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ORIGINAL ARTICLE

Partially ordered left almost semihypergroups

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Abstract The aim of this paper is to study the concept of ordered LA-semihypergroup. Here we consider some LA-semihypergroups and define a binary relation on them such that to become partially ordered LA-semihypergroups.

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1. Introduction and preliminaries

Hyperstructure theory was introduced in 1934, when Marty [1] defined hypergroups, began to analyze their properties and applied them to groups. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Several books and papers have been written on hyperstructure theory, see [2–6].

Let H be a non-empty set, then the map $\circ : H \times H \to \mathcal{P}^*(H)$ is called hyperoperation or join operation on the set H, where $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$ denotes the set of all non-empty subsets of H. A hypergroupoid is a set H together with a (binary) hyperoperation.

If A and B are two non-empty subsets of H, then we denote

$$A \circ B = \bigcup_{a\in A, b\in B} a \circ b, \quad a \circ B = \{a\} \circ B \text{ and } A \circ b = A \circ \{b\}.$$

There are several authors who study the ordering of hyperstructures, for instance, Bakhshi and Borzooei [7], Chvalina [8], Chvalina and Moučka [9], Heidari and Davvaz [10], Hošková [11], Kondo and Lekkoksung [12] and Novák [13].

Recently, Hila and Dine [14] introduced the notion of LA-semihypergroups as a generalization of semigroups, semihypergroups, and LA-semigroups. Yaqoob, Corsini and Yousafzai [15] extended the work of Hila and Dine and characterized intra-regular left almost semihypergroups by their hyperideals using pure left identity.

A hypergroupoid $(H, \circ)$ is called an LA-semihypergroup if for every $x, y, z \in H$, we have $(x \circ y) \circ z = (z \circ y) \circ x$. The law $(x \circ y) \circ z = (z \circ y) \circ x$ is called a left invertive law. An element $e \in H$ is called a left identity (resp., pure left identity) if for all $x \in H, x \in e \circ x$ (resp., $x = e \circ x$). In an LA-semihypergroup, the medial law $(x \circ y) \circ (z \circ w) = (x \circ z) \circ (y \circ w)$ holds for all $x, y, z, w \in H$. An LA-semihypergroup may or may not contain a left identity and pure left identity. In an LA-semihypergroup H with pure left identity, the paramedial law $(x \circ y) \circ (z \circ w) =$

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$(w \circ z) \circ (y \circ x)$ holds for all $x, y, z, w \in H$. If an LA-semihypergroup contains a pure left identity, then by using medial law, we get $x \circ (y \circ z) = y \circ (x \circ z)$ for all $x, y, z \in H$. (cf. [15]).

Lemma 1 [15]. *If H is an LA-semihypergroup with left identity, then $H \circ H = H$.*

Definition 1 [15]. Let H be an LA-semihypergroup. A non-empty subset A of H is called a sub LA-semihypergroup of H if $x \circ y \subseteq A$ for every $x, y \in A$.

Definition 2 [15]. A subset I of an LA-semihypergroup H is called a right (left) hyperideal of H if $I \circ H \subseteq I$ ($H \circ I \subseteq I$) and is called a hyperideal if it is two-sided hyperideal.

Definition 3 [15]. By a bi-hyperideal of an LA-semihypergroup H, we mean a sub LA-semihypergroup B of H such that $(B \circ H) \circ B \subseteq B$.

Definition 4 [15]. A non-empty subset Q of an LA-semihypergroup H is called a quasi-hyperideal of H if $Q \circ H \cap H \circ Q \subseteq Q$.

Let $(H, \circ)$ be an LA-semihypergroup and σ an equivalence relation on H. If A and B are non-empty subsets of H, then $A \hat{\sigma} B$ means that for all $a \in A$, there exists $b \in B$ such that $a \sigma b$ and for all $b\prime \in B$, there exists $a\prime \in A$ such that $a\prime \sigma b\prime$. Also, $A \hat{\hat{\sigma}} B$ means that for all $a \in A$ and $b \in B$, we have $a \sigma b$.

Definition 5. The equivalence relation σ is called

(1) regular on the right (resp., on the left) if for all $x \in H$, from $a \sigma b$, it follows that $(a \circ x) \hat{\sigma} (b \circ x)$ (resp., $(x \circ a) \hat{\sigma} (x \circ b)$);
(2) strongly regular on the right (resp., on the left) if for all $x \in H$, from $a \sigma b$, it follows that $(a \circ x) \hat{\hat{\sigma}} (b \circ x)$ (resp., $(x \circ a) \hat{\hat{\sigma}} (x \circ b)$);
(3) σ is called regular (resp., strongly regular) if it is regular (resp., strongly regular) on the right and on the left.

A partial order is a binary relation σ on a set X which satisfies the conditions of reflexivity, anti-symmetry and transitivity.

2. Partially ordered left almost semihypergroups

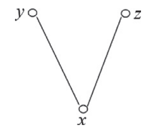
Here we introduce the concept of partially ordered left almost semihypergroups and discuss their related properties.

Definition 6. An ordered LA-semihypergroup $(H, \circ, \leqslant)$ is a poset $(H, \leqslant)$ at the same time an LA-semihypergroup $(H, \circ)$ such that: for any $a, b, x \in H, a \leqslant b$ implies $x \circ a \leqslant x \circ b$ and $a \circ x \leqslant b \circ x$.

If A and B are non-empty subsets of H, then we say that $A \leqslant B$ if for every $a \in A$ there exists $b \in B$ such that $a \leqslant b$.

Example 1. Let $H = \{x, y, z\}$. The binary hyperoperation "$\circ$", the order "$\leqslant$" and the corresponding Hasse diagram are given as follows:

$\circ$	x	y	z
x	x	y	z
y	z	$\{y,z\}$	$\{y,z\}$
z	y	$\{y,z\}$	$\{y,z\}$



$\leqslant \; := \{(x,x),(x,y),(x,z),(y,y),(z,z)\}$.

It is easy to verify that $(H, \circ, \leqslant)$ is an ordered LA-semihypergroup.

Definition 7. If $(H, \circ, \leqslant)$ is an ordered LA-semihypergroup and $A \subseteq H$, then $(A]$ is the subset of H defined as follows:

$(A] = \{t \in H : t \leqslant a, \text{for some } a \in A\}$.

Lemma 2. *Let $(H, \circ, \leqslant)$ be an ordered LA-semihypergroup. Then the following assertions hold:*

(i) $A \subseteq (A]$ for every $A \subseteq H$.
(ii) If $A \subseteq B$, then $(A] \subseteq (B]$ for every $A, B \subseteq H$.
(iii) $(A] \circ (B] \subseteq (A \circ B]$ for every $A, B \subseteq H$.
(iv) $((A]] = (A]$ for every $A \subseteq H$.
(v) $((A] \circ (B]] = (A \circ B]$ for all $A, B \subseteq H$.
(vi) If $A, B, C \subseteq H$ such that $A \subseteq B$, then $C \circ A \subseteq C \circ B$ and $A \circ C \subseteq B \circ C$.

Proof. The proof is straightforward. □

Definition 8. A non-empty subset A of an ordered LA-semihypergroup $(H, \circ, \leqslant)$ is called a sub LA-semihypergroup of H if $(A \circ A] \subseteq (A]$.

Definition 9. A non-empty subset A of an ordered LA-semihypergroup $(H, \circ, \leqslant)$ is called a left (resp., right) hyperideal of H if the following conditions hold:

(i) $H \circ A \subseteq A$ (resp., $A \circ H \subseteq A$);
(ii) If $a \in A$ and $b \leqslant a$, then $b \in A$ for every $b \in H$.

A is called a hyperideal of H if it is a left and a right hyperideal.

Definition 10. A sub LA-semihypergroup B of an ordered LA-semihypergroup $(H, \circ, \leqslant)$ is called a bi-hyperideal of H if the following conditions hold:

(i) $(B \circ H) \circ B \subseteq B$;
(ii) If $a \in B$ and $b \leqslant a$, then $b \in B$ for every $b \in H$.

Definition 11. A non-empty subset Q of an ordered LA-semihypergroup $(H, \circ, \leqslant)$ is called a quasi-hyperideal of H if the following conditions hold:

(i) $Q \circ H \cap H \circ Q \subseteq Q$;
(ii) If $a \in Q$ and $b \leqslant a$, then $b \in Q$ for every $b \in H$.

Definition 12. A non-empty subset P of an ordered LA-semihypergroup $(H, \circ, \leqslant)$ is called a prime hyperideal of H if the following conditions hold:

(*i*) $A \circ B \subseteq P \Rightarrow A \subseteq P$ or $B \subseteq P$ for any two hyperideals A and B of H;
(*ii*) If $a \in P$ and $b \leqslant a$, then $b \in P$ for every $b \in H$.

Definition 13. A non-empty subset I of an ordered LA-semihypergroup $(H, \circ, \leqslant)$ is called a semiprime hyperideal of H if the following conditions hold:

(*i*) $A \circ A \subseteq I \Rightarrow A \subseteq P$ for any hyperideal A of H;
(*ii*) If $a \in I$ and $b \leqslant a$, then $b \in I$ for every $b \in H$.

Proposition 1. *Let $(H, \circ, \leqslant)$ be an ordered LA-semihypergroup such that $H = H \circ H$, then every right hyperideal of H is a hyperideal.*

Proof. Let A be a right hyperideal of H. Let $x \in H \circ A$ which implies that $x \in y \circ z$ for some $y \in H$ and $z \in A$ with $z \leqslant i$ for some $i \in A$. Now as $H = H \circ H$ so $y \in b \circ c$ for some $b, c \in H$. Therefore

$$x \leqslant y \circ i \subseteq (b \circ c) \circ i \subseteq (i \circ c) \circ b \in (A \circ H) \circ H \subseteq A.$$

This implies that $x \in A$. Also if $a \in A$ and $b \leqslant a$, then $b \in A$ for every $b \in H$ holds obviously. Thus A is left hyperideal of H. Hence A is a hyperideal. □

Theorem 1. *The intersection of two hyperideals of an ordered LA-semihypergroup H, if it is non-empty, is a hyperideal of H.*

Proof. The proof is straightforward. □

Lemma 3. *Let $(H, \circ, \leqslant)$ be an ordered LA-semihypergroup with pure left identity such that $H = H \circ H$, then $(H \circ a]$ is a left hyperideal of H, for all $a \in H$.*

Proof. First we will show that $(H \circ a]$ is a left hyperideal of H, i.e $H \circ (H \circ a] \subseteq (H \circ a]$. Let $x \in H \circ (H \circ a]$ then $x \in y \circ b$ for some y in H and b in $(H \circ a]$ where $b \leqslant c \circ a$ for some $c \in H$. Since $H = H \circ H$, so let $y \in z_1 \circ z_2$. We have

$$\begin{aligned} x &\leqslant y \circ (c \circ a) \subseteq (z_1 \circ z_2) \circ (c \circ a) \\ &= (a \circ c) \circ (z_2 \circ z_1), \text{by paramedial law} \\ &= ((z_2 \circ z_1) \circ c) \circ a, \text{by left invertive law} \\ &\subseteq H \circ a. \end{aligned}$$

Therefore $x \in (H \circ a]$. For the second condition, let x be any element in $(H \circ a]$, then $x \leqslant b \circ a$ for some $b \circ a$ in $H \circ a$. Let y be any other element of H such that $y \leqslant x \leqslant b \circ a$, which implies that y is in $(H \circ a]$. Hence $(H \circ a]$ is a left hyperideal of H. □

Lemma 4. *Let $(H, \circ, \leqslant)$ be an ordered LA-semihypergroup with pure left identity and let A be a left hyperideal of H then $(A \circ A]$ is a hyperideal of H.*

Proof. First we show that $(A \circ A] \circ H \subseteq (A \circ A]$. For this, let $x \in (A \circ A] \circ H$, which implies that $x \in y \circ z$ for some $y \in (A \circ A]$ and $z \in H$, where $y \leqslant a \circ b$ for some $a \circ b \subseteq A \circ A$. Now we consider

$$x \leqslant y \circ z \subseteq (a \circ b) \circ z = (z \circ b) \circ a \subseteq (H \circ A) \circ A \subseteq A \circ A.$$

Thus $(A \circ A] \circ H \subseteq (A \circ A]$. Next we show that $H \circ (A \circ A] \subseteq (A \circ A]$. For this, let us consider $x \in H \circ (A \circ A]$, which implies that $x \in y \circ z$ for some $y \in H$ and $z \in (A \circ A]$, where $z \leqslant a \circ b$ for some $a \circ b \subseteq A \circ A$. Now consider $x \leqslant y \circ z \subseteq y \circ (a \circ b)$. Now using the fact that $(H, \circ, \leqslant)$ be an ordered LA-semihypergroup with pure left identity, we have

$$x \leqslant y \circ z \subseteq y \circ (a \circ b) = a \circ (y \circ b) \subseteq A \circ (H \circ A) \subseteq A \circ A.$$

Again let $x \in (A \circ A]$ then $x \leqslant a \circ b$ for some $a \circ b \subseteq A \circ A$. Let w be any other element of H such that $w \leqslant x \leqslant a \circ b$ then $w \in A \circ A$. Hence $(A \circ A]$ is a hyperideal of H. □

Theorem 2. *An ordered LA-semihypergroup H is an ordered semihypergroup if and only if $x \circ (y \circ z) = (z \circ y) \circ x$ for all $x, y, z \in H$.*

Proof. The proof is straightforward. □

Lemma 5. *Let H be an ordered LA-semihypergroup with pure left identity and $a \in H$. Then $< a >= (H \circ a]$.*

Proof. As H is an ordered LA-semihypergroup with pure left identity, so we have $H \circ (H \circ a] \subseteq (H \circ a]$, which shows that $(H \circ a]$ is a left hyperideal of H containing a. Let I be another left hyperideal containing a. Thus $H \circ a \subseteq I$, so $(H \circ a] \subseteq I$. □

Definition 14. Let H be an ordered LA-semihypergroup. A non-empty subset M of H is called an M-hypersystem of H if for each $a, b \in M$, there exist $x \in H$ and $c \in M$ such that $c \leqslant a \circ (x \circ b)$ or equivalently $c \in (a \circ (H \circ b))$].

Definition 15. Let H be an ordered LA-semihypergroup. A non-empty subset N of H is called an N-hypersystem of H if for each $a \in N$, there exist $x \in H$ and $c \in N$ such that $c \leqslant a \circ (x \circ a)$ or equivalently $c \in (a \circ (H \circ a))$].

Remark 1. Every M-hypersystem of H is an N-hypersystem of H.

Definition 16. A left hyperideal P of an ordered LA-semihypergroup H is called quasi-prime hyperideal if for all left hyperideals $A; B$ of $H, A \circ B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.

Definition 17. A left hyperideal P of an ordered LA-semihypergroup H is called quasi-semiprime hyperideal if for any left hyperideal A of $H, A \circ A \subseteq P$ implies $A \subseteq P$.

Remark 2. Every quasi-prime hyperideal of H is a quasi-semiprime hyperideal.

Lemma 6. *Let I be a left hyperideal of an ordered LA-semihypergroup H with pure left identity e. Then I is quasi-prime hyperideal if and only if for all $a, b \in H, a \circ (H \circ b) \subseteq I$ implies $a \in I$ or $b \in I$.*

Proof. Suppose that $a \circ (H \circ b) \subseteq I$. We get $H \circ (a \circ (H \circ b)) \subseteq H \circ I \subseteq I$. Consider

$$\begin{aligned} H\circ(a\circ(H\circ b)) &= (H\circ H)\circ(a\circ(H\circ b)) = (H\circ a)\circ(H\circ(H\circ b))\\ &= (H\circ a)\circ((H\circ H)\circ(H\circ b))\\ &= (H\circ a)\circ((b\circ H)\circ(H\circ H))\\ &= (H\circ a)\circ((b\circ H)\circ H) = (H\circ a)\circ((H\circ H)\circ b)\\ &= (H\circ a)\circ(H\circ b). \end{aligned}$$

Now since I is a left hyperideal of H, so

$$(H\circ a]\circ(H\circ b]\subseteq((H\circ a)\circ(H\circ b)] = (H\circ(a\circ(H\circ b))]\subseteq I.$$

Since $(H\circ a]$ and $(H\circ b]$ are left hyperideals of H and I is quasi-prime hyperideal, $(H\circ a]\subseteq I$ or $(H\circ b]\subseteq I$. By Lemma 5, $a\in I$ or $b\in I$. Conversely, let A and B be left hyperideals of H such that $A\circ B\subseteq I$ and $A\nsubseteq I$. Then there exist $x\in A$ and $x\notin I$. Now for all $y\in B$, we have $x\circ(H\circ y)\subseteq A\circ(H\circ B)\subseteq A\circ B\subseteq I$. Hence by assumption, $y\in I$ for all $y\in B$. Hence $B\subseteq I$, this implies that I is quasi-prime hyperideal. □

Theorem 3. *Let I be a left hyperideal of an ordered LA-semihypergroup H with pure left identity e. Then I is quasi-prime hyperideal if and only if $H\setminus I$ is an M-hypersystem.*

Proof. Let I be quasi-prime hyperideal and let $a,b\in H\setminus I$. Assume that $c\notin(a\circ(H\circ b)]$ for all $c\in H\setminus I$. Then $(a\circ(H\circ b)]\subseteq I$. This implies that $a\circ(H\circ b)\subseteq I\Rightarrow a\in I$ or $b\in I$, which contradicts the assumption that $a,b\in H\setminus I$. So $c\in(a\circ(H\circ b)]$ for all $c\in H\setminus I$. Hence $H\setminus I$ is an M-hypersystem.

Conversely assume that $H\setminus I$ is an M-hypersystem. Assume that $a\circ(H\circ b)\subseteq I$. Suppose that $a,b\in H\setminus I$, so there exist some $c\in H\setminus I$ and $x\in H$ such that $c\leqslant a\circ(x\circ b)$ *ubseteq*$(a\circ(H\circ b)]$, which implies that $c\in I$, it contradicts the assumption $c\in H\setminus I$. Hence $a\circ(H\circ b)\subseteq I$ implies that $a\in I$ or $b\in I$. Hence I is quasi-prime hyperideal. □

Lemma 7. *Let I be a left hyperideal of an ordered LA-semihypergroup H with pure left identity e. Then I is quasi-semiprime hyperideal if and only if for all $a\in H, a\circ(H\circ a)\subseteq I$ implies $a\in I$.*

Proof. The proof is straightforward. □

Theorem 4. *Let I be a left hyperideal of an ordered LA-semihypergroup H with pure left identity e. Then I is quasi-semiprime hyperideal if and only if $H\setminus I$ is an N-hypersystem.*

Proof. The proof is straightforward. □

Theorem 5. *If N is an N-hypersystem of an ordered LA-semihypergroup H and $a\in N$,then there exists an M-hypersystem M of H such that $a\in M\subseteq N$.*

Proof. Let N be an N-hypersystem of an ordered LA-semihypergroup H and $a\in N$, then by definition there exists some $c_1\in N$ such that $c_1\in(a\circ(H\circ a)]$, so $(a\circ(H\circ a)]\cap N\neq\emptyset$. Take $a_1\in(a\circ(H\circ a)]\cap N$ and again using the definition of N-hypersystem there exist $c_2\in N$ such that $c_2\in(a_1\circ(H\circ a_1)]$, so $(a_1\circ(H\circ a_1)]\cap N\neq\emptyset$. Continuing in this way, we take $a_i\in(a_{i-1}\circ(H\circ a_{i-1})]\cap N\neq\emptyset$. Take $a=a_0$ and let $M=\{a_0,a_1,\ldots\}$ then this set M is an M-hypersystem and $a\in M\subseteq N$. □

Definition 18. A left hyperideal I of an ordered LA-semihypergroup H is called quasi-irreducible hyperideal if for all left hyperideals $A;B$ of $H, A\cap B\subseteq I$ implies $A\subseteq I$ or $B\subseteq I$.

Definition 19. Let H be an ordered LA-semihypergroup with pure left identity. A non-empty subset I of H is called an I-hypersystem of H if for each $a,b\in I$, $(<a>\cap<b>)\cap I\neq\emptyset$.

Theorem 6. *Let I be a left hyperideal of an ordered LA-semihypergroup H with pure left identity e. Then I is quasi-irreducible hyperideal if and only if $H\setminus I$ is an I-hypersystem.*

Proof. Let I be a quasi-irreducible hyperideal of H and suppose that for each $a,b\in H\setminus I$, such that $(<a>\cap<b>)\cap H\setminus I=\emptyset$. This implies that $(<a>\cap<b>)\subseteq I$. So $a,b\in I$, which is a contradiction to the assumption that $a,b\in H\setminus I$. Hence $(<a>\cap<b>)\cap H\setminus I\neq\emptyset$, so $H\setminus I$ is an I -hypersystem.

Conversely let for any left hyperideals $A;B$ of $H, A\cap B\subseteq I$. Suppose that $A\nsubseteq I$ or $B\nsubseteq I$ and let $a\in A$ and $b\in B$, which implies that $a,b\in H\setminus I$. Since $H\setminus I$ is an I-hypersystem so there exist some $c\in<a>\cap<b>$ and $c\in H\setminus I$, which shows that $c\in<a>\cap<b>\subseteq A\cap B\subseteq I$, which is not possible. Thus $A\subseteq I$ or $B\subseteq I$. Hence I is quasi-irreducible hyperideal. □

Definition 20. Let $(H_1,\circ_1,\leqslant_1)$ and $(H_2,\circ_2,\leqslant_2)$ be two ordered LA-semihypergroups. Then $(H_1\times H_2,\circ)$ is an ordered LA-semihypergroup, where the hyperoperation $\circ$ defined as follows: $(x_1,x_2)\circ(y_1,y_2)=(x_1\circ_1 y_1, x_2\circ_2 y_2)$.

The order relation defined on $H_1\times H_2$ as follows: $(x_1,x_2)\leqslant(y_1,y_2)$ if and only if $x_1\leqslant_1 y_1$ or $x_1=y_1$ and $x_2\leqslant_2 y_2$. In the following we prove that $(H_1\times H_2,\circ,\leqslant)$ is an ordered LA-semihypergroup and is called the direct product of ordered LA-semihypergroup $(H_1,\circ_1,\leqslant_1)$ and $(H_2,\circ_2,\leqslant_2)$.

Theorem 7. *Let $(H_1,\circ_1,\leqslant_1)$ and $(H_2,\circ_2,\leqslant_2)$ be two ordered LA-semihypergroups. Then $(H_1\times H_2,\circ,\leqslant)$ is an ordered LA-semihypergroup.*

Proof. Suppose that $(x_1,x_2)\leqslant(y_1,y_2)$ for $(x_1,x_2),(y_1,y_2)\in H_1\times H_2$ and $(t_1,t_2)\in(h_1,h_2)\circ(x_1,x_2)$ for $(h_1,h_2)\in H_1\times H_2$. Then $t_1\in h_1\circ_1 x_1$ and $t_2\in h_2\circ_2 x_2$. Since $(x_1,x_2)\leqslant(y_1,y_2)$, so we have two cases:

Case (i) $x_1\leqslant_1 y_1$. Then $t_1\in h_1\circ_1 x_1\leqslant_1 h_1\circ_1 y_1$ so there exists $s_1\in h_1\circ_1 y_1$ such that $t_1\leqslant_1 s_1$. Now, if $s_2\in h_2\circ_2 y_2$ then $(t_1,t_2)\leqslant(s_1,s_2)\in(h_1,h_2)\circ(y_1,y_2)$.

Case (ii) $x_1=y_1$ and $x_2\leqslant_2 y_2$. Then $t_2\in h_2\circ_2 x_2\leqslant_2 h_2\circ_2 y_2$ so there exists $s_2\in h_2\circ_2 y_2$ such that $t_2\leqslant_2 s_2$. $(t_1,t_2)\leqslant(s_1,s_2)\in(h_1,h_2)\circ(y_1,y_2)$. Therefore, $(H_1\times H_2,\circ,\leqslant)$ is an ordered LA-semihypergroup. □

3. Regular ordered LA-semihypergroups

In this section we present some results on regular ordered LA-semihypergroup.

Definition 21. Let $(H, \circ, \leqslant)$ be an ordered LA-semihypergroup, and $a \in H$. Then a is said to be regular element of H if there exists an element $x \in H$ such that $a \leqslant (a \circ x) \circ a$, or equivalently $a \leqslant (a \circ H) \circ a$. If every element of H is regular then H is said to be a regular ordered LA-semihypergroup.

Lemma 8. *Every right hyperideal of a regular ordered LA-semihypergroup H is a hyperideal.*

Proof. Let $(A]$ be any right hyperideal of a regular ordered LA-semihypergroup H, then for each $a \in H$ there exist $x \in H$ such that $a \leqslant (a \circ x) \circ a$. Let $y \in A$, then

$$a \circ y \leqslant ((a \circ x) \circ a) \circ y \subseteq (y \circ a) \circ (a \circ x) \subseteq A,$$

which shows that $(A]$ is a left hyperideal of H. Hence $(A]$ is a hyperideal of H. $\square$

Lemma 9. *Let $(H, \circ, \leqslant)$ be an ordered LA-semihypergroup. If H is regular ordered LA-semihypergroup, then $(A \circ B] = (A \cap B]$ for right hyperideal A and left hyperideal B of H.*

Proof. The proof is straightforward. $\square$

Theorem 8. *Every hyperideal of a regular ordered LA-semihypergroup H is prime hyperideal if and only if it is irreducible hyperideal of H.*

Proof. Suppose that P is a prime hyperideal of H and let $(A \circ B] \subseteq P$. Then by Lemma 9, $(A \circ B] = (A \cap B]$ so $(A \cap B] \subseteq P$ which implies that either $(A] \subseteq P$ or $(B] \subseteq P$. Hence P is irreducible hyperideal of H.

Conversely, suppose that P is an irreducible hyperideal of H. Then $(A \cap B] \subseteq P$ implies either $(A] \subseteq P$ or $(B] \subseteq P$. Again by above Lemma 9, $(A \circ B] = (A \cap B]$. Hence P is prime hyperideal. $\square$

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References

[1] F. Marty, Sur une generalization de la notion de groupe, 8^{iem} Congres Math. Scandinaves. Stockholm., 1934, pp. 45–49.
[2] P. Corsini, Prolegomena of hypergroup theory, Aviani editor, 1993.
[3] T. Vougiouklis, Hyperstructures and their Representations, Hadronic Press, Florida, 1994.
[4] S. Abdullah, M. Aslam, T. Anwar, A note on M-hypersystems and N-hypersystems in Γ-semihypergroups, Quasigr. Relat. Syst. 19 (2) (2011) 169–172.
[5] S. Abdullah, K. Hila, M. Aslam, On bi-Γ-hyperideals of Γ-semihypergroups, UPB Sci. Bull. Ser. A 74 (4) (2012) 79–90.
[6] A. Ahmed, M. Aslam, S. Abdullah, Interval valued (α, β)-fuzzy hyperideals of semihyperrings, UPB Sci. Bull. Ser. A 72 (2) (2013) 69–86.
[7] M. Bakhshi, R.A. Borzooei, Ordered polygroups, Ratio Math. 24 (2013) 31–40.
[8] J. Chvalina, Commutative hypergroups in the sense of Marty and ordered sets, in: Proc. Summer School, Gen. Algebra Ordered Sets, Olomouc (Czech Republic), 1994, pp. 19–30.
[9] J. Chvalina, J. Moučka, Hypergroups determined by orderings with regular endomorphism monoids, Ital. J. Pure Appl. Math. 16 (2004) 227–242.
[10] D. Heidari, B. Davvaz, On ordered hyperstructures, U.P.B. Sci. Bull. Ser. A 73 (2) (2011) 85–96.
[11] Š. Hošková, Upper order hypergroups as a reflective subcategory of subquasiorder hypergroups, Ital. J. Pure Appl. Math. 20 (2006) 215–222.
[12] M. Kondo, N. Lekkoksung, On intra-regular ordered Γ-semihypergroups, Int. J. Math. Anal. 7 (28) (2013) 1379–1386.
[13] M. Novák, *EL*-hyperstructures: an overview, Ratio Math. 23 (2012) 65–80.
[14] K. Hila, J. Dine, On hyperideals in left almost semihypergroups, ISRN Algebra, Article ID 953124, 2011, p. 8.
[15] N. Yaqoob, P. Corsini, F. Yousafzai, On intra-regular left almost semihypergroups with pure left identity, J. Math. (2013) 10. Article ID 510790.