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**ORIGINAL ARTICLE**

# Further properties on M-continuity

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**Abstract** The concept of M-open sets [1] can be applied in modifications of rough set approximations [2,3] which is widely applied in many application fields. The aim of this paper is to introduce and investigate some new classes of topological mappings called M-continuous mappings via M-open sets. Also,

M-irresolute mappings which are stronger than M-continuous mappings are studied and the relationships between these mappings are investigated. Several properties of these new notions have been discussed and the connections between them are studied.

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## 1. Introduction and preliminaries

Throughout this paper $(X, \tau)$ and $(Y, \sigma)$ (simply, $X$ and $Y$) represent non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure of subset $A$ of $X$, the interior of $A$ and the complement of $A$ is denoted by cl($A$), int($A$) and $A^c$ or $X\backslash A$, respectively.

$A$ subset $A$ of a space $(X, \tau)$ is called regular open [4] if $A = \text{int}(\text{cl}(A))$. $A$ point $x \in X$ is said to be a $\theta$-interior point of $A$ [5] if there exists an open set U containing $x$ such that $U \subseteq \text{cl}(U) \subseteq A$. The set of all $\theta$-interior points of $A$ is said to be the $\theta$-interior set and a subset $A$ of $X$ is called $\theta$-open if $A = \text{int}_\theta(A)$. $A$ subset $A$ of $X$ is called $\delta$-open [5] if it is the union of regular open sets. The complement of $\delta$-open set is called $\delta$-closed. The $\delta$-interior of a subset $A$ of $X$ is the union of all $\delta$-open sets of $X$ contained in $A$. For a subset $A$ of a space $(X, \tau)$, the closure of $A$, the interior of $A$, the $\delta$-interior of $A$, the $\theta$-interior of $A$ and the complement of $A$ is denoted by cl($A$), int($A$), $\text{int}_\delta(A)$, $\text{int}_\theta(A)$ and $A^c$ or $X\backslash A$, respectively. $A$ subset $A$ of a space $(X, \tau)$ is called preopen [6] or locally dense [7] (resp. $\delta$-preopen [8], $\alpha$-open [9], $\beta$-open [10], b-open [11] or $\gamma$-open [12], semi-open[13], $\delta$-semi-open [14], e-open [15], $\theta$-semi-open [16] if $A \subseteq \text{int}(\text{cl}(A))$ (resp. $A \subseteq \text{int}(\text{cl}_\delta(A))$, $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$, $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$, $A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$, $A \subseteq \text{cl}(\text{int}(A))$, $A \subseteq \text{cl}(\text{int}_\delta(A))$, $A \subseteq \text{cl}(\text{int}_\delta(A)) \cup \text{int}(\text{cl}_\delta(A))$ and $A \subseteq \text{cl}(\text{int}_\theta(A))$. The complement of a $\delta$-semi-open (resp. $\delta$-preopen, $\theta$-semi-open) set is called $\delta$-semiclosed (resp. $\delta$-preclosed, $\theta$-semiclosed). The family of all $\theta$-semi-open (resp. preopen, $\delta$-preopen, e-open, e*-open,$\theta$-open, $\alpha$-open, $\beta$-open, $\gamma$-open) is

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denoted by $\theta$-SO($X$) (resp. PO($X$), $\delta$-PO($X$), e-O($X$), e*-O($X$), $\theta$-O($X$), $\alpha$-O($X$), $\beta$-O($X$), $\gamma$-O($X$)).

The study of rough sets on an approximation space was initiated by [2,3]. Rough set theory is one of the new methods that connect information systems and data processing to mathematics in general and especially to the theory of topological structures and spaces. A large number of authors [17–24] had turned their attention to the generalization of approximation spaces which is widely applied in many applications fields.

The purpose of this paper is to introduce and investigate some new classes of topological mappings called M-continuous mappings and M-irresolute via M-open sets. Also, some properties and characterizations of these notions are discussed.

The following definitions and results were introduced and studied in [1].

**Definition 1.1.** Let $(X, \tau)$ be a topological space. Then a subset $A$ of $X$ is said to be:

(i) an M-open set, if $A \subseteq \mathrm{cl}(\mathrm{int}_\theta(A)) \cup \mathrm{int}(\mathrm{cl}_\delta(A))$,
(ii) an M-closed set if $\mathrm{int}(\mathrm{cl}_\theta(A)) \cap \mathrm{cl}(\mathrm{int}_\delta(A)) \subseteq A$.

The family of all M-open (resp. M-closed) subsets of a space $(X, \tau)$ will be as always denoted by MO($X$) (resp. MC($X$)).

This concept can be applied in modifications of rough set approximations [2,3] which is widely applied in many application fields.

**Definition 1.2.** Let $(X, \tau)$ be a topological space and $A \subseteq X$. Then:

(i) the M-interior of $A$ is the union of all M-open sets contained in $A$ and is denoted by M-int($A$),
(ii) the M-closure of $A$ is the intersection of all M-closed sets containing $A$ and is denoted by M-cl($A$).

**Definition 1.3.** For a space $(X, \tau)$, a point $p \in X$ is called an M-limit point of $A$ if for every M-open set $G$ containing $p$ contains a point of $A$ other than $p$. The set of all M-limit points of $A$ is called M-derived set of $A$ and is denoted by M-d($A$).

**Definition 1.4.** $A$ subset N of a space $(X, \tau)$ is called an M-neighbourhood (briefly, M-nbd) of a point $p \in X$ if there exists an M-open set $W$ such that $p \in W \subseteq N$.

**Definition 1.5.** $A$ mapping $f$: $(X, \tau) \to (Y, \sigma)$ is called:

(i) $\theta$-continuous [25] or strongly $\theta$-continuous [26] if, for each $V \in \sigma, f^{-1}(V) \in \theta - O(X)$,
(ii) $\theta$-semicontinuous [16] if for each $V \in \sigma$, $f^{-1}(V) \in \theta - \mathrm{SO}(X)$,
(iii) precontinuous [6] if for each $V \in \sigma, f^{-1}(V) \in \mathrm{PO}(X)$,
(iv) $\delta$-precontinuous (equivalently, $\delta$-almost continuous) [8] if for each $V \in \sigma, f^{-1}(V) \in \delta - \mathrm{PO}(X)$,
(v) $\delta$-semicontinuous [27] if for each $V \in \sigma$, $f^{-1}(V) \in \delta - \mathrm{SO}(X)$,
(vi) e-continuous [15] if for each $V \in \sigma, f^{-1}(V) \in e\ O(X)$,
(vii) $\beta$-continuous [10] if for each $V \in \sigma, f^{-1}(V) \in \beta - O(X)$,
(viii) b-continuous or $\gamma$-continuous [12] if for each $V \in \sigma$, $f^{-1}(V) \in \gamma - O(X)$,
(ix) semicontinuous [13] if for each $V \in \sigma, f^{-1}(V) \in \mathrm{SO}(X)$,
(x) $\delta$-continuous [26] if, $f^{-1}(V)$ is $\delta$-open in $X$ for every regular open set $V$ of $Y$,
(xi) $e^*$-continuous [28] if for each $V \in \sigma, f^{-1}(V) \in e^* - O(X)$,
(xii) $\alpha$-continuous [29] if for each $V \in \sigma, f^{-1}(V) \in \alpha - O(X)$.

**Lemma 1.1.** *For a topological space $(X, \tau)$ and $A \subseteq X$, then the following statements are hold:*

(i) *If $A \subseteq F_i$, $F_i$ is an M-closed set of $X$, then $A \subseteq M-cl(A) \subseteq F_i$,*
(ii) *If $G_i \subseteq A$, $G_i$ is an M-open set of $X$, then $G_i \subseteq M-int(A) \subseteq A$.*

**Proposition 1.1.** *Let $(X, \tau)$ be a topological space and $A \subseteq X$. Then, the following statements are hold:*

(i) *A is M-closed if and only if it contains each of its M-limit points,*
(ii) M-cl($A$) = $A \cup$ M-d($A$),
(iii) M-b($A$) = M-cl($A$)$\backslash$M-int($A$),
(iv) M-Bd($A$) = $A\backslash$M-int($A$).

*Where the set of M-boundary (resp. M-border) of A is denoted by M-b(A) (resp. M-Bd(A)).*

**Definition 1.6.** $A$ mapping $f$: $(X, \tau) \to (Y, \sigma)$ is called

(i) irresolute [30] if, $f^{-1}(V)$ is semi-open in $X$ for every semi-open set $V$ of $Y$,
(ii) $\theta$-irresolute [31] or quasi $\theta$-continuous [32] if, $f^{-1}(V)$ is $\theta$-open in $X$ for every $\theta$-open set $V$ of $Y$,
(iii) quasi-irresolute [13] if, $f^{-1}(V)$ is $\theta$-semi-open in $X$ for every $\theta$-semi-open set $V$ of $Y$,
(iv) pre-irresolute [6] if, $f^{-1}(V)$ is preopen in $X$ for every preopen set $V$ of $Y$,
(v) $\delta$-pre-irresolute [33] if, $f^{-1}(V)$ is $\delta$-preopen in $X$ for every $\delta$-preopen set $V$ of $Y$,
(vi) e-irresolute [34] if, $f^{-1}(V)$ is e-open in $X$ for every e-open set $V$ of $Y$,
(vii) $e^*$-irresolute [34] if, $f^{-1}(V)$ is $e^*$-open in $X$ for every $e^*$-open set $V$ of $Y$,
(viii) b-irresolute or $\gamma$-irresolute [12] if, $f^{-1}(V)$ is b-open in $X$ for every b-open set $V$ of $Y$,
(ix) $\alpha$-irresolute [35] if, $f^{-1}(V)$ is $\alpha$-open in $X$ for every $\alpha$-open set $V$ of $Y$,
(x) $\beta$-irresolute [36] if, $f^{-1}(V)$ is $\beta$-open in $X$ for every $\beta$-open set $V$ of $Y$,
(xi) $\delta$-semi-irresolute [37] if, $f^{-1}(V)$ is $\delta$-semi-open in $X$ for every $\delta$-semi-open set $V$ of $Y$.

**Definition 1.7.** [31] $A$ mapping $f$: $(X, \tau) \to (Y, \sigma)$ is called: $\theta$-open if the image of every open set of $(X, \tau)$ is $\theta$-open in $(Y, \sigma)$,

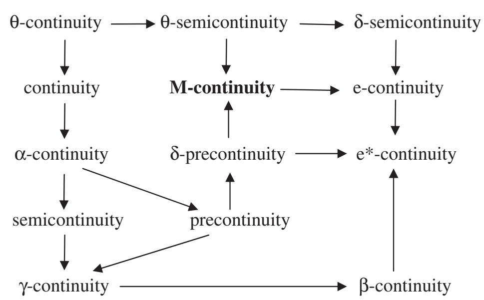
**Proposition 1.2.** [31] *A mapping $f$: $(X, \tau) \to (Y, \sigma)$ is called $\delta$-open if and only if $f^{-1}(cl_\delta(B)) \subseteq cl_\delta(f^{-1}(B))$, for each $B \subseteq Y$.*

**Lemma 1.2.** [32]. *For a space $(X, \tau)$, every dense set is preopen.*

## 2. M-continuous mappings

**Definition 2.1.** A mapping $f$: $(X, \tau) \to (Y, \sigma)$ is called M-continuous if $f^{-1}(V)$ is M-open in $X$, for every open set $V$ of $Y$.

**Remark 2.1.** The implication between some types of mappings of Definitions 1.5, 2.1, are given by the following diagram.




The converse of these implications is not true, in general by [28,6,29,7,13,14,10,27,12,25,31,16] and the following examples.

**Example 2.1.** Let $X = Y = \{a, b, c\}$ with topologies $\tau_x = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_y = \{Y, \phi, \{a\}, \{a, b\}\}$. Then a mapping $f$: $(X, \tau_x) \to (Y, \tau_y)$ which defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$ is M-continuous but not $\theta$-semicontinuous. Since, $f^{-1}(\{a\}) = \{b\}$ is not $\theta$-semi-open of $X$.

**Example 2.2.** In Example 2.1, and $(Y, \sigma)$ is a discrete topology, where $Y = \{a, b\}$, the mapping $f$: $(X, \tau) \to (Y, \sigma)$ which defined by $f(a) = b$ and $f(b) = f(c) = a$ is M-continuous but not $\delta$-precontinuous. Since, $f^{-1}(\{a\}) = \{b, c\}$ is not $\delta$-preopen of $X$.

**Example 2.3.** Let $X = \{a, b, c, d\}$ and $Y = \{a, b\}$ with topologies $\tau_x = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_y = \{Y, \phi, \{a\}, \{b\}\}$, then a mapping $f$: $(X, \tau_x) \to (Y, \tau_y)$ which defined by $f(a) = f(d) = b$ *and* $f(b) = f(c) = a$ is e-continuous but not M-continuous. Since, $f^{-1}(\{b\}) = \{a, d\}$ is not M-open of $X$.

The following example is an application of the concept of M-open sets in the rough set approximations.

**Example 2.4.** If we have the following information system. The objects $\{x_1, x_2, x_3, x_4\}$ represent the ID of students, the attributes $\{EL(1), MA, AL(1)\}$ are three salyets studied by the students, EL(1) is English language (1), $MA$ is Mathematics and $AL(1)$ is Arabic language(1). The values are the numbers scored by the students in an exam in the following table.

| Object(U) | $a_1$ $EL(1)$ | $a_2$ $MA$ | $a_3$ $AL(1)$ |
|---|---|---|---|
| $x_1$ | 86 | 83 | 77 |
| $x_2$ | 93 | 85 | 81 |
| $x_3$ | 89 | 60 | 78 |
| $x_4$ | 88 | 60 | 82 |

And consider the relation $R_i$ on the set of objects defined by:

$x\, R_i\, y$ iff $|a_i(x) - a_i(y)| \leqslant 2$, $i = 1, 2, 3$. Then we can get the following classifications corresponding to every subclass of attributes:

$$S_{EL(1)} = \{\{x_4\}, \{x_2, x_4\}, \{x_3, x_4\}\}, S_{MA} = \{\{x_1\}, \{x_3\}, \{x_1, x_3\}\}, S_{AL(1)} = \{\{x_1\}, \{x_2\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}\}$$

and hence the topologies generated by the above classes are: $\tau_{El(1)} = \{U, \phi, \{x_4\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_2, x_3, x_4\}\}$, $\tau_{MA} = \{U, \phi, \{x_1\}, \{x_3\}, \{x_1, x_3\}\}$ and $\tau_{AL(1)} = \{U, \phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}\}$. Hence, the identity mapping $f$: $(U, \tau_{El(1)}) \to (U, \tau_{MA})$ is M-continuous.

**Proposition 2.1.** *An M-continuous mapping is $\delta$-precontinuous, if for any subset $A$ of $X$ is nowhere dense.*

**Proof.** Let $V \in \sigma$ and $f$ be an M-continuous mapping. Then $f^{-1}(V) \in MO(X)$. If we put $f^{-1}(V) = A \in MO(X)$. Hence $A \subseteq cl(int_\theta(A)) \cup int(cl_\delta(A))$. But $int_\theta(A) \subseteq int(A) \subseteq cl(A)$, then $int_\theta(A) \subseteq int(cl(A))$. Since, $A$ is nowhere dense and by Lemma 1.2, we have $int_\theta(A) \subseteq \phi$. Therefore, $f$ is $\delta$-precontinuous.

In the following theorems, we introduce some characterizations on M-continuous mappings. □

**Theorem 2.1.** *Let $f$: $(X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent:*

(i) *$f$ is M-continuous,*
(ii) *For each $x \in X$ and each neighbourhood $W$ of $f(x)$ in $Y$, there exists an M-neighbourhood $V$ of $x$ in $X$ such that $f(V) \subseteq W$,*
(iii) *The inverse image of each closed set in $Y$ is M-closed in $X$,*
(iv) *$int(cl_\theta(f^{-1}(B))) \cap cl(int_\delta(f^{-1}(B))) \subseteq f^{-1}(cl(B))$, for each $B \subseteq Y$,*
(v) *$f^{-1}(int(B)) \subseteq cl(int_\theta(f^{-1}(B))) \cup int(cl_\delta(f^{-1}(B)))$, for each $B \subseteq Y$,*
(vi) *If $f$ is a bijective, then $int(f(A)) \subseteq f(cl(int_\theta(A))) \cup f(int(cl_\delta(A)))$, for each $A \subseteq X$,*
(vii) *If $f$ is a bijective, then $f(int(cl_\theta(A))) \cap f(cl(int_\delta(A))) \subseteq cl(f(A))$, for each $A \subseteq X$.*

**Proof.**

(i) → (ii). Let $W$ be a neighbourhood of $f(x)$ in $Y$. Then there exists an open set $G \subseteq Y$ such that $f(x) \in G \subseteq W$. Thus $x \in f^{-1}(G) \subseteq f^{-1}(W)$ for all $x \in X$. If we Put $f^{-1}(W) = V$. Since $f$ is M-continuous, then $f^{-1}(G)$ is an M-open set. So, $x \in f^{-1}(G) \subseteq V$. Hence, $V$ is an M-neighbourhood of $x$ in $X$. Since, $f^{-1}(W) = V$, then $f(V) \subseteq W$.

(ii) → (i). Let $G \subseteq Y$ be open set and for all $x \in f^{-1}(G)$. Then $f(x) \in G$ and there exists an M-neighbourhood $V$ of $x$ such that $f(V) \subseteq G$. Hence, $x \in V \subseteq f^{-1}f(V) \subseteq f^{-1}(G)$. Thus $f^{-1}(G)$ is an M-neighbourhood for all $x \in X$, then by Definition 1.4, $f^{-1}(G)$ is M-open in $X$. Therefore, $f$ is M-continuous.

(i) → (iii). Obvious.

(iii) → (iv). Since $B \subseteq \text{cl}(B) \subseteq Y$ which is a closed set, then by hypothesis $f^{-1}(\text{cl}(B))$ is M-closed in $X$. So, by Definition 1.1, $f^{-1}(\text{cl}(B)) \supseteq \text{int}(\text{cl}_\theta(f^{-1}(\text{cl}(B)))) \cap \text{cl}(\text{int}_\delta(f^{-1}(\text{cl}(B)))) \supseteq \text{int}(\text{cl}_\theta(f^{-1}(B))) \cap \text{cl}(\text{int}_\delta(f^{-1}(B)))$,

(iv) → (v). By replacing $Y\backslash B$ instead of $B$ in (iv), we have $\text{int}(\text{cl}_\theta(f^{-1}(Y\backslash B))) \cap \text{cl}(\text{int}_\delta(f^{-1}(Y\backslash B))) \subseteq f^{-1}(\text{cl}(Y\backslash B))$, this implies that $\text{int}(\text{cl}_\theta(X\backslash f^{-1}(B))) \cap \text{cl}(\text{int}_\delta(X\backslash f^{-1}(B))) \subseteq X\backslash f^{-1}(\text{int}(B))$, then $\text{int}(X\backslash \text{int}_\theta(f^{-1}(B))) \cap \text{cl}(X \backslash \text{cl}_\delta(f^{-1}(B))) \subseteq X\backslash f^{-1}(\text{int}(B))$, hence $X\backslash \text{cl}(\text{int}_\theta(f^{-1}(B))) \cap X\backslash \text{int}(\text{cl}_\delta(f^{-1}(B))) \subseteq X\backslash f^{-1}(\text{int}(B))$. Therefore, $\text{cl}(\text{int}_\theta(f^{-1}(B))) \cup \text{int}(\text{cl}_\delta(f^{-1}(B))) \supseteq f^{-1}(\text{int}(B))$.

(v) → (vi). Follows directly by replacing $f(A)$ instead of $B$ in (v) and applying the condition bijective of $f$,

(vi) → (vii). We put $X\backslash A$ replacing by $A$ in (vi) and by using the condition of a bijective of $f$, we have

$$\text{int}(f(X \setminus A)) \subseteq f(\text{cl}(\text{int}_\theta(X \setminus A))) \cup f(\text{int}(\text{cl}_\delta(X \setminus A))), \text{int}(f(X) \setminus f(A)) \subseteq f(\text{cl}(X \setminus \text{cl}_\theta(A))) \cup f(\text{int}(X \setminus \text{int}_\delta(A))), Y \setminus \text{cl}(f(A)) \subseteq f(X \setminus \text{int}(\text{cl}_\theta(A))) \cup f(X \setminus \text{cl}(\text{int}_\delta(A))), Y \setminus \text{cl}(f(A)) \subseteq f(X) \setminus f(\text{int}(\text{cl}_\theta(A))) \cup f(X) \setminus f(\text{cl}(\text{int}_\delta(A))), Y \setminus \text{cl}(f(A)) \subseteq Y \setminus f(\text{int}(\text{cl}_\theta(A))) \cup Y \setminus f(\text{cl}(\text{int}_\delta(A))), Y \setminus \text{cl}(f(A)) \subseteq Y \setminus [f(\text{int}(\text{cl}_\theta(A))) \cap f(\text{cl}(\text{int}_\delta(A)))], \text{cl}(f(A)) \supseteq f(\text{int}(\text{cl}_\theta(A))) \cap f(\text{cl}(int_\delta(A)))$$

(vii) → (i). Let $V \in \sigma$ and we put $W = Y\backslash V$. Then by hypothesis, $f\,[\text{int}(\text{cl}_\theta(f^{-1}(W)))] \cap f\,[\text{cl}(\text{int}_\delta(f^{-1}(W)))] \subseteq \text{cl}(ff^{-1}(W)) \subseteq \text{cl}(W) = W$. So, $\text{int}(\text{cl}_\theta(f^{-1}(W))) \cap \text{cl}[\text{int}_\delta(f^{-1}(W))] \subseteq f^{-1}(W)$, so by Definition 1.1, $f^{-1}(W)$, is M-closed in $X$. Therefore, $f$ is M-continuous.

**Remark 2.2.** The bijective condition in above theorem in parts (vi), (vii) is necessary as showed by the following example.

**Example 2.4.** Let $X = Y = \{a, b, c, d\}$ with topologies $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, d\}, \{a, b, c\}\}$. Then a mapping $f$: $(X, \tau) \to (Y, \sigma)$ which defined by $f(a) = b$, $f(b) = f(c) = a$ and $f(d) = c$, is:

(i) satisfying condition (vi) but not M-continuous. Since, $A = \{b, c\}$ is open in $Y$ but $f^{-1}(A) = f^{-1}(\{b, c\}) = \{a, d\}$ is not M-open of $X$,
(ii) satisfying condition (vii) but not M-continuous. Since, $A = \{a, d\}$ is closed in $Y$ but $f^{-1}(A) = f^{-1}(\{a, d\}) = \{b, c\}$ is not M-closed of $X$.

**Definition 2.2.** A mapping $f$: $(X, \tau) \to (Y, \sigma)$ is called M-continuous at point $p \in X$ if the inverse image of each neighbourhood of $f(p)$ is an M-neighbourhood of $p$ in $X$.

**Theorem 2.2.** *A mapping $f$: $(X, \tau) \to (Y, \sigma)$ is M-continuous if and only if it is M-continuous at every point $x \in X$.*

**Proof.** Obvious from Theorem 2.1.

**Theorem 2.3.** *Let $f$: $(X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent:*

(i) *$f$ is M-continuous,*
(ii) $\text{M-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$, for each $B \subseteq Y$,
(iii) $f(\text{M-cl}(A)) \subseteq \text{cl}(f(A))$, for each $A \subseteq X$,
(iv) $f^{-1}(\text{int}(B)) \subseteq \text{M-int}(f^{-1}(B))$, for each $B \subseteq Y$,
(v) *$\text{M-Bd}(f^{-1}(B)) \subseteq f^{-1}(Bd(B))$, for each $B \subseteq Y$,*
(vi) $\text{M-b}(f^{-1}(B)) \subseteq f^{-1}(\text{b}(B))$, for each $B \subseteq Y$.

**Proof.**

(i) → (ii). Since $B \subseteq \text{cl}(B) \subseteq Y$ which is a closed set. Then by hypothesis, $f^{-1}(\text{cl}(B))$ is M-closed in $X$. Hence, by Lemma 1.1, $\text{M-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for each $B \subseteq Y$.

(ii) → (iii). Let $A \subseteq X$. Then $f(A) \subseteq Y$, hence by hypothesis, $\text{M-cl}(A) \subseteq \text{M-cl}(f^{-1}(f(A))) \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore, $f(\text{M-cl}(A)) \subseteq f f^{-1}(\text{cl}(f(A))) \subseteq \text{cl}(f(A))$,

(iii) → (i). Let $V \subseteq Y$ be a closed set. Then, $f^{-1}(V) \subseteq X$. Hence, by (iii), $f(\text{M-cl}(f^{-1}(V))) \subseteq \text{cl}(f(f^{-1}(V))) \subseteq \text{cl}(V) = V$. Thus $\text{M-cl}(f^{-1}(V)) \subseteq f^{-1}(V)$ and hence $f^{-1}(V) \in \text{MC}(X)$. Hence, $f$ is M-continuous,

(i) → (iv). Let $\text{int}(B) \subseteq B \subseteq Y$ be open. Then by hypothesis, $f^{-1}(\text{int}(B))$ is an M-open set in $X$. Hence, by Lemma 1.1, $f^{-1}(\text{int}(B)) \subseteq \text{M-int}(f^{-1}(B))$, for each $B \subseteq Y$.

(iv) → (i). Let $U \subseteq Y$ be an open set. Then by assumption, $f^{-1}(U) = f^{-1}(\text{int}(U)) \subseteq \text{M-int}(f^{-1}(U))$. Hence, $f^{-1}(U)$ is M-open in $X$. Therefore, $f$ is M-continuous.

(iv) → (v). Let $V \subseteq Y$. Then by hypothesis, $f^{-1}(\text{int}(V)) \subseteq \text{M-int}(f^{-1}(V))$ and so $f^{-1}(V)\backslash \text{M-int}(f^{-1}(V)) \subseteq f^{-1}(V)\backslash f^{-1}(\text{int}(V))$. By Proposition 1.1, $\text{M-Bd}(f^{-1}(V)) \subseteq f^{-1}(\text{Bd}(V))$.

(v) → (iv). Let $V \subseteq Y$. Then by hypothesis, $f^{-1}(V)\backslash \text{M-int}(f^{-1}(V)) \subseteq f^{-1}(V)\backslash f^{-1}(\text{int}(V))$. Therefore, $f^{-1}(\text{int}(V)) \subseteq \text{M-int}(f^{-1}(V))$.

(iv) → (vi). Let $B \subseteq Y$. Then by (iv), $f^{-1}(\text{int}(B)) \subseteq \text{M-int}(f^{-1}(B))$. Hence by (ii), $\text{M-cl}(f^{-1}(B))\backslash \text{M-int}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))\backslash f^{-1}(\text{int}(B))$. So, by Proposition 1.1, $\text{M-b}(f^{-1}(B)) \subseteq f^{-1}(\text{b}(B))$, for each $B \subseteq Y$.

(vi) → (iv). Let $B \subseteq Y$. Then by Proposition 1.1, $\text{M-b}(f^{-1}(B)) = \text{M-cl}(f^{-1}(B)) \backslash \text{M-int}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))\backslash f^{-1}(\text{int}(B))$ this implies that $f^{-1}(\text{int}(B)) \subseteq \text{M-int}(f^{-1}(B))$, for each $B \subseteq Y$.

**Theorem 2.4.** *If, $f$: $(X, \tau) \to (Y, \sigma)$ is a mapping, then the following statements are equivalent:*

(i) *$f$ is M-continuous,*
(ii) $f(\text{M-d}(A)) \subseteq \text{cl}(f(A))$, for each $A \subseteq X$.

**Proof.**

(i) → (ii). Since $f$ is M-continuous, then By Theorem 2.3, $f(\text{M-cl}(A)) \subseteq \text{cl}(f(A))$, for each $A \subseteq X$. So, by Proposition 1.1, $f(\text{M-d}(A)) \subseteq f(\text{M-cl}(A)) \subseteq \text{cl}(f(A))$,

(ii) → (i). Let $U \subseteq Y$ be a closed set. Then, $f^{-1}(U) \subseteq X$. Hence, by hypothesis, $f(\text{M-d}(f^{-1}(U))) \subseteq \text{cl}(f(f^{-1}(U))) \subseteq \text{cl}(U) = U$. Thus, $\text{M-d}(f^{-1}(U)) \subseteq f^{-1}(U)$. Then by Proposition 1.1, $f^{-1}(U)$ is M-closed in $X$. Therefore, $f$ is M-continuous. □

**Definition 2.3.** A mapping $f$: $(X, \tau) \to (Y, \sigma)$ is called $\theta$-bicontinuous if, $f$ is $\theta$-open and $\theta$-continuous mapping.

**Theorem 2.5.** *The inverse image of each M-open set in $(Y, \sigma)$ under $\theta$-bicontinuous mapping $f$: $(X, \tau) \to (Y, \sigma)$ is M-open in $(X, \tau)$.*

**Proof.** Let $f$ be a $\theta$-bicontinuous mapping and $B \in \text{MO}(Y)$. Then $f^{-1}(B) \subseteq f^{-1}(\text{cl}(\text{int}_\theta(B))) \cup f^{-1}(\text{int}(\text{cl}_\delta(B))) \subseteq \text{cl}(f^{-1}(\text{int}_\theta(B))) \cup f^{-1}(\text{int}(\text{cl}_\delta(B)))$. By Definitions 1.5, 1.7, $f$ is $\delta$-precontinuous and $\theta$-semicontinuous, then $f^{-1}(B) \subseteq \text{cl}(\text{int}_\theta(f^{-1}(\text{int}_\theta(B)))) \cup \text{int}(\text{cl}_\delta(f^{-1}(\text{int}(\text{cl}_\delta(B))))) \subseteq \text{cl}(\text{int}_\theta(f^{-1}(\text{int}_\theta(B)))) \cup \text{int}(\text{cl}_\delta(f^{-1}(\text{cl}_\delta(B)))) \subseteq \text{cl}(\text{int}_\theta(f^{-1}(B))) \cup \text{int}(\text{cl}_\delta(f^{-1}(B)))$. Therefore, $f^{-1}(B) \in \text{MO}(X)$.

**Remark 2.3.** According the above theorem, it is clear that the inverse image of each $\delta$-preopen (resp. $\theta$-semi-open) set in $(Y, \sigma)$ is M-open of $(X, \tau)$ under $\theta$-bicontinuous mapping.

**Remark 2.4.** The restriction of M-continuous mapping is not M-continuous.

**Example 2.5.** Let $X = Y = \{a, b, c, d\}$ with topologies $\tau = \{X, \phi, \{b, c, d\}, \{c, d\}, \{d\}, \{b, d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $A = \{a, b, c\} \subseteq X$. Hence, the identity mapping $f$: $(X, \tau) \to (Y, \sigma)$ is M-continuous. But, $f_A$: $(A, \tau_A) \to (Y, \sigma)$ is not M-continuous. Since, $\{a\} \in \sigma$ but, $f^{-1}(\{a\}) \notin \text{MO}(A)$.

The next theorem gives the conditions under which the restriction of M-continuous mappings is M-continuous.

**Theorem 2.6.** *In a space $(X, \tau)$, if $A \in \theta\text{-}O(X, \tau)$ and $B \in MO(X, \tau)$, then $A \cap B \in MO(X, \tau_A)$.*

**Proof.** Since $A \cap B \subseteq \theta\text{-int}(A) \cap (\text{cl}(\theta\text{-int}(B)) \cup \text{int}(\delta\text{-cl}(B))) = (\theta\text{-int}(A) \cap \text{cl}(\theta\text{-int}(B))) \cup (\theta\text{-int}(A)) \cap \text{int}(\delta\text{-cl}(B)) \subseteq \text{cl}(\theta\text{-int}(A)) \cap \theta\text{-int}(B) \cup \text{int}(\theta\text{-int}(A) \cap \text{int}(\delta\text{-cl}(B)))$. Since $A \cap B \subseteq A$, then $A \cap B \subseteq (A \cap \text{cl}(\theta\text{-int}(A) \cap \theta\text{-int}(B))) \cup (A \cap \text{int}(\theta\text{-int}(A) \cap \text{int}(\delta\text{-cl}(B)))) \subseteq \text{cl}_A(\theta\text{-int}(A) \cap \theta\text{-int}(B)) \cup \text{int}_A(\theta\text{-int}(A) \cap \text{int}(\delta\text{-cl}(B))) \subseteq \text{cl}_A(\theta\text{-int}(A) \cap \theta\text{-int}(B)) \cup \text{int}_A(\delta\text{-cl}(\theta\text{-int}(A) \cap \delta\text{-cl}(B))) \subseteq \text{cl}_A(\theta\text{-int}(A) \cap \theta\text{-int}(B)) \cup \text{int}_A(\delta\text{-cl}(\delta\text{-cl}(\theta\text{-int}(A) \cap B)))$. Since, $\theta\text{-int}(A) \cap \theta\text{-int}(B) \subseteq \theta\text{-int}(A) \subseteq A$ which is $\theta$-open in $A$, then $A \cap B \subseteq \text{cl}_A\, \theta\text{-int}_A(\theta\text{-int}(A) \cap \theta\text{-int}(B)) \cup \text{int}_A(A \cap \delta\text{-cl}(\theta\text{-int}(A) \cap B)) \subseteq \text{cl}_A\, \theta\text{-int}_A(A \cap B) \cup \text{int}_A\, \delta\text{-cl}_A(\theta\text{-int}(A) \cap B) \subseteq \text{cl}_A\, \theta\text{-int}_A(A \cap B) \cup \text{int}_A\, \delta\text{-cl}_A(A \cap B)$. Therefore, $A \cap B \in \text{MO}(X, \tau_A)$.

**Remark 2.5.** The composition of two M-continuous mappings need not be M-continuous as showed by the following example.

**Example 2.6.** Let $X = Y = Z = \{a, b, c, d, e\}$, with topologies $\tau_x = \{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$, $\tau_y$ is an indiscrete topology and $\tau_z = \{Z, \phi, \{a, e\}\}$. Then the identity mappings $f$: $(X, \tau_x) \to (Y, \tau_y)$ and $g$: $(Y, \tau_y) \to (Z, \tau_z)$ are M-continuous, but $g o f$ is not M-continuous. Since $f^{-1}(\{a, e\})$ is not M-open of $X$.

The next theorem gives the conditions under which the composition of two M-continuous mappings is M-continuous.

**Theorem 2.7.** *If, $f$: $(X, \tau_x) \to (Y, \tau_y)$ and $g$: $(Y, \tau_y) \to (Z, \tau_z)$ are mappings, then*

(i) *$g\ o\ f$ is M-continuous if, $f$ is M-continuous and $g$ is continuous,*
(ii) *$g\ o\ f$ is M-continuous if, $f$ is $\theta$-bicontinuous and $g$ is M-continuous.*

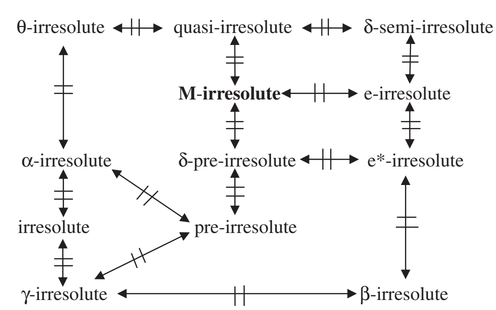
**Proof.**

(i) Let $V \in \tau_z$ and $g$ be continuous. Then $g^{-1}(V) \in \tau_y$. But $f$ is M-continuous, then $f^{-1}(g^{-1}(V)) = (g o f)^{-1}(V) \in \text{MO}(X, \tau_x)$. Hence, $g o f$ is M-continuous.
(ii) Let $V \in \tau_z$ and $g$ be M-continuous, then $g^{-1}(V) \in \text{MO}(Y, \tau_y)$. By Theorem 2.6, then $(g o f)^{-1}(V) \in \text{MO}(X, \tau_x)$. Hence, $g o f$ is M-continuous.

## 3. M-Irresolute mappings

**Definition 3.1.** *A* mapping $f$: $(X, \tau) \to (Y, \sigma)$ is called M-irresolute, if $f^{-1}(U) \in MO(X)$, for each $U \in MO(Y)$.

**Remark 3.1.** The implication between some types of mappings of Definitions 1.6, 3.1, are given by the following diagram.




By using the [31,13,6,33–35,12,30,36,37] and the following examples, we can showed that the above remark.

**Example 3.1.** Let $X = Y = \{a, b, c, d\}$, with topologies $\tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\}$. Then a mapping $f$: $(X, \tau) \to (Y, \sigma)$ which defined by the identity mapping is M-irresolute but, not quasi-irresolute (resp. e-irresolute) mapping. Since, $\{a, b, d\} \in \theta\text{-SO}(Y)$ but $f^{-1}(\{a, b, d\}) = \{a, b, d\} \notin \theta\text{-SO}(X)$. Also, $\{c, d\} \in \text{e-O}(Y)$ but $f^{-1}(\{c, d\}) = \{c, d\} \notin \text{e-O}(X)$.

**Example 3.2.** Let $X = Y = \{a, b, c, d\}$, with topologies $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Then a mapping $f$: $(X, \tau) \to (Y, \sigma)$ which defined by $f(a) = f(c) = a$, $f(b) = d$, $f(d) = b$ is quasi-irresolute but not M-irresolute. Since, $\{b\} \in \text{MO}(Y)$ but $f^{-1}(\{b\}) = \{d\} \notin \text{MO}(X)$.

**Example 3.3.** Let $X = \{a, b, c, d\}$ and $Y = \{a, b, c\}$ with topologies $\tau_x = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and $\tau_y = \{Y, \phi, \{b\}, \{c\}, \{b, c\}\}$. Then a mapping $f$: $(X, \tau) \to (Y, \sigma)$ which defined by $f(a) = b$, $f(b) = c$, $f(c) = a$, $f(d) = d$ is e-irresolute but not M-irresolute. Since, $\{a, b\} \in \text{MO}(Y)$ but, $f^{-1}(\{a, b\}) = \{a, c\} \notin \text{MO}(X)$.

**Example 3.4.** Let $X = Y = \{a, b, c, d\}$, with topologies $\tau = \{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then a mapping $f: (X, \tau) \to (Y, \sigma)$ which defined by the identity mapping is M-irresolute but, not $\delta$-pre-irresolute. Since, $\{a, b, d\} \in \delta$-PO($Y$) but $f^{-1}(\{a, b, d\}) = \{a, b, d\}$ is not $\delta$-preopen of $(X, \tau)$. In the following theorems, we introduce some characterizations on M-irresolute mappings.

**Theorem 3.1.** *Let $f: (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent:*

(i) *$f$ is M-irresolute,*
(ii) *The inverse image of each M-closed in $(Y, \sigma)$ is M-closed in $(X, \tau)$,*
(iii) *M-cl($f^{-1}(B)$) $\subseteq f^{-1}$(M-cl($B$)), for each $B \subseteq Y$,*
(iv) $f$(M-cl($A$)) $\subseteq$ M-cl($f(A)$), for each $A \subseteq X$,
(v) $f^{-1}$(M-int($B$)) $\subseteq$ M-int($f^{-1}(B)$), for each $B \subseteq Y$,
(vi) M-Bd($f^{-1}(B)$) $\subseteq f^{-1}$(M-Bd($B$)), for each $B \subseteq Y$,
(vii) M-b($f^{-1}(B)$) $\subseteq f^{-1}$(M-b($B$)), for each $B \subseteq Y$,
(viii) $f$(M-b($A$)) $\subseteq$ M-b($f(A)$), for each $A \subseteq X$,
(ix) $f$(M-d($A$)) $\subseteq$ M-cl($f(A)$), for each $A \subseteq X$,
(x) *For each $x \in X$ and each M-neighbourhood $U$ containing $f(x)$, there exists an M-neighbourhood $V$ containing $x$ such that $f(V) \subseteq U$.*

**Proof.**

(i) → (ii). Obvious.

(ii) → (iii). Let $B \subseteq Y$ and $B \subseteq$ M-cl($B$) $\subseteq Y$ which is M-closed. Then by (ii), $f^{-1}$(M-cl($B$)) $\subseteq X$ is M-closed. Then by Lemma 1.1, we have M-cl($f^{-1}(B)$) $\subseteq$ M-cl($f^{-1}$(M-cl($B$))) $= f^{-1}$(M-cl($B$)).

(iii) → (iv). Since $A \subseteq X$, then $f(A) \subseteq Y$. But, $f(A) \subseteq$ M-cl($f(A)$) which is M-closed in $Y$, hence by (ii), $f^{-1}$(M-cl($f(A)$)) $\subseteq X$ is M-closed in $X$. Then $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}$(M-cl($f(A)$)) and by Lemma 1.1, we have $A \subseteq$ M-cl($A$) $\subseteq f^{-1}$(M-cl($f(A)$)). Hence $f$(M-cl($A$)) $\subseteq$ M-cl($f(A)$).

(iv) → (i). Let $W \in$ MO($Y$) and $F = Y \backslash W$. Then by (iv), $f$(M-cl($f^{-1}(F)$)) $\subseteq$ M-cl($f(f^{-1}(F))$) $\subseteq$ M-cl($F$) $= F$. So, M-cl($f^{-1}(F)$) $\subseteq f^{-1}(F)$ and therefore $f^{-1}(W) \in$ MO($X$). Hence, $f$ is M-irresolute.

(v) → (vi). Let $B \subseteq Y$. Then by hypothesis, $f^{-1}(B) \backslash$M-int($f^{-1}(B)$) $\subseteq f^{-1}(B) \backslash f^{-1}$(M-int($B$)). By Proposition 1.1, M-Bd($f^{-1}(B)$) $\subseteq f^{-1}$(M-Bd($B$)).

(vi) → (v). Let $B \subseteq Y$. Then by hypothesis, we have $f^{-1}(B) \backslash$M-int($f^{-1}(B)$) $\subseteq f^{-1}(B) \backslash f^{-1}$(M-int($B$)). Therefore, $f^{-1}$(M-int($B$)) $\subseteq$ M-int($f^{-1}(B)$).

(i) → (v). Since M-int($B$) $\subseteq B$ which is M-open in $Y$, then by (i), $f^{-1}$(M-int($B$)) $\subseteq X$ is M-open. By Lemma 1.1, $f^{-1}$(M-int($B$)) $\subseteq$ M-int($f^{-1}(B)$).

(v) → (i). Let $B \in$ MO($Y$). Then B = M-int($B$). Hence by (v), we have $f^{-1}(B) = f^{-1}$(M-int($B$)) $\subseteq$ M-int($f^{-1}(B)$). Thus, $f^{-1}(B) \in$ MO($X$). So, $f$ is M-irresolute,

(i) → (vii). Let $B \subseteq Y$. Then by(iii), M-b($f^{-1}(B)$) = M-cl($f^{-1}(B)$)$\backslash$M-int($f^{-1}(B)$) $\subseteq f^{-1}$(M-cl($B$))$\backslash$M-int($f^{-1}(B)$) $\subseteq f^{-1}$[M-b($B$) $\cup$ M-int($B$)]$\backslash$M-int ($f^{-1}(B)$) $\subseteq [f^{-1}$(M-b($B$)) $\cup f^{-1}$(M-int($B$))]$\backslash$M-int($f^{-1}$(M-int($B$))). Hence by (i), M-b ($f^{-1}(B)$) $\subseteq f^{-1}$(M-b($B$)) $\cup f^{-1}$(M-int($B$))$\backslash f^{-1}$ (M-int($B$)) $= f^{-1}$(M- b($B$)),

(vii) → (viii). Follows directly by replacing $f(A)$ instead of $B$ in (vii).

(viii) → (vii). Let $B \subseteq Y$. Then by hypothesis, $f$(M-b($f^{-1}(B)$)) $\subseteq$ M-b($f(f^{-1}(B))$) $\subseteq$ M-b($B$) and therefore M-b($f^{-1}(B)$) $\subseteq f^{-1}$(M-b($B$)).

(vii) → (i). Let $B \in$ MO($Y$). Then $B$ = M-int($B$), by hypothesis, M-cl($f^{-1}(B)$)$\backslash$M-int($f^{-1}(B)$) $\subseteq f^{-1}$(M-cl($B$))$\backslash f^{-1}$(M-int($B$)). Then M-cl ($f^{-1}(B)$)$\backslash$M-int($f^{-1}(B)$) $\subseteq f^{-1}$(M-cl($B$))$\backslash f^{-1}(B)$, by (iii) we have $f^{-1}(B) \subseteq$ M-int($f^{-1}(B)$), hence, $f^{-1}(B) \in$ MO($X$). So, $f$ is M-irresolute.

(i) → (ix). Let $A \subseteq X$. Then by(iv), $f$(M-d($A$)) $\subseteq f$(M-cl($A$)) $\subseteq$ M-cl($f(A)$),

(ix) → (i). Let $F \subseteq Y$ be M-closed. Hence by hypothesis, $f$(M-d($f^{-1}(F)$)) $\subseteq$ M-cl($f(f^{-1}(F))$) $\subseteq$ M-cl($F$) $= F$, then M-d($f^{-1}(F)$) $\subseteq f^{-1}(F)$. Then by Proposition 1.1, $f^{-1}(F)$ is M-closed set in $X$. Therefore, $f$ is M-irresolute.

(i) → (x). Since U is M-neighbourhood of $f(x)$, then there exists an M-open set G containing $f(x)$ such that $f(x) \in$ G $\subseteq$ U, hence $x \in f^{-1}$(G) $\subseteq f^{-1}$(U). Put $f^{-1}$(U) $= V$. Then by hypothesis, there exists M-neighbourhood $V$ containing $x$ such that $f(V) = f f^{-1}$(U) $\subseteq$ U.

(x) → (i). Let U be an M-open set of $Y$ for every $x \in f^{-1}$(U). Then $f(x) \in f f^{-1}$(U) $\subseteq$ U, hence U is M-neighbourhood of $f(x)$. By hypothesis, there exists M-neighbourhood $V$ containing $x$ such that $f(V) \subseteq$ U, then $V \subseteq f^{-1}f(V) \subseteq f^{-1}$(U). By Definition 1.4, $f^{-1}$(U) is M-neighbourhood of $x$, for every $x \in f^{-1}$(U) and hence, $f^{-1}$(U) is M-open in $X$. Therefore, $f$ is M-irresolute.

**Theorem 3.2.** *If, $f: (X, \tau_x) \to (Y, \tau_y)$ and $g: (Y, \tau_y) \to (Z, \tau_z)$ are mappings, then*

(i) *$g$ o $f$ is M-irresolute, if both $f$ and $g$ are M-irresolute,*
(ii) *$g$ o $f$ is M-continuous, if $f$ is M-irresolute and $g$ is M-continuous.*

**Proof.**

(i) Let U $\in$ MO(Z, $\tau_z$). Since $g$ is M-irresolute, then $g^{-1}$(U) $\in$ MO($Y$, $\tau_y$). But $f$ is M-irresolute, then $f^{-1}(g^{-1}$(U)) $\in$ MO($X$, $\tau_x$). Hence, $g$ o $f$ is M-irresolute.
(ii) Let U $\in \tau_z$. Since, $g$ is M-continuous, then $g^{-1}$(U) $\in$ MO($Y$, $\tau_y$). But $f$ is M-irresolute, then $f^{-1}(g^{-1}$(U)) $\in$ MO($X$, $\tau_x$). Hence, $g$ o $f$ is M-continuous.

## 4. Conclusion

Maps have always been tremendous importance in all branches of mathematics and the whole science. In the other hand, topology plays a significant role in quantum physics, high energy and super string theory [38,39]. Thus we have obtained a new class of mappings called M-continuous which may have possible application in quantum physics, high energy and superstring theory. Also, the new concepts initiated in this paper can be applied in modifications of rough set approximations [24] which is widely applied in many application fields.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