4.1 in view of Examples 3.2, 3.3, 3.4, 3.7. □

Now we state our next theorem by using an implicit function due to Singh and Jain [5].

Theorem 4.3. *Let* A, B, S *and* T *be four self-mappings of a fuzzy metric space* $(X,M,*)$ *satisfying*

$$\phi(M(Ax,By,kt),M(Sx,Ty,t),M(Ax,Sx,t), M(By,Ty,kt)) \geqslant 0, \quad (4.9)$$

$$\phi(M(Ax,By,kt),M(Sx,Ty,t),M(Ax,Sx,kt), M(By,Ty,t)) \geqslant 0. \quad (4.10)$$

for all $x,y \in X, \phi \in \Phi, k \in (0,1)$ *and* $t > 0$. *Suppose that the pairs* (A,S) *and* (B,T) *satisfy the* (CLR_{ST}) *property, then the pairs* (A,S) *and* (B,T) *have a coincidence point each. Further,* A,B,S *and* T *have a unique common fixed point provided that both the pairs* (A,S) *and* (B,T) *are weakly compatible.*

Proof. The proof of this theorem can be completed on the lines of the proof of Theorem 4.1 (in view of Lemma 2.1). Due to paucity of the space, we omitted the details. □

By putting A, B, S and T suitably in earlier proved results, one can drive a multitude of common fixed point theorems for a pair or triod of mappings. As a sample, we get the following natural result for a pair of self-mappings.

Corollary 4.2. *Let* A *and* S *be two self-mappings of a fuzzy metric space* $(X,M,*)$. *Suppose the following:*

(1) *the pair* (A,S) *enjoys the* (CLR_S) *property,*
(2) *there exists* $\psi \in \Psi$ *such that*

$$\psi(M(Ax,By,t),M(Sx,Ty,t),M(Ax,Sx,t), M(By,Ty,t)) \geqslant 0, \quad (4.11)$$

for all $x,y \in X$ *and* $t > 0$. *Then* A *and* S *have a coincidence point. Further,* A *and* S *have a unique common fixed point provided that the pair* (A,S) *is weakly compatible.*

As an application of Theorem 4.2, we have the following result involving four finite families of self-mappings.

Theorem 4.4. *Let* $\{A_i\}_{i=1}^m, \{B_r\}_{r=1}^n, \{S_k\}_{k=1}^p$ *and* $\{T_g\}_{g=1}^q$ *be four self-mappings of a fuzzy metric space* $(X,M,*)$ *such that* $A = A_1A_2 \ldots A_m, B = B_1B_2 \ldots B_n, S = S_1S_2 \ldots S_p$ *and* $T = T_1T_2 \ldots T_q$ *which satisfy the inequality (4.1). If the pairs* (A,S) *and* (B,T) *enjoy the* (CLR_{ST}) *property, then* (A,S) *and* (B,T) *have a coincidence point each. Moreover,* $\{A_i\}_{i=1}^m, \{B_r\}_{r=1}^n, \{S_k\}_{k=1}^p$ *and* $\{T_g\}_{g=1}^q$ *have a unique common fixed point provided the pairs* $(A_1A_2 \ldots A_m, S_1S_2 \ldots S_p)$ *and* $(B_1B_2 \ldots B_n, T_1T_2 \ldots Tq)$ *commute pairwise.*

Proof. The proof of this theorem is similar to that of Theorem 4.1 contained in Imdad et al. [41], hence details are omitted. □

The importance of Theorem 4.4 is that it can be utilized to derive common fixed point theorems for any finite number of mappings. As a sample for five mappings, we can derive the following by setting one family of two members while the remaining three of single members:

Corollary 4.3. *Let* A,B,R,S *and* T *be five self-mappings of a fuzzy metric space* $(X,M,*)$. *Suppose that*

(1) *the pairs* (A,SR) *and* (B,T) *satisfy the* $(CLR_{(SR)(T)})$ *property,*
(2) *there exists* $\psi \in \Psi$ *such that*

$$\psi(M(Ax,By,t),M(SRx,Ty,t),M(Ax,SRx,t), M(By,Ty,t)) \geqslant 0, \quad (4.12)$$

for all $x,y \in X$ *and* $t > 0$. *Then the pairs* (A,SR) *and* (B,T) *have a coincidence point each. Moreover,* A,B,R,S *and* T *have a unique common fixed point provided the pairs* (A,SR) *and* (B,T) *commute pairwise (i.e.,* $AS = SA, AR = RA, SR = RS$ *and* $BT = TB$*).*

Similarly, we can derive a common fixed point theorem for six self-mappings by setting two families of two members while the rest two of single members:

Corollary 4.4. *Let* A,B,S,R,T *and* H *be six self-mappings of a fuzzy metric space* $(X,M,*)$. *Suppose that*

(1) *the pairs* (A,SR) *and* (B,TH) *satisfy the* $(CLR_{(SR)(TH)})$ *property,*
(2) *there exists* $\psi \in \Psi$ *such that*

$$\psi(M(Ax,By,t),M(SRx,THy,t),M(Ax,SRx,t), M(By,THy,t)) \geqslant 0, \quad (4.13)$$

for all $x,y \in X$ *and* $t > 0$. *Then the pairs* (A,SR) *and* (B,TH) *have a coincidence point each. Further,* A,B,R,S,H *and* T *have a unique common fixed point provided the pairs* (A,SR) *and* (B,TH) *commute pairwise (i.e.,* $AS = SA, AR = RA, SR = RS, BT = TB, BH = HB$ *and* $TH = HT$*).*

By setting $A_1 = A_2 = \cdots = A_m = A, B_1 = B_2 = \cdots = B_n = B, S_1 = S_2 = \cdots = S_p = S$ and $T_1 = T_2 = \cdots = T_q = T$ in Theorem 4.4, we deduce the following:

Corollary 4.5. *Let A, B, S and T be four self-mappings of a fuzzy metric space $(X, M, *)$ such that the pairs (A^m, S^p) and (B^n, T^q) (wherein m, n, p, q are fixed positive integers) satisfy the (CLR_{S^p,T^q}) property. Suppose that there exists $\psi \in \Psi$ such that*

$$\psi(M(A^m x, B^n y, t), M(S^p x, T^q y, t), M(A^m x, S^p x, t), M(B^n y, T^q y, t)) \geqslant 0, \tag{4.14}$$

for all $x, y \in X$ and $t > 0$. Then the pairs (A, S) and (B, T) have a coincidence point each. Further, A, B, S and T have a unique common fixed point provided both the pairs (A^m, S^p) and (B^n, T^q) commute pairwise.

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