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ORIGINAL ARTICLE

Tanh–coth scheme for traveling wave solutions for Nonlinear Wave Interaction model

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Abstract The coupled 1D nonlinear Schrödinger Zakharov System (sch-zakh) is considered as the model equation for Nonlinear Wave Interactions model. Tanh–coth Scheme is used to derive abundant solitary wave solutions for the model equations. The obtained solutions include soliton solutions.

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1. Introduction

Nonlinearity is a fascinating element of nature and many scientists see nonlinear science as the most important frontier for the fundamental understanding of nature. Many complex physical phenomena are frequently described and modeled by nonlinear evolution equation, so the exact or analytical solutions of the discussed nonlinear evolution equation become more and more important, which not only is considered a valuable tool in checking the accuracy of computational dynamics, but also gives us a good help to readily understand the essentials of complex physical phenomenon, e.g., collision of two solitary solutions. Looking for exact solitary wave solutions to nonlinear evolution equations have long been a major concern for both mathematicians and physicists. These solutions may well describe various phenomena in physics and other fields, such as solitons and propagation with a finite speed, and thus they may give more insight into the physical aspects of the problems. Modern theories of nonlinear science have been highly developed over the last half century.

At the classical level, a set of coupled nonlinear wave equations describing the interaction between high-frequency Langmuir waves and low-frequency ion-acoustic waves were firstly derived by Zakharov [1]. Since then, this system has been the subject of a large number of studies. In one dimension, the Zakharov Equations (ZE) may be written as

$$\begin{aligned} &iE_t + E_{xx} - nE = 0, \\ &n_{tt} - n_{xx} - |E|^2_{xx} = 0 \end{aligned} \tag{1}$$

where E is the envelope of the high-frequency electric field, n is the plasma density measured from its equilibrium value. The system can be derived from a hydrodynamic description of the plasma [2,3]. However, some important effects such as transit-time damping and ion nonlinearities, which are also implied by the fact that the values used for the ion damping

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have been anomalously large from the point of view of linear ion-acoustic wave dynamics, have been ignored in the ZE. This is equivalent to say that, the ZE are simplified model of strong Langmuir turbulence. Thus we have to generalize the ZE by taking more elements into account. Starting from the dynamical plasma equations with the help of relaxed Zakharov simplification assumptions, and through taking use of the time-averaged two-time-scale two-fluid plasma description, the ZE are generalized to contain the self-generated magnetic field [4]. The generalized Zakharov equations (GZE) are a set of coupled equations and may be written as [5]

$$\begin{aligned} &iE_t + E_{xx} - 2\beta|E|^2E + 2nE = 0, \\ &n_{tt} - n_{xx} + |E|^2_{xx} = 0 \end{aligned} \tag{2}$$

where E is the envelope of the high-frequency electric field, and n is the plasma density measured from its equilibrium value. This system is reduced to the classical Zakharov equations whenever $\beta = 0$. Due to the fact that the GZE is a realistic model in plasma, it makes sense to study the solitary wave solutions of the GZE. Recently various powerful mathematical methods such as homotopy perturbation method [6], variational iteration method [7–13], Adomian decomposition method [14] and others [15–19] have been proposed to obtain exact and approximate analytic solutions for nonlinear problems.

2. Description of tanh–coth method

We now present briefly the main steps of the tanh–coth strategy that will be applied. A PDE

$$P(u, u_t, u_x, u_{tt}, u_{xx}\ldots) = 0 \tag{3}$$

can be converted to an ODE

$$P(u, u', u'', u'''\ldots) = 0 \tag{4}$$

upon using a wave variable $\xi = x - ct$ Eq. (4) is then integrated as long as all terms contain derivatives where integration constants are considered zeros. Introducing a new independent variable $Y = \tanh\ (\xi)$ leads to the change of derivatives:

$$\frac{d}{d\xi} = (1 - Y^2)\frac{d}{dY}$$

$$\frac{d^2}{d\xi^2} = (1 - Y^2)\left\{(1 - Y^2)\frac{d^2}{dY^2} - 2Y\frac{d}{dY}\right\}$$

$$\frac{d^3}{d\xi^3} = 2(1 - Y^2)(3Y^2 - 1)\frac{d}{dY} - 6Y(1 - Y^2)^2\frac{d^2}{dY^2} + (1 - Y^2)^3\frac{d^3}{dY^3}$$

The tanh–coth method admits the use of the finite expansion

$$u(\xi) = S(Y) = \sum_{i=0}^{m} a_i Y^i + \sum_{i=1}^{m} \frac{b_i}{Y^i} \tag{5}$$

where m is a positive integer, for this method, that will be determined. Expansion (5) reduces to the standard tanh method $b_i = 0$, $1 \leqslant i \leqslant m$. The parameter is usually obtained, as stated before, by balancing the linear terms of the highest order in the resulting equation with the highest order nonlinear terms. If m is not an integer, and then a transformation formula should be used to overcome this difficulty. Substituting (5) into the ODE results is an algebraic system of equations in the powers of that will lead to the determination of the parameters $a_i = 0, (i = 0, \ldots, m) b_i = 0, (i = 1, \ldots, m)$ and c.

3. Application tanh–coth method for generalized Zakharov equations

We introduce a transformation for (GZE) Eq. (2)

$$\begin{aligned} &E(x,t) = U(\xi)e^{i\theta}, \quad \eta(x,t) = V(\xi), \\ &\quad \theta = kx + \omega t, \quad \xi = p(x - 2kt) \end{aligned}$$

where k, ω and p are real constant. Put these transformation in Eq. (2), we have the ordinary differential equation (ODE) for $U\ (\xi)$ and $V(\xi)$

$$\begin{aligned} &U(\xi)(k^2 + \omega) - p^2U''(\xi) + 2\beta U^3(\xi) - 2U(\xi)V(\xi) = 0, \\ &(4k^2 - 1)V''(\xi) + U''(\xi) = 0B \end{aligned} \tag{6}$$

where prime denotes the differential with respect to ξ. Integration of second equation of Eq. (6) twice with respect to ξ.

$$V(\xi) = \frac{C - U^2(\xi)}{(4k^2 - 1)} \tag{7}$$

where C is second integration constant and the first one is taken as zero. The value of $V(\xi)$ is put in first Eq. (6)

$$\begin{aligned} &U(\xi)\left(k^2 + \omega - \frac{2C}{4k^2 - 1}\right) - p^2U''(\xi) \\ &\quad + 2\left(\beta + \frac{1}{4k^2 - 1}\right)U^3(\xi) = 0 \end{aligned} \tag{8}$$

Obtain after integrating the ODE once and setting the constant of integration equal to zero. Balancing U'' with U^3 in Eq. (8) gives $m + 2 = 3m$, i.e. $m = 1$. The tanh–coth method (5) admits the use of the finite expansion

$$u(\xi) = S(Y) = a_0 + a_1 Y + \frac{b_1}{Y} \tag{9}$$

Substituting Eq. (9) into Eq. (8), we get

$$\begin{aligned} &\alpha\left(a_0 + a_1 Y + \frac{b_1}{Y}\right) - 2p^2b_1\left\{\frac{(1-Y^2)^2}{y^3} - 2(1-Y^2)Y\left(a_1 - \frac{b_1}{Y^2}\right)\right\} \\ &\quad + 2\left(\beta + \frac{1}{4k^2 - 1}\right)\left(a_0 + a_1 Y + \frac{b_1}{Y}\right)^3 = 0 \\ &\alpha\left(a_0 + a_1 Y + \frac{b_1}{Y}\right) - 2p^2b_1\left(\frac{1}{Y^3} + Y - \frac{2}{Y}\right) \\ &\quad + 2p^2\left((a_1 + b_1)Y - a_1Y^3 - \frac{b_1}{Y}\right) \\ &\quad + 2\left(\beta + \frac{1}{4k^2 - 1}\right)\left(a_0 + a_1 Y + \frac{b_1}{Y}\right)^3 = 0 \end{aligned}$$

Comparing the coefficients of Y^2 and Y^3 we get $a_0\ a_1 = 0$, $a_0 b_1 = 0$. So for a non-zero a_0 only trivial solution exists. To get a non-trivial solution, we get the following four sets of solutions

(1) The First set

$$a_0 = 0, \quad a_1 = 0,$$

$$b_1 = \sqrt{\frac{-p^2(4k^2 - 1)}{(\beta(4k^2 - 1) + 1)\{2p^2 - 1 + \alpha\}}}, \quad 2p^2 + \alpha = 0$$

(2) The Second set

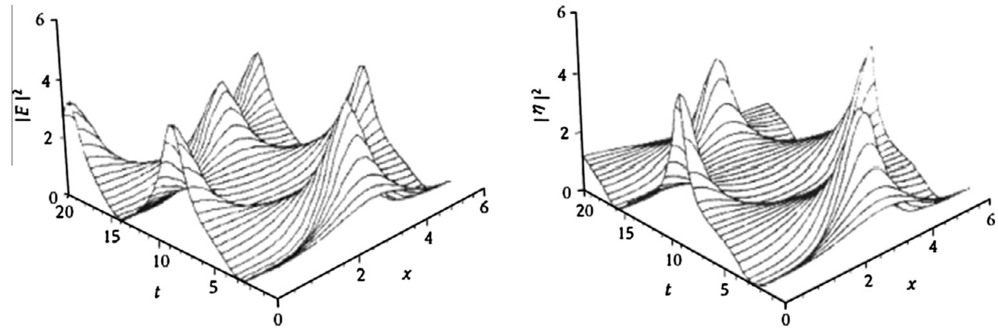

Figure 1 Plot of one set of solution of theoretical model for $C = \alpha = \beta = p = k = 1$.

$$a_0 = 0, \quad a_1 = \sqrt{\frac{-p^2(4k^2-1)}{(\beta(4k^2-1)+1)\{2p^2-1+\alpha\}}},$$
$$b_1 = 0, \quad 2p^2 + \alpha = 0$$

(3) The Third set

$$a_0 = 0, \quad a_1 = \sqrt{\frac{-p^2(4k^2-1)}{(\beta(4k^2-1)+1)\{8p^2-1+\alpha\}}}$$
$$b_1 = -\sqrt{\frac{-p^2(4k^2-1)}{(\beta(4k^2-1)+1)\{8p^2-1+\alpha\}}}, \quad 8p^2 + \alpha = 0$$

(4) The Fourth set

$$a_0 = 0, \quad a_1 = \sqrt{\frac{p^2(4k^2-1)}{(\beta(4k^2-1)+1)\{4p^2+1-\alpha\}}},$$
$$b_1 = -\sqrt{\frac{p^2(4k^2-1)}{(\beta(4k^2-1)+1)\{4p^2+1-\alpha\}}}, \quad 4p^2 - \alpha = 0$$

where $\alpha = k^2 + \omega - \frac{2C}{4k^2-1}$

In view of this we obtain the following kink shaped solitary wave solution (see Fig. 1)

$$E_1(x,t) = \sqrt{\frac{-p^2(4k^2-1)}{(\beta(4k^2-1)+1)\{2p^2-1+\alpha\}}} \coth\{p(x-2kt)\}e^{i(kx+\omega t)},$$
$$\eta_1(x,t) = \frac{C}{(4k^2-1)} + \frac{p^2}{(\beta(4k^2-1)+1)\{2p^2-1+\alpha\}} \coth^2\{p(x-2kt)\}$$
$$E_2(x,t) = \sqrt{\frac{-p^2(4k^2-1)}{(\beta(4k^2-1)+1)\{2p^2-1+\alpha\}}} \tanh\{p(x-2kt)\}e^{i(kx+\omega t)},$$
$$\eta_2(x,t) = \frac{C}{(4k^2-1)} + \frac{p^2}{(\beta(4k^2-1)+1)\{2p^2-1+\alpha\}} \tanh^2\{p(x-2kt)\}$$
$$E_3(x,t) = \sqrt{\frac{-p^2(4k^2-1)}{(\beta(4k^2-1)+1)\{8p^2-1+\alpha\}}} [\tanh\{p(x-2kt)\} - \coth\{p(x-2kt)\}]e^{i(kx+\omega t)},$$
$$\eta_3(x,t) = \frac{C}{(4k^2-1)} + \frac{p^2}{(\beta(4k^2-1)+1)\{8p^2-1+\alpha\}} [\tanh\{p(x-2kt)\} - \coth\{p(x-2kt)\}]^2$$
$$E_4(x,t) = \sqrt{\frac{p^2(4k^2-1)}{(\beta(4k^2-1)+1)\{4p^2+1-\alpha\}}} [\tanh\{p(x-2kt)\} - \coth\{p(x-2kt)\}]e^{i(kx+\omega t)},$$
$$\eta_4(x,t) = \frac{C}{(4k^2-1)} - \frac{p^2}{(\beta(4k^2-1)+1)\{4p^2+1-\alpha\}} [\tanh\{p(x-2kt)\} - \coth\{p(x-2kt)\}]^2$$

4. Conclusions

By using the tanh–coth method, traveling wave solutions are derived for Generalized Zakharov equations (GZE). The transformation formulae were used for nonlinearity to show that the analysis is applicable to a nonlinear problem. The present method is readily applicable to a large variety of such nonlinear equations. The obtained solutions include soliton solutions, periodic solutions and rational solutions.

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