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ORIGINAL ARTICLE

Topological and non-topological soliton solutions of Hamiltonian amplitude equation by He's semi-inverse method and ansatz approach

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Abstract This paper obtains the exact 1-soliton solution to the Hamiltonian amplitude equation. There are two types of integration architectures that are implemented in this paper. They are the He's semi-inverse method and the ansatz method. These soliton solutions are obtained. There are constraint conditions that also fall out which must remain valid in order for the solitons and other solutions to exist.

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1. Introduction

It is well known that exact traveling wave solutions of nonlinear evolution equations (NLEEs) play an important role in the study of nonlinear wave phenomena. The wave phenomena are observed in fluid dynamics, plasma, elastic media, optical fibers, etc. In recent years, many powerful integration architectures have been developed to construct exact solutions of NLEEs [1–36]. One of the most effective direct methods to develop the exact traveling wave solutions of nonlinear partial differential equations is the He's variational principle [1]. This method is introduced by He [1] and it is known He's semi-inverse variational principle. In last years, it is used commonly to obtain soliton solutions of NLPDEs and systems by many authors [2–11]. Biswas et al. [5–9] obtained optical solitons and soliton solutions with higher order dispersion by using the He's variational principle.

Taghizadeh and Mirzazadeh [12] obtained exact solutions of the Hamiltonian amplitude equation by using the first integral method. The aim of this paper is to find new exact solutions of the Hamiltonian amplitude equation by using the He's semi-inverse variational principle method and the ansatz method [13,14].

2. Governing equation

In this paper, we consider the Hamiltonian amplitude equation [15,16]

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$$iq_x + q_{tt} + 2\sigma|q|^2 q - \varepsilon q_{xt} = 0, \quad (1)$$

where $\sigma = \pm 1$ and $\varepsilon \ll 1$. This is an equation which governs certain instabilities of modulated wave trains, with the additional term $-\varepsilon q_{xt}$ overcoming the ill-posedness of the unstable nonlinear Schrödinger equation. It is a Hamiltonian analog of the Kuramoto–Sivashinsky equation which arises in dissipative systems and is apparently not integrable.

The nonlinear Schrödinger's equation describes numerous nonlinear physical phenomena in the field of applied sciences such as solitons in nonlinear optical fibers, solitons in the mean-field theory of Bose–Einstein condensates, and rogue waves in oceanography.

3. The semi-inverse variational principle method

Let us consider a general nonlinear PDE in the form

$$P\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial t \partial x}, \frac{\partial^2 u}{\partial t^2}, \ldots\right) = 0, \quad (2)$$

where P is a polynomial in its arguments. Jabbari et al. [11] have been written the He's semi-inverse method in the following steps:

Step 1: Seek solitary wave solutions of Eq. (2) by taking $u(x,t) = U(\xi),\ \xi = x - ct$, and transform Eq. (2) to the ordinary differential equation (ODE)

$$Q\left(U, \frac{dU}{d\xi}, \frac{d^2U}{d\xi^2}, \ldots\right) = 0. \quad (3)$$

Step 2: If possible, integrating Eq. (3) term by term one or more times. This yields constant(s) of integration. For simplicity, the integration constant(s) can be set to zero.

Step 3: According to He's semi-inverse method, we construct the following trial-functional, we construct the following trial-functional

$$J(U) = \int L d\xi, \quad (4)$$

where L is an unknown function of U and its derivatives.

Step 4: By the Ritz method, we can obtain different forms of solitary wave solutions, such as

$$U(\xi) = A\,\mathrm{sech}(B\xi), U(\xi) = A\,\mathrm{csch}(B\xi),$$
$$U(\xi) = A\tanh(B\xi), U(\xi) = A\coth(B\xi) \quad (5)$$

and so on. For example in this paper, we search a solitary wave solution in the form

$$U(\xi) = A\ \mathrm{sech}(B\xi), \quad (6)$$

where A and B are constants to be further determined. Substituting Eq. (6) into Eq. (4) and making J stationary with respect to A and B results in

$$\frac{\partial J}{\partial A} = 0 \quad (7)$$

$$\frac{\partial J}{\partial B} = 0 \quad (8)$$

Solving Eqs. (7) and (8), we obtain A and B. Hence the solitary wave solution (6) is well determined.

3.1. Application to the Hamiltonian amplitude equation

In order to solve Eq. (1), we use the following wave transformation

$$q(x,t) = U(\xi)e^{i\Phi(x,t)} \quad (9)$$

where $U(\xi)$ represents the shape of the pulse and

$$\xi = \mu(x - vt), \quad (10)$$

$$\Phi(x,t) = -\kappa x + \omega t + \theta. \quad (11)$$

In Eq. (9), the function $\Phi(x,t)$ is the phase component of the soliton. Then, in Eq. (11), κ is the soliton frequency, while ω is the wave number of the soliton and θ is the phase constant. Finally in Eq. (10), v is the velocity of the soliton. By replacing Eq. (9) into Eq. (1) and separating the real and imaginary parts of the result, we have

$$1 - 2v\omega - \varepsilon\kappa v - \varepsilon\omega = 0, \quad (12)$$

and

$$\mu^2(v^2 + \varepsilon v)U'' - (\varepsilon\kappa\omega + \omega^2 - \kappa)U + 2\sigma U^3 = 0. \quad (13)$$

Using Eq. (12), we get

$$v = \frac{1 - \varepsilon\omega}{2\omega + \varepsilon\kappa}. \quad (14)$$

By He's semi-inverse principle [8,10], we can obtain the following variational formulation

$$J = \int_0^\infty \left[-\frac{\mu^2(v^2 + \varepsilon v)}{2}(U')^2 - \frac{(\varepsilon\kappa\omega + \omega^2 - \kappa)}{2}U^2 + \frac{\sigma}{2}U^4\right]d\xi. \quad (15)$$

By a Ritz-like method, we search a solitary wave solution in the form

$$U(\xi) = A\,\mathrm{sech}(B\xi), \quad (16)$$

where A and B are unknown constants to be further determined. Substituting Eq. (16) into Eq. (15), we have

$$J = \int_0^\infty \left[-\frac{\mu^2 A^2 B^2 (v^2 + \varepsilon v)}{2}\mathrm{sech}^2(B\xi)\tanh^2(B\xi) - \frac{(\varepsilon\kappa\omega + \omega^2 - \kappa)A^2}{2}\mathrm{sech}^2(B\xi) + \frac{\sigma A^4}{2}\mathrm{sech}^4(B\xi)\right]d\xi$$
$$= -\frac{\mu^2 A^2 B(v^2 + \varepsilon v)}{6} - \frac{(\varepsilon\kappa\omega + \omega^2 - \kappa)A^2}{2B} + \frac{\sigma A^4}{3B}. \quad (17)$$

Making J stationary with A and B yields

$$\frac{\partial J}{\partial A} = -\frac{\mu^2 AB(v^2 + \varepsilon v)}{3} - \frac{(\varepsilon\kappa\omega + \omega^2 - \kappa)A}{B} + \frac{4\sigma A^3}{3B} = 0, \quad (18)$$

$$\frac{\partial J}{\partial B} = -\frac{\mu^2 A^2(v^2 + \varepsilon v)}{6} + \frac{(\varepsilon\kappa\omega + \omega^2 - \kappa)A^2}{2B^2} - \frac{\sigma A^4}{3B^2} = 0. \quad (19)$$

From Eqs. (18) and (19), we have

$$A = \pm\sqrt{\frac{\varepsilon\kappa\omega + \omega^2 - \kappa}{\sigma}}, \quad B = \pm\sqrt{\frac{\varepsilon\kappa\omega + \omega^2 - \kappa}{\mu^2(v^2 + \varepsilon v)}}. \quad (20)$$

Using the traveling wave transformation (9), we have the following **bright (bell-shaped) soliton solutions** of the Eq. (1):

$$q(x,t) = \pm\sqrt{\frac{\varepsilon\kappa\omega + \omega^2 - \kappa}{\sigma}}\,\text{sech}\left[\pm\sqrt{\frac{\varepsilon\kappa\omega + \omega^2 - \kappa}{(v^2 + \varepsilon v)}} \times \left(x - \left\{\frac{1 - \varepsilon\omega}{2\omega + \varepsilon\kappa}\right\}t\right)\right] \times e^{i\{-\kappa x + \omega t + \theta\}}, \quad (21)$$

where v is given by (14).

4. Ansatz approach

This section will utilize the ansatz method to solve the Hamiltonian amplitude equation. The bright soliton, dark soliton and singular soliton solutions to Eq. (1) will be obtained by the aid of ansatz method. The starting point is the assumption

$$q(x,t) = p(x,t)e^{i\phi(x,t)}, \quad (22)$$

where $P(x,t)$ is the amplitude part and the phase component $\phi(x,t)$ is given by

$$\phi(x,t) = -\kappa x + \omega t + \theta. \quad (23)$$

In Eq. (23), κ represents the soliton wave number, while ω is the frequency and θ is the phase constant. The amplitude component dictates the type of soliton in question, namely bright or dark or singular. Thus from (22), we have

$$q_x = \left(\frac{\partial P}{\partial x} - i\kappa P\right)e^{i\phi}, \quad (24)$$

$$q_{tt} = \left(\frac{\partial^2 P}{\partial t^2} + 2i\omega\frac{\partial P}{\partial t} - \omega^2 p\right)e^{i\phi}, \quad (25)$$

and

$$q_{xt} = \left(\frac{\partial^2 P}{\partial t\partial x} + i\omega\frac{\partial P}{\partial x} - i\kappa\frac{\partial P}{\partial t} + \kappa\omega p\right)e^{i\phi}. \quad (26)$$

Substituting Eqs. (24)–(26) into Eq. (1) and decomposing into real and imaginary parts respectively yields the following set of relations

$$\frac{\partial^2 P}{\partial t^2} - \left(\varepsilon\kappa\omega + \omega^2 - \kappa\right) - \varepsilon\frac{\partial^2 P}{\partial t\partial x} + 2\sigma p^3 = 0 \quad (27)$$

and

$$(1 - \varepsilon\omega)\frac{\partial P}{\partial x} + (2\omega + \kappa\varepsilon)\frac{\partial P}{\partial t} = 0. \quad (28)$$

Since the amplitude portion $p(x,t)$ is of the form $U(\mu(x - vt))$, Eq. (28) reduces to (14). It is now Eq. (27) that will be studied in a detailed fashion in the following subsections where the three types of soliton solutions will be obtained.

4.1. Bright soliton solution

For bright soliton, the hypothesis is

$$P(x,t) = A\,\text{sech}^p\,\tau, \quad (29)$$

where

$$\tau = B(x - vt). \quad (30)$$

The value of the unknown exponent p will fall out during the course of derivation of the soliton solutions. Also A and B are free parameters, while v is the speed of the soliton. Thus from (29), we have

$$\frac{\partial^2 P}{\partial t^2} = Ap^2B^2v^2\,\text{sech}^p\,\tau - AB^2v^2p(1+p)\,\text{sech}^{p+2}\,\tau, \quad (31)$$

$$\frac{\partial^2 P}{\partial t\partial x} = -Ap^2B^2v\,\text{sech}^p\,\tau + AB^2vp(1+p)\,\text{sech}^{p+2}\,\tau, \quad (32)$$

and

$$P^3 = A^3\text{sech}^{3p}\tau. \quad (33)$$

Substitution of (29) into the real part equation given by (27) leads to

$$\left(\varepsilon\kappa\omega + \omega^2 - \kappa - p^2B^2v^2 - \varepsilon p^2B^2v\right)\text{sech}^p\,\tau + B^2\left(v^2 + \varepsilon v\right)p(1+p)\,\text{sech}^{p+2}\,\tau - 2\sigma A^2\,\text{sech}^{3p}\,\tau = 0. \quad (34)$$

By virtue of balancing principle, on equating the exponents $3p$ and $p+2$, from (34), gives

$$p = 1. \quad (35)$$

Next, from (34) setting the coefficients of the linearly independent functions to zero implies

sech^1 Coeff.:

$$\varepsilon\kappa\omega + \omega^2 - \kappa - B^2\left(v^2 + \varepsilon v\right) = 0, \quad (36)$$

sech^3 Coeff.:

$$2B^2\left(v^2 + \varepsilon v\right) - 2\sigma A^2 = 0.$$

Solving the above equations yields

$$A = \pm\sqrt{\frac{\varepsilon\kappa\omega + \omega^2 - \kappa}{\sigma}}, \quad (37)$$

and

$$B = \pm\sqrt{\frac{\varepsilon\kappa\omega + \omega^2 - \kappa}{(v^2 + \varepsilon v)}}. \quad (38)$$

Eqs. (37) and (38) prompts the constraints

$$\sigma\left(\varepsilon\kappa\omega + \omega^2 - \kappa\right) > 0, \quad (39)$$

and

$$\left(\varepsilon\kappa\omega + \omega^2 - \kappa\right)\left(v^2 + \varepsilon v\right) > 0, \quad (40)$$

respectively. Thus, the bright 1-soliton solution to Eq. (1) is given by

$$q(x,t) = A\,\text{sech}\,[B(x - vt)]e^{i\{-\kappa x + \omega t + \theta\}}, \quad (41)$$

where the free parameters A and B are respectively given by (37) and (38) with the constraints (39) and (40). The velocity of the soliton is seen in (14).

4.2. Topological (dark) soliton solution

The starting hypothesis for dark 1-soliton solution to (27) is

$$P(x,t) = A\tanh^p\tau, \quad (42)$$

where τ is the same as (30). However, for dark solitons the parameters A and B are indeed free soliton parameters, although v still represents the velocity of the dark soliton. Thus from (42), we have

$$\frac{\partial^2 P}{\partial t^2} = AB^2v^2p(p-1)\tanh^{p-2}\tau - 2AB^2v^2p^2\tanh^p\tau + AB^2v^2p(p+1)\tanh^{p+2}\tau, \quad (43)$$

$$\frac{\partial^2 P}{\partial t \partial x} = -AB^2 vp(p-1)\tanh^{p-2}\tau + 2AB^2 vp^2 \tanh^p \tau - AB^2 vp(p+1)\tanh^{p+2}\tau, \tag{44}$$

and

$$P^3 = A^3 \tanh^{3p}\tau. \tag{45}$$

In this case, substituting this hypothesis (42) into (27) leads to

$$B^2(v^2+\varepsilon v)p(p-1)\tanh^{p-2}\tau - \left(\varepsilon\kappa\omega + \omega^2 - \kappa + 2p^2B^2v^2 + 2\varepsilon p^2 B^2 v\right)\tanh^p\tau + B^2(v^2+\varepsilon v)p(p+1)\tanh^{p+2}\tau + 2\sigma A^2\tanh^{3p}\tau = 0. \tag{46}$$

By balancing the power of $\tanh^{p+2}$ and $\tanh^{3p}$ in (46) we have:

$$p = 1. \tag{47}$$

Now, from (46), setting the coefficients of the linearly independent functions $\tanh^{(p+j)}\tau$ to zero, where $j = 0, 2$, gives

$\tanh^1$ Coeff.:

$$\varepsilon\kappa\omega + \omega^2 - \kappa + 2B^2(v^2+\varepsilon v) = 0, \tag{48}$$

$\tanh^3$ Coeff.:

$$2B^2(v^2+\varepsilon v) + 2\sigma A^2 = 0.$$

Solving the above equations yields

$$A = \pm\sqrt{\frac{\varepsilon\kappa\omega + \omega^2 - \kappa}{2\sigma}}, \tag{49}$$

and

$$B = \pm\sqrt{\frac{\kappa - \varepsilon\kappa\omega - \omega^2}{2(v^2+\varepsilon v)}}. \tag{50}$$

Eqs. (49) and (50) prompts the constraints

$$\sigma(\varepsilon\kappa\omega + \omega^2 - \kappa) > 0, \tag{51}$$

and

$$(\kappa - \varepsilon\kappa\omega - \omega^2)(v^2+\varepsilon v) > 0, \tag{52}$$

respectively. Thus, the topological 1-soliton solution to Eq. (1) is given by

$$q(x,t) = A\ \tanh[B(x-vt)]e^{i\{-\kappa x + \omega t + \theta\}}, \tag{53}$$

where the free parameters A and B are respectively given by (49) and (50) with the constraints (51) and (52). The velocity of the soliton is seen in (14).

4.3. Singular soliton solution

For singular soliton, the hypothesis is

$$P(x,t) = A\,\mathrm{csch}^p\tau, \tag{54}$$

where τ is the same as (30). The value of the unknown exponent p will fall out during the course of derivation of the soliton solutions. Also A and B are free parameters, while v is the speed of the soliton. Substitution of (54) into the real part equation given by (27) leads to

$$\left(\varepsilon\kappa\omega + \omega^2 - \kappa + p^2B^2v^2 + \varepsilon p^2B^2v\right)\mathrm{csch}^p\tau + B^2(v^2+\varepsilon v)p(p+1)\mathrm{csch}^{p+2}\tau + 2\sigma A^2\mathrm{csch}^{3p}\tau = 0. \tag{55}$$

From (55), the balancing principle yields

$$p = 1. \tag{56}$$

Next, from (55) setting the coefficients of the linearly independent functions to zero implies

$$A = \pm\sqrt{\frac{\kappa - \varepsilon\kappa\omega - \omega^2}{\sigma}}, \tag{57}$$

and

$$B = \pm\sqrt{\frac{\omega^2 + \varepsilon\kappa\omega - \kappa}{(v^2+\varepsilon v)}}. \tag{58}$$

Eqs. (57) and (58) prompts the constraints

$$\sigma(\kappa - \varepsilon\kappa\omega - \omega^2) > 0, \tag{59}$$

and

$$(\varepsilon\kappa\omega + \omega^2 - \kappa)(v^2+\varepsilon v) > 0, \tag{60}$$

respectively. Thus, the bright 1-soliton solution to Eq. (1) is given by

$$q(x,t) = A\ \mathrm{csch}\ [B(x-vt)]e^{i\{-\kappa x + \omega t + \theta\}}, \tag{61}$$

where the free parameters A and B are respectively given by (57) and (58) with the constraints (59) and (60). The velocity of the soliton is seen in (14).

5. Conclusions

In this paper, the He's semi-inverse variational principle method and the ansatz method have been applied to obtain the new exact solutions of the Hamiltonian amplitude equation. The results show that these methods are powerful tool for obtaining the exact solutions of complex nonlinear partial differential equations. It may be concluded that, these methods can be easily extended to all kinds of complex nonlinear partial differential equations.

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