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ORIGINAL ARTICLE

Reliability equivalence factors of a system with mixture of *n* independent and non-identical lifetimes with delay time

Abdelfattah Mustafa [a,b,*], Adel A. El-Faheem [c]

[a] Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt
[b] Mathematics Department, College of Science and Humanity Studies, Salman bin Abdulaziz University, P.O. Box 83, Alkharj 11942, Saudi Arabia
[c] Department of Mathematics, Faculty of Science, Aswan University, Aswan 81528, Egypt

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Abstract Recently, [1,2] generalized the reliability equivalence technique to a system with mixed of two non-identical lifetimes with delay time. The aim of this study is to generalize reliability equivalence technique to apply it to a system of mixture of n independent and non-identical lifetimes with delay time. We shall improve the system by using some reliability techniques: (i) reducing the failure for some lifetimes; (ii) add hot duplication components; (iii) add cold duplication components; and (iv) add cold duplication components with imperfect switches. We start by establishing two different types of reliability equivalence factors, the survival reliability equivalence (SRE) and mean reliability equivalence (MRE) factors. Also, we introduced some numerical results and conclusions.

MSC: 60K10, 62N05, 62P30, 90B25

1. Introduction

Operations Research, in its various fields, is concerned with the problem of system performance in the best possible way. In reliability theory, one way to improve the performance of a system is to use the redundancy method. There are two main such methods: hot and cold duplications method; in theses cases, it is assumed that some of the system components are duplicated in parallel (via a perfect switch). Unfortunately, for many different reasons, such as space limitation and high

* Corresponding author at: Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt. Tel.: +20 1114549473.
E-mail address: abdelfatah_mustafa@yahoo.com (A. Mustafa).
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cost, it is not always possible to improve a system by duplicating some or all of its components. In such cases where duplication is not possible, the engineer turns to another well-known method in reliability theory, the so-called reduction method. In this method, it is assumed that the failure rates of some of the system components are reduced by a factor , $0 < \rho < 1$. Now, once the reduction method is adopted, the main problem facing the engineer is to decide to what degree the failure rate should be decreased in order to improve the system. To solve this problem, one can make equivalence between the reduction method and the duplication method based on some reliability measures. The comparison of the designs produces the so-called reliability equivalence factors [3].

Different vectors of the reliability equivalence factors of a series introduced in [4]. Three different types of the reliability equivalence factors of a parallel–series system consists of four independent and identical components are introduced in [5]. The lifetime of the system component is assumed to follow exponential distribution. The concept of the reliability equivalence on n components parallel system with non-constant failure rates is applied in [6]. The concept of reliability equivalence when the components have mixture Weibull failure rates introduced in [7]. The equivalence factors of a general series–parallel system introduced in [8], but [9] introduced equivalence factors of a general parallel–series system and assumed that all components are independent and follow the exponential distribution with the same parameter, $\lambda > 0$. The reliability equivalence factor of a series system studied in [10] when the failure rates of the system components are functions of time t and introduced two cases of non-constant failure rates (i) Weibull distribution (ii) linear increasing failure rate distribution. There are two methods are used to improve the given system, but [7] introduced reliability equivalence factors for some systems with mixture Weibull failure rates and studied two cases (i) the mixture of two stages of life time distribution with Weibull failure rates, (ii) the mixture of two stages failure rates with Weibull distribution. The reliability equivalence factor of the system such that the failure rates of the system's components are functions of time t are introduced in [11], studied two cases (i) the life time distribution of a components has two stages with increasing failure rates, (ii) the failure rates of the components have the two stages. The reliability equivalence techniques to a system consists of n independent and non-identical components connected in series system, which have mixture constant failure rates are introduced in [12]. Ref. [1] generalized reliability equivalence technique to apply it to a system consists of m independent and non-identical lifetimes distributions, with mixture failure lifetimes $f_1(t), f_2(t), \ldots, f_m(t)$. [2] generalized reliability equivalence technique to apply it to a system consists of two independent and non-identical lifetimes distributions, with mixed failure lifetimes and delay time. In this article, we genralized [2], consider a system with mixture of n independent and non-identical lifetimes with delay time. This model is applicable when each component or product experiences more failure modes. For example, a mechanical component, such as a load-carrying bearing or a cutting tool, may fail due to wear-out or when the applied stress exceeds the design strength of component material. Since the component or the tool can fail in either of the failure modes, it is then appropriate to describe the hazard rate by a mixed model, it is expressed as follows, [13,14],

$$f(t) = p_1 f_1(t) + p_2 f_2(t), \tag{1.1}$$

where $0 \leqslant p_i \leqslant 1$, $i = 1, 2$, $p_1 + p_2 = 1$, the quantity p_1 is the probability that the component or the tool fails in the first failure mode and p_2 is the probability that it fails in the second failure, if the second failure mode occurs after a delay time δ from the first failure mode, [15,16], then:

$$f_d(t) = p_1 f_1(t) + p_2 f_2(t - \delta), \quad 0 \leqslant \delta \leqslant t. \tag{1.2}$$

We derive the REF and MREF for a system with mixture of n non-identical lifetimes distribution, delay times. Assuming the failure rates and delay time of the types of the lifetimes are, λ_i, δ_i $i = 1, 2, \ldots, n$, that is the failure time for the system is given as

$$f(t) = \sum_{i=1}^{n} p_i \lambda_i \exp\{-\lambda_i (t - \delta_i)\}, \tag{1.3}$$

where $0 \leqslant p_i \leqslant 1$, $\sum_{i=1}^{n} p_i = 1$, $0 \leqslant \delta_i \leqslant t$, $\forall 1 \leqslant i \leqslant n$.

2. The original system

The reliability function $R(t)$, for the original system can be obtained as follows

$$R(t) = \sum_{i=1}^{n} p_i \exp\{-\lambda_i (t - \delta_i)\}. \tag{2.1}$$

From Eq. (2.1), one can easily obtain the mean time to failure, say MTTF as follows

$$\mathrm{MTTF} = \sum_{i=1}^{n} \frac{p_i \exp\{\lambda_i \delta_i\}}{\lambda_i}. \tag{2.2}$$

3. The improved systems

The quality of the system reliability can be improved using four different methods of the system improvements.

3.1. Reduction method

Let $R_{A,\rho}(t)$ denotes the reliability function of the improved system when the set A of the failure rate of the mixture lifetimes are reduced by the factor ρ_i, $0 < \rho_i < 1$, $i = 1, 2, \ldots, n$. One can obtain the function $R_{A,\rho}(t)$, as follows

$$\begin{aligned} R_{A,\rho}(t) &= \sum_{i \in A} p_i \exp\{-\rho_i \lambda_i (t - \delta_i)\} + \sum_{i \in \overline{A}} p_i \exp\{-\lambda_i (t - \delta_i)\} \\ &= \sum_{i \in A} p_i [\exp\{-\rho_i \lambda_i (t - \delta_i)\} - \exp\{-\lambda_i (t - \delta_i)\}] \\ &\quad + \sum_{i=1}^{n} p_i \exp\{-\lambda_i (t - \delta_i)\}, \end{aligned} \tag{3.1}$$

where $\overline{A} = \{1, 2, 3, \ldots, n\} \setminus A$.

From Eq. (3.1), the MTTF of the improved system, say $\mathrm{MTTF}_{A,\rho}$, becomes

$$\begin{aligned}\mathrm{MTTF}_{A,\rho} &= \sum_{i\in A}\frac{p_i}{\rho_i\lambda_i}[\exp\{\lambda_i\rho_i\delta_i\}-\exp\{\lambda_i\delta_i\}] \\ &\quad+\sum_{i=1}^{n}\frac{p_i\exp\{\lambda_i\delta_i\}}{\lambda_i} \\ &= \mathrm{MTTF}+\sum_{i\in A}\frac{p_i}{\rho_i\lambda_i}[\exp\{\lambda_i\rho_i\delta_i\}-\exp\{\lambda_i\delta_i\}]. \qquad (3.2)\end{aligned}$$

3.2. Hot duplication method

Let $R^H(t)$ be the reliability function of the improved system obtained by assuming hot duplications of the system component. The function $R^H(t)$ can be obtained as follows, see [17].

$$\begin{aligned}R^H(t) &= 2\sum_{i=1}^{n}p_i\exp\{-\lambda_i(t-\delta)\}-\sum_{i=1}^{n}\sum_{j=1}^{n}p_ip_j\exp\{-(\lambda_i \\ &\quad+\lambda_j)t\}\exp\{\lambda_i\delta_i+\lambda_j\delta_j\}. \qquad (3.3)\end{aligned}$$

Let MTTF^H be the MTTF of improved system assuming hot duplication method. Using Eq. (3.3), one can deduce MTTF^H as

$$\begin{aligned}\mathrm{MTTF}^H &= 2\sum_{i=1}^{n}\frac{p_i\exp\{\lambda_i\delta_i\}}{\lambda_i}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{p_ip_j\exp\{\lambda_i\delta_i+\lambda_j\delta_j\}}{\lambda_i+\lambda_j} \\ &= \mathrm{MTTF} \\ &\quad+\left[\sum_{i=1}^{n}\frac{p_i\exp\{\lambda_i\delta_i\}}{\lambda_i}-\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{p_ip_j\exp\{\lambda_i\delta_i+\lambda_j\delta_j\}}{\lambda_i+\lambda_j}\right]. \qquad (3.4)\end{aligned}$$

That is, hot duplication of a single component increases the meantime to system failure by the amount $\left[\sum_{i=1}^{n}\frac{p_i\exp\{\lambda_i\delta_i\}}{\lambda_i}-\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{p_ip_j\exp\{\lambda_i\delta_i+\lambda_j\delta_j\}}{\lambda_i+\lambda_i}\right]$

3.3. Cold duplication method

Let $R^C(t)$ be the reliability function of the improved system obtained by assuming cold duplications of the mixing system components. The function $R^C(t)$ can be obtained as follows, see [18].

$$\begin{aligned}R^C(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{p_ip_j}{\lambda_i-\lambda_j}[\lambda_i\exp\{-\lambda_jt\}-\lambda_j\exp\{-\lambda_it\}] \\ &\quad\times\exp\{\lambda_i\delta_i+\lambda_j\delta_j\}. \qquad (3.5)\end{aligned}$$

From Eq. (3.5), the MTTF of the improved system, say MTTF^C, assuming cold duplications method is given as

$$\begin{aligned}\mathrm{MTTF}^C &= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(\lambda_i+\lambda_j)p_ip_j}{\lambda_i\lambda_j}\exp\{\lambda_i\delta_i+\lambda_j\delta_j\} \\ &= \mathrm{MTTF} \\ &\quad+\sum_{i=1}^{n}\frac{p_i\exp\{\lambda_i\delta_i\}}{\lambda_i}\left[\sum_{j=1}^{n}\frac{(\lambda_i+\lambda_j)p_j}{\lambda_j}\exp\{\lambda_j\delta_j\}-1\right]. \qquad (3.6)\end{aligned}$$

That is, cold duplication of the system component increases the mean time to system failure by the amount $\left[\sum_{i=1}^{n}\frac{p_i\exp\{\lambda_i\delta_i\}}{\lambda_i}\left[\sum_{j=1}^{n}\frac{(\lambda_i+\lambda_j)p_j}{\lambda_j}\exp\{\lambda_j\delta_j\}-1\right]\right]$.

3.4. Imperfect switch duplication method

Let us consider now that, the system reliability can be improved assuming cold duplication method with imperfect switch. In such method, it is assumed that the component is connected by a cold redundant standby component via a random switch having a constant failure rate, say β.

Let $R^I(t)$ be the reliability function of the improved system when the system component is improved according to the cold duplication method with imperfect switch for the mixing components. The function $R^I(t)$, is given by

$$\begin{aligned}R^I(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}p_ip_j\lambda_i\left\{\frac{\lambda_j}{\lambda_i+\beta-\lambda_j}\left[\frac{\exp\{-\lambda_jt\}}{\lambda_j}-\frac{\exp\{-(\lambda_i+\beta)t\}}{\lambda_i+\beta}\right]\right. \\ &\quad\left.+\frac{\beta}{\lambda_i-\beta-\lambda_j}\left[\frac{\exp\{-(\lambda_j+\beta)t\}}{\lambda_j+\beta}-\frac{\exp\{-\lambda_it\}}{\lambda_i}\right]\right\} \\ &\quad\times\exp\{\lambda_i\delta_i+\lambda_j\delta_j\} \qquad (3.7)\end{aligned}$$

From Eq. (3.7), the MTTF of the improved system, say MTTF^I is given by

$$\begin{aligned}\mathrm{MTTF}^I &= \sum_{i=1}^{n}\sum_{j=1}^{n}p_ip_j(\lambda_i+\beta+\lambda_j)\left\{\frac{\lambda_i}{\lambda_j(\lambda_i+\beta)^2}+\frac{\beta}{\lambda_i(\lambda_j+\beta)^2}\right\} \\ &\quad\times\exp\{\lambda_i\delta_i+\lambda_j\delta_j\}=\mathrm{MTTF} \\ &\quad+\sum_{i=1}^{n}\frac{p_i\exp\{\lambda_i\delta_i\}}{\lambda_i}\left\{\sum_{j=1}^{n}p_j\lambda_i(\lambda_i+\beta+\lambda_j)\right. \\ &\quad\left.\times\left[\frac{\lambda_i}{\lambda_j(\lambda_i+\beta)^2}+\frac{\beta}{\lambda_i(\lambda_j+\beta)^2}\right]\exp\{\lambda_j\delta_j\}-1\right\}. \qquad (3.8)\end{aligned}$$

That is, imperfect switch duplication of the system component increases the mean time to system failure by the amount

$$\left[\sum_{i=1}^{n}\frac{p_i\exp\{\lambda_i\delta_i\}}{\lambda_i}\left\{\sum_{j=1}^{n}p_j\lambda_i(\lambda_i+\beta+\lambda_j)\left[\frac{\lambda_i}{\lambda_j(\lambda_i+\beta)^2}+\frac{\beta}{\lambda_i(\lambda_j+\beta)^2}\right]\right.\right. \\ \left.\left.\times\exp\{\lambda_j\delta_j\}-1\right\}\right]$$

4. The α-fractiles

This section presents the α-fractiles of the original and improved systems. Let $L(\alpha)$ be the α-fractile of the original system and $L^D(\alpha)$, $D = H, C, I$, are the α-fractiles of the improved systems. The α-fractiles $L(\alpha)$ and $L^D(\alpha)$ are defined as the solution of the following equations, respectively,

$$R\left(\frac{L(\alpha)}{\Lambda}\right)=\alpha,\ R^D\left(\frac{L^D(\alpha)}{\Lambda}\right)=\alpha, \qquad (4.1)$$

where $\Lambda=\sum_{i=1}^{n}\lambda_i$.

It follows from Eqs. (2.1) and the first Eq. (4.1) that $L = L(\alpha)$, satisfies the following equation

$$\sum_{i=1}^{n}p_i\exp\left\{-\lambda_i\left(\frac{L}{\Lambda}-\delta_i\right)\right\}=\alpha. \qquad (4.2)$$

From the second equation of (4.1), when $D = H$, and Eq. (3.3), one can verify that $L = L^H(\alpha)$ satisfies the following equation

$$\left[2-\sum_{j=1}^{n}p_j\exp\left\{-\lambda_j\left(\frac{L}{\Lambda}-\delta_j\right)\right\}\right]\sum_{i=1}^{n}p_i\exp\left\{-\lambda_i\left(\frac{L}{\Lambda}-\delta_i\right)\right\}=\alpha. \tag{4.3}$$

Similarly, from Eq. (3.5) and the second equation of (4.1), when $D = C$, $L = L^C(\alpha)$ can be obtained by solving the following equation

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{p_ip_j}{\lambda_i-\lambda_j}\left[\lambda_i\exp\left\{-\frac{\lambda_jL}{\Lambda}\right\}-\lambda_j\exp\left\{-\frac{\lambda_iL}{\Lambda}\right\}\right]\exp\{\lambda_i\delta_i+\lambda_j\delta_j\}=\alpha. \tag{4.4}$$

Finally, from Eq. (3.7) and the second equation of (4.1), when $D = I$, $L = L^I(\alpha)$ satisfies the following equation

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}p_ip_j\lambda_i\left\{\frac{\lambda_j}{\lambda_i+\beta-\lambda_j}\left[\frac{1}{\lambda_j}\exp\left\{-\frac{\lambda_jL}{\Lambda}\right\}\right.\right.\\&\left.-\frac{1}{\lambda_i+\beta}\exp\left\{-\frac{(\lambda_i+\beta)L}{\Lambda}\right\}\right]+\frac{\beta}{\lambda_i-\beta-\lambda_j}\\&\left.\times\left[\frac{1}{\lambda_j+\beta}\exp\left\{-\frac{(\lambda_j+\beta)L}{\Lambda}\right\}-\frac{1}{\lambda_i}\exp\left\{-\frac{\lambda_iL}{\Lambda}\right\}\right]\right\}\\&\times\exp\{\lambda_i\delta_i+\lambda_j\delta_j\}=\alpha.\end{aligned} \tag{4.5}$$

Eqs. 4.2, (4.3)–(4.5) have no closed form solutions and can be solved using some numerical program such as Mathematica Program System.

5. Reliability equivalence factors

In this section, we derive survival reliability equivalence factor (SREF) and mean reliability equivalence factor (MREF) of the system component.

5.1. The SREF

We shall derive the SREF, when the failure of mixture of lifetime of the system component are reduced by the factor ρ, these factors will be denoted by $\rho_A^D(\alpha), D = H, C, I, A \subseteq \{1, 2, \ldots, n\}$. The factor $\rho_A^D(\alpha)$ is defined as the solution of the equation

$$R^D(t) = R_{A,\rho}(t) = \alpha. \tag{5.1}$$

Using Eq. (3.1) together with Eq. (5.1), one can verify that $\rho = \rho_A^D$ satisfies the following system of equations

$$\left.\begin{aligned}&\sum_{i\in A}p_i\exp\{-\rho_i\lambda_i(t-\delta_i)\}+\sum_{i\in\overline{A}}p_i\exp\{-\lambda_i(t-\delta_i)\}=\alpha,\\&R^D(t)=\alpha.\end{aligned}\right\} \tag{5.2}$$

By using Eqs. (5.2), together with Eqs. (3.3), (3.5), (3.7), one can verify that the factor $\rho_A^H, \rho_A^C, \rho_A^I, A \subseteq \{1, 2, \ldots, n\}$ satisfies the systems of equations which have no closed form solutions and can be solved using some numerical program such as Mathematica Program System.

5.2. The MREF

The MREF, say ξ_A^D, for D = H, C, I, $A \subseteq \{1, 2, \ldots, n\}$ can be obtained by solving the following equation

$$\mathrm{MTTF}_{A,\rho} = \mathrm{MTTF}^D. \tag{5.3}$$

Using Eq. (3.2) together with Eq. (5.3), one can verify that $\xi = \xi_A^D$ satisfies the equation

$$\sum_{i\in A}\frac{p_i}{\xi_i\lambda_i}[\exp\{\lambda_i\xi_i\delta_i\}-\exp\{\lambda_i\delta_i\}] = \mathrm{MTTF}^D - \mathrm{MTTF}. \tag{5.4}$$

Eq. (5.4) can be solved numerically by using Mathematica Program System, to get ξ_i^D for given λ_i, δ_i and MTTF^D. The MTTF^D are given, for $D = H,C$ and I, from Eqs. 3.4, 3.6 and 3.8, respectively.

6. Numerical results

To explain how one can utilize the previously obtained theoretical results, we introduce a numerical example. In such example, we calculate the two different reliability equivalence factors of a system of one component with two non-identical mixing lifetimes, under the following assumptions:

1. The system component has three mixture of lifetimes, $n = 3$.
2. The failure rates of the mixture lifetimes are $\lambda_1 = 0.07$, $\lambda_2 = 0.08$, $\lambda_3 = 0.09$.
3. The components of the probability vector are $p_1 = 0.35$, $p_2 = 0.25$, $p_3 = 0.40$.
4. The system reliability will be improved when the system component of mixing lifetimes is improved according to one of the previous duplication methods.
5. In the reduction method, we improve the system reliability when the failure rates of the set $A \subseteq \{1, 2, 3\}$ of mixture lifetime are reducing by the factor ρ_i, $i \in A$.
6. In the imperfect switch duplication method = 0.03.
7. The delay times for the mixing lifetimes are $\delta_1 = 0$, $\delta_2 = 0.03$, $\delta_3 = 0.06$.

For this example, we have found that:

The mean time to failure of the original and improved systems assuming hot, imperfect and cold duplication methods are presented in Table 1.

From Table 1, one can conclude that:

$$\mathrm{MTTF} < \mathrm{MTTF}^H < \mathrm{MTTF}^I < \mathrm{MTTF}^C.$$

Table 1 The MTTF of the original and improved systems.

MTTF	$\mathrm{MTTF}^{\mathrm{H}}$	$\mathrm{MTTF}^{\mathrm{I}}$	$\mathrm{MTTF}^{\mathrm{C}}$
12.601	18.921	21.791	25.369

Table 2 The α-fractiles of the original and improved systems.

α	L	L^H	L^I	L^C
0.1	6.9651	9.0165	10.199	11.7936
0.2	4.8509	6.7991	7.8042	9.0527
0.3	3.6222	5.4667	6.3361	7.3637
0.4	2.754	4.4879	5.2418	6.1005
0.5	2.0826	3.6947	4.3444	5.0619
0.6	1.5354	3.0091	3.561	4.1533
0.7	1.0736	2.3843	2.8409	3.3164
0.8	0.6742	1.7812	2.1409	2.5016
0.9	0.3225	1.1438	1.3985	1.6356

Table 3 The SREF $\rho_A^D(\alpha)$.

α	A = {1,2}			A = {2,3}			A = {1,2,3}		
	ρ^H	ρ^I	ρ^C	ρ^H	ρ^I	ρ^C	ρ^H	ρ^I	ρ^C
0.1	0.6989	0.6003	0.5074	0.6714	0.5676	0.4722	0.7723	0.6827	0.5904
0.2	0.6061	0.5043	0.4189	0.5983	0.4932	0.4051	0.7132	0.6212	0.5355
0.3	0.5237	0.4229	0.3442	0.5338	0.4307	0.3489	0.6621	0.5712	0.4914
0.4	0.4424	0.3446	0.2726	0.4705	0.371	0.2955	0.6129	0.5247	0.4508
0.5	0.3571	0.2641	0.1989	0.4047	0.3101	0.241	0.5628	0.4785	0.4105
0.6	0.2633	0.1765	0.1187	0.3327	0.2445	0.1823	0.5089	0.4299	0.3685
0.7	0.1545	0.0758	0.0263	0.2499	0.1696	0.1152	0.4484	0.3761	0.3221
0.8	0.0189	NA	NA	0.1477	0.0776	0.0327	0.3757	0.3123	0.2671
0.9	NA	NA	NA	0.0027	NA	NA	0.2768	0.2261	0.1931

The α-fractiles $L(\alpha)$,$L^D(\alpha)$ and the reliability equivalence factors $\rho^D(\alpha)$, $D = H, C, I$ are calculated using Mathematica Program System according to the previous theoretical formulae. In such calculations, the level α is chosen to be $0.1, 0.2, \ldots, 0.9$.

Table 2 represents the α-fractiles of the original and improved systems that are obtained by improving the system component according to the previously mentioned methods.

Based on the results presented in Table 2, it seems that:

$L(\alpha) < L^H(\alpha) < L^I(\alpha) < L^C(\alpha)$ in all studied cases.

This is confirmed by the results obtained for MTTF.

Table 3 shows the SREF of the improved systems using each duplication method.

According to the results presented in Table 3, it may be observed that:

1. Hot duplication of the mixing lifetimes will increase $L(0.1)$ from $\frac{6.9651}{A}$ to $\frac{9.0165}{A}$, see Table 2. The same effect on $L(0.1)$ can occur by reducing the failure rates of (i) the first and second type of mixing lifetimes, $A = \{1,2\}$, by the factor $\rho^H = 0.6989$, (ii) the second and third type of mixing lifetimes, $A = \{2,3\}$, by the factor $\rho^H = 0.6714$, (iii) All types of mixing lifetimes, $A = \{1,2,3\}$ by the factor $\rho^H = 0.7723$, see Table 3.
2. Imperfect duplication of the mixing lifetimes will increase $L(0.1)$ from $\frac{6.9651}{A}$ to $\frac{10.1990}{A}$, see Table 2. The same effect on $L(0.1)$ can occur by reducing the failure rates of (i) the first and second type of mixing lifetimes, $A = \{1,2\}$, by the factor $\rho^I = 0.6003$, (ii) the second and third types of mixing lifetimes, $A = \{2,3\}$ by the factor $\rho^I = 0.5676$, (iii) All types of mixing lifetimes, $A = \{1,2,3\}$ by the factor $\rho^I = 0.6827$, seeTable 3,
3. Cold duplication of the mixing lifetime will increase $L(0.1)$ from $\frac{16.9651}{A}$ to $\frac{11.7936}{A}$, see Table 2. The same effect on $L(0.1)$ can occur by reducing the failure rates of (i) the first and second type of mixing lifetimes, $A = \{1,2\}$ by the factor $\rho^C = 0.5074$, (ii) the second and third type of mixing lifetimes, $A = \{2,3\}$ by the factor $\rho^C = 0.4722$, (iii) All types of mixing lifetimes, $A = \{1,2,3\}$ by the factor $\rho^C = 0.5904$, see Table 3,
4. In the same manner, one can read the rest of results presented in Tables 3,

Table 4 The MREF, ξ^D.

A	ξ^H	ξ^I	ξ^C
{1,2}	0.5625	0.4693	0.3889
{2,3}	0.5449	0.4516	0.3722
{1,2,3}	0.6654	0.5777	0.4961

5. The notation NA means that there is no equivalence between the two improved systems: one obtained by reducing the failure rates of the only one type of the mixing lifetime and the other obtained by improving the system component according to the duplication methods.

Table 4 shows the MREF of the improved systems using each duplication method.

Based on the results presented in Table 4, one can conclude that:

1. The improved system that can be obtained by improving the mixing lifetime according to hot duplication method has the same meantime to failure of that system which can be obtained by reducing (i) the first and second types of the mixing failure rates, $A = \{1,2\}$ by the factor $\xi^H = 0.5625$, (ii) the second and third types of the mixing failure rates, $A = \{2,3\}$, by the factor $\xi^H 0.5449$, (iii) All types of the mixing failure rates, $A = \{1,2,3\}$, by the factor $\xi^H = 0.6654$, see Table 4.
2. The improved system that can be obtained by improving the mixing lifetime according to imperfect duplication method has the same mean time to failure of that system which can be obtained by reducing (i) the first and second types of the mixing failure rates, $A = \{1,2\}$, by the factor $\xi^I = 0.4693$, (ii) the second and third types of the mixing failure rates, $A = \{2,3\}$, by the factor $\xi^I = 0.4516$, (iii) All types of the mixing failure rates, $A = \{1,2,3\}$ by the factor $\xi^I = 0.5777$, see Table 4.
3. The improved system that can be obtained by improving mixing lifetime according to cold duplication method, has the same meantime to failure of that system which can be obtained by reducing (i) the first and second types of the mixing failure rates, $A = \{1,2\}$, by the factor $\xi^C = 0.3889$, (ii) the second andthird types of the mixing

failure rates, $A = \{2,3\}$, by the factor $\xi^C = 0.3722$, (iii) All types of the mixing failure rates, $A = \{1,2,3\}$, by the factor $\xi^C = 0.4961$, see Table 4.

7. Conclusions

This paper discusses the reliability equivalence factors of a system with mixture of n independent and non-identical lifetimes with delay time. The system studied here generalizes several well-known systems such as a system of mixture of $2(m)$ independent and non-identical lifetimes, Mustafa and El-Faheem [1]. We derived two types of the reliability equivalence factors of the system. We presented a numerical example to illustrate how the theoretical results derived in the paper can be applied.

Indeed, there are several possible extensions of this work. As an example, the case of a series (parallel) system with mixture of n independent and non-identical lifetimes with delay time, system (series, parallel, etc.) with mixture of lifetimes with non-constant failure rates can be studied.

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