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Exact solutions for nonlinear integro-partial differential equations using the generalized Kudryashov method

Khaled A. Gepreel[a,b,*], Taher A. Nofal[b,c], Ameara A. Alasmari[b]

[a] Mathematics Department, Faculty of Science, Zagazig University, Zagazig, Egypt
[b] Mathematics Department, Faculty of Science, Taif University, Saudi Arabia
[c] Mathematics Department, Faculty of Science, El-Minia University, Egypt

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ABSTRACT

In this research, we construct the traveling wave solutions for some nonlinear evolution equations in mathematical physics. New solutions such as soliton solutions are found. The method used is the generalized Kudryashov method (GKM). We apply the method successfully to find the exact solutions of the following nonlinear integro-partial differential equations: the $(1+1)$-dimensional integro-differential Ito equation, $(2+1)$-dimensional integro-differential Sawada–Kotera equation and two members of integro-differential Kadomtsev–Petviashvili (KP) hierarchy equations. These equations have numerous important applications in mathematical physics as well as in engineering. This method is efficient, powerful and simple.

1. Introduction

Nonlinear partial differential equations (NPDEs) have a very important role in describing many nonlinear phenomena in chemistry, physics, mathematical biology, and many other fields of science and engineering. There are many methods for finding the exact solutions to nonlinear PDEs such as homogenous balance method [1,2], Darboux transform method [3,4], first integral method [5,6], tanh function method [7], modified simple equation method [8,9,10], auxiliary equations method [11,12], (G'/G)-expansion method [13,14], F-expansion method [15], Jacobi elliptic function method [16] and so on (see for example [17-26]). Recently Kudryashov [27] has presented a direct method namely truncated expansion method to discuss the analytic solutions for nonlinear evaluation equations. Kabir [28] has improved the truncated expansion method for nonlinear PDEs which called the improved of Kudryashov method. More recently Kaplan [29] has generalized the Kudryashov method to solve the nonlinear PDEs. In this paper, we have applied the generalized Kudryashov method to find the traveling wave solutions for the following nonlinear integro-partial differential equations:

(i) The $(1+1)$-dimensional integro-differential Ito equation [30]

$$u_{tt}+u_{xxxt}+3(2u_x u_t+u u_{xt})+3u_{xx}\partial_x^{-1}(u_t)=0\,. \tag{1}$$

(ii) The $(2+1)$-dimensional integro-differential Sawada–Kotera equation [31]

$$u_t=\left(u_{xxxx}+5uu_{xx}+\frac{5}{3}u^3+u_{xy}\right)_x-5\partial_x^{-1}(u_{yy})+5uu_y+5u_x\partial_x^{-1}(u_y). \tag{2}$$

(iii) The first integro-differential KP hierarchy equation [32–34]

$$u_t=\frac{1}{2}u_{xxy}+\frac{1}{2}\partial_x^{-2}[u_{yyy}]+2u_x\partial_x^{-1}[u_y]+4uu_y. \tag{3}$$

(iv) The second integro-differential KP hierarchy equation [32–34]

$$u_t=\frac{1}{16}u_{xxxxx}+\frac{5}{4}\partial_x^{-1}[uu_{yy}]+\frac{5}{4}\partial_x^{-1}\left[{u_y}^2\right]+\frac{5}{16}\partial_x^{-3}[u_{yyyy}]+\frac{5}{4}u_x\partial_x^{-2}[u_{yy}]+\frac{5}{2}u\partial_x^{-1}[u_{yy}]+\frac{5}{2}u_y\partial_x^{-1}[u_y]+\frac{15}{2}u^2\,u_x+\frac{5}{2}\,u_x\,u_{xx}+\frac{5}{4}u\,u_{xxx}+\frac{5}{8}\,u_{xyy}, \tag{4}$$

where $\partial_x^{-1}=\int_{-\infty}^{x}dx$.

* Corresponding author.
E-mail addresses: kagepreel@yahoo.com (K.A. Gepreel), nofal_ta@yahoo.com (T.A. Nofal), aamalasmari@hotmail.com (A.A. Alasmari).

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2. Description of the generalized Kudryashov method for NPDE

In this section, we consider the following nonlinear partial differential equation:-

$$P(v, v_t, v_x, v_{tt}, v_{xx}, v_{xt}, \ldots) = 0, \tag{5}$$

where P is a polynomial in the unknown function $v = v(x, t)$ and its partial derivatives.

The basic steps in the application of the GKM detailed in the following [29]:

Step 1. We assume that:-

$$v(x, t) = V(\xi),\ \xi = x - kt, \tag{6}$$

where k is an arbitrary constant. Eq. (6) leads to get:

$$P\left(V, V', V'', \ldots\right) = 0. \tag{7}$$

Step 2. We suppose the exact solution of Eq. (7) to be in the following rational form:-

$$V(\xi) = \frac{\sum_{i=0}^{N} a_i Q^i(\xi)}{\sum_{j=0}^{M} b_j Q^j(\xi)} = \frac{A[Q(\xi)]}{B[Q(\xi)]}, \tag{8}$$

where $a_i (i = 0, 1, 2, \ldots, N)$ and $b_j (j = 0, 1, 2, \ldots, M)$ are constants to be determined later such that $a_N \neq 0$, $b_M \neq 0$ and $Q = \frac{1}{1 \pm e^{\xi}}$. The function Q has then to satisfy the first order Bernoulli differential equation:

$$Q'(\xi) = Q^2(\xi) - Q(\xi). \tag{9}$$

Step 3. Determine the positive integer numbers N and M in Eq. (8) by balancing the highest order derivatives and the nonlinear terms in Eq. (7).

Step 4. Substituting Eqs. (8) and (9) into Eq. (7), we obtain a polynomial in Q^{i-j}, $(i, j = 0, 1, 2, \ldots)$. Setting all coefficients of this polynomial to be zero, we obtain a system of algebraic equations which can be solved by the Maple or Mathematica software package to get the unknown parameters $a_i (i = 0, 1, 2, \ldots, N)$ and $b_j (j = 0, 1, 2, \ldots, M)$. Consequently, we obtain the exact solutions of Eq. (5).

3. Applications of the generalized Kudryashov method for nonlinear integro-PDEs

Zhao et al. [30] have used the extended monoclinic test for study the exact solutions for Ito equations. Yu [31] have study the exact solution of integro-differential Sawada–Kotera by using the bilinear method. Gepreel [34] and Mohamed et al. [23] have studied the exact and approximate solutions to the integro-differential KP hierarchy equations by using (f/g) expansion method and reduced differential transform method respectively. In this section, we use the generalized Kudryashov method to find the traveling wave solutions for some nonlinear evolution equations. We shall namely solve the $(1 + 1)$-dimensional integro-differential Ito equation, the $(2 + 1)$-dimensional integro-differential Sawada–Kotera equation, and two members of the integro-differential KP hierarchy equations. These equations have very important applications in the area of mathematical physics.

3.1. The generalized Kudryashov method for the (1 + 1)-dimensional integro-differential Ito equation

In this subsection, we apply the generalized Kudryashov method to construct the exact solution to the $(1 + 1)$-dimensional integro-differential Ito Eq. (1). We use the transformation

$$u(x, t) = v_x(x, t). \tag{10}$$

This transformation changes the $(1 + 1)$-dimensional integro-differential Ito Eq. (1) to the following nonlinear partial differential equation:

$$v_{ttx} + v_{xxxxt} + 6v_{xx}v_{xt} + 3v_x v_{xxt} + 3v_{xxx}v_t = 0. \tag{11}$$

The traveling wave transformation (6) converts Eq. (11) to the following ODE:-

$$k^2V''' - kV^{(5)} - 6k\left(V''\right)^2 - 6kV'V''' = 0. \tag{12}$$

By integrating twice, we obtain:

$$kV' - V''' - 3\left(V'\right)^2 + C_1 = 0, \tag{13}$$

where $' = \frac{d}{d\xi}$ and C_1 is the integration constant. The relation between N and M in Eq. (8) is given by balancing the highest order V''' and the nonlinear term $(V')^2$ in (13), as follows:

$$N = M + 1. \tag{14}$$

Eq. (14) has an infinite number of solutions. For the special case in which we choose $M = 1$, we obtain:

$$V(\xi) = \frac{a_0 + a_1 Q + a_2 Q^2}{b_0 + b_1 Q}, \tag{15}$$

where a_0, a_1, a_2, b_0 and b_1 are constants to be determined later. Now substituting Eq. (15) into Eq. (13) and cleaning the denominator, we get a polynomial in $Q(\xi)$. Setting each power coefficient of $Q(\xi)$ to be zero, we get a system of nonlinear of algebraic equations. With the help of Maple, we solve the system of algebraic equation to get the following results:

Case 1.

$$C_1 = 0,\ a_0 = -\frac{1}{2}a_1,\ a_2 = 4b_0,\ b_1 = -2b_0,\ k = 4. \tag{16}$$

When, we substitute (16) into (15), we get:

$$V_1(\xi) = -\frac{1}{2}\left[\frac{-8b_0 - 2a_1\left(1 \pm e^{\xi}\right) + a_1\left(1 \pm e^{\xi}\right)^2}{b_0\left(1 \pm e^{\xi}\right)\left(-1 \pm e^{\xi}\right)}\right] \tag{17}$$

where $\xi = x - 4t$. Consequently, the solution of the $(1 + 1)$-dimensional integro-differential Ito equation takes the form:

$$u_1(x, t) = -\frac{8\left(\pm e^{[2x-8t]}\right)}{\left(1 \pm e^{[x-4t]}\right)^2\left(-1 \pm e^{[x-4t]}\right)^2}. \tag{18}$$

Case 2.

$$C_1 = 0,\ a_0 = \frac{b_0(a_1 + 2\ b_0)}{b_1},\ a_2 = -2b_1,\ k = 1. \tag{19}$$

Eqs. (20) and (16) lead to get

$$V_2(\xi) = \frac{-2b_1 + (a_1 + 2b_0)\left(1 \pm e^{\xi}\right)}{b_1\left(1 \pm e^{\xi}\right)}, \tag{20}$$

where $\xi = x - t$. Hence the traveling wave solution for the $(1 + 1)$-dimensional integro-differential Ito equation takes the form:

$$u_2(x, t) = \frac{2\left(\pm e^{[x-t]}\right)}{\left(1 \pm e^{[x-t]}\right)^2}. \tag{21}$$

Case 3.

$$C_1 = 0,\quad a_1 = \frac{a_0 b_1 - 2{b_0}^2 - 2b_0 b_1}{b_0},\ a_2 = 0,\ k = 1. \tag{22}$$

Substitute (22) into (15), we have

$$V_3(\xi) = \frac{a_0 b_1 - 2{b_0}^2 - 2b_0 b_1 + a_0 b_0\left(1 \pm e^{\xi}\right)}{b_0\left(b_0\left(1 \pm e^{\xi}\right) + b_1\right)}, \tag{23}$$

where $\xi = x - t$. Then, we deduce the exact solution of the $(1 + 1)$-dimensional integro-differential Ito equation as follows:

$$u_3(x, t) = \frac{2b_0(b_0 + b_1)\left(\pm e^{[x-t]}\right)}{\left(\pm b_0 e^{[x-t]} + b_0 + b_1\right)^2}. \tag{24}$$

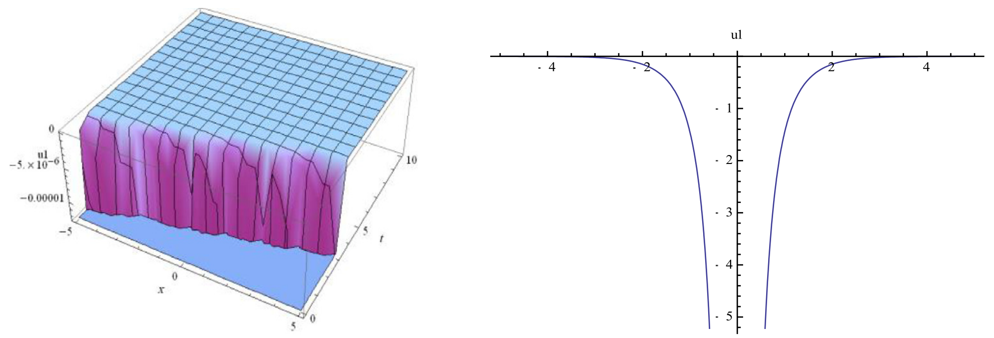

Fig. 1. The exact solution u_1 of Eq. (1) and its projection at $t = 0$.

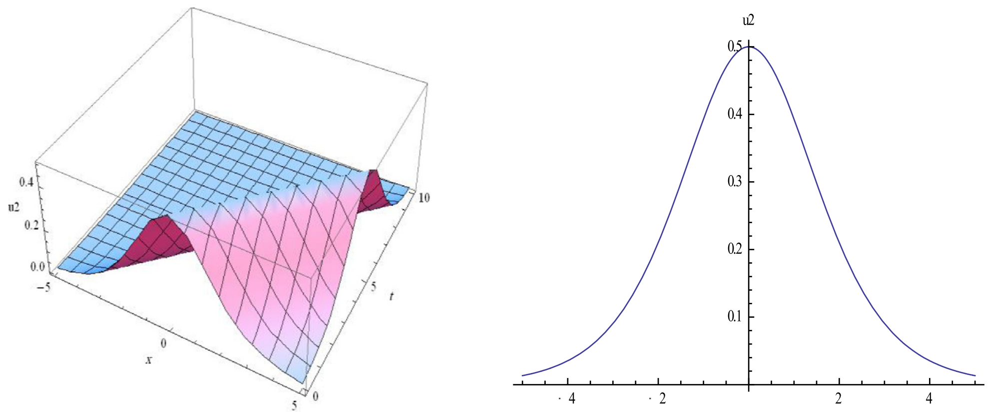

Fig. 2. The exact solution u_2 of Eq. (1) and its projection at $t = 0$.

3.2. Numerical solutions for the exact solutions of the (1 + 1)-dimensional integro-differential Ito equation

In this subsection, we present some figures to illustrate the behavior of the exact solutions, which obtained in Section 3.1. To this end, we select some special values of the parameters to show the behavior of extended rational generalized Kudryashov method for (1 + 1)-dimensional integro-differential Ito equation (Figs. 1 and 2).

3.3. The generalized Kudryashov method for the (2 + 1)-dimensional integro-differential Sawada–Kotera equation

In this subsection, we will discuss the exact traveling wave solutions of (2 + 1)-dimensional integro-differential Sawada–Kotera Eq. (2) by using the generalized Kudryashov method. Eq. (10) transforms the (2 + 1)-dimensional integro-differential Sawada–Kotera Eq. (2) to the following nonlinear PDE:

$$v_{xt} = v_{xxxxxx} + 5(v_x v_{xxx})_x + \frac{5}{3}\left(v_x^3\right)_x + v_{xxxy} - 5v_{yy} + 5v_x v_{xy} + 5v_{xx}v_y. \tag{25}$$

We assume that

$$v(x, y, t) = V(\eta), \quad \eta = x + y - kt, \tag{26}$$

where k is an arbitrary constant. Eq. (26) leads to get:

$$-kV'' = V^{(6)} + 5\left(V'V'''\right)' + \frac{5}{3}[\left(V'\right)^3]' + V^{(4)} - 5V'' + 10V'V'', \tag{27}$$

where $' = \frac{d}{d\eta}$. By integrating and take the transformation $W = V'$, we obtain:

$$W^{(4)} + 5WW'' + \frac{5}{3}W^3 + W'' + 5W^2 + (k-5)W + C_2 = 0, \tag{28}$$

where C_2 is the integration constant. Considering the homogeneous balance between the highest order derivative $W^{(4)}$ and nonlinear term W^3 in (28), we get:

$$N = M + 2 \tag{29}$$

In the special case if $M = 1$, then

$$W(\eta) = \frac{a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3}{b_0 + b_1 Q} \tag{30}$$

Substituting Eq. (30) into Eq. (28) and cleaning the denominator, we get a polynomial in $Q(\eta)$. Setting the coefficient of same power of $Q(\eta)$ to be zero, we obtain the system of nonlinear of algebraic equations. Using Maple software package to solve the system of algebraic equation to get the following results:

$$C_2 = -\frac{28}{75}, \; a_0 = -\frac{14}{5}b_0, \; a_1 = 12b_0 - \frac{14}{5}b_1, \; a_2 = -12b_0 + 12\,b_1, \; a_3 = -12\,b_1, \; k = \frac{29}{5}. \tag{31}$$

We substitute (31) into (30), we get:

$$W(\eta) = -\frac{2}{5}\left[\frac{30 - 30(1 \pm e^{\eta}) + 7(1 \pm e^{\eta})^2}{(1 \pm e^{\eta})^2}\right], \tag{32}$$

where $\eta = x + y - \frac{29}{5}t$. Consequently, the solution of (2 + 1)-dimensional integro-differential Sawada–Kotera equation takes the form:

$$u_4(x, y, t) = -\frac{2\left(30 - 30\left(1 \pm e^{\left[x+y-\frac{29}{5}t\right]}\right) + 7\left(1 \pm e^{\left[x+y-\frac{29}{5}t\right]}\right)^2\right)}{5\left(1 \pm e^{\left[x+y-\frac{29}{5}t\right]}\right)^2}. \tag{33}$$

There are many cases are omitted for convenience.

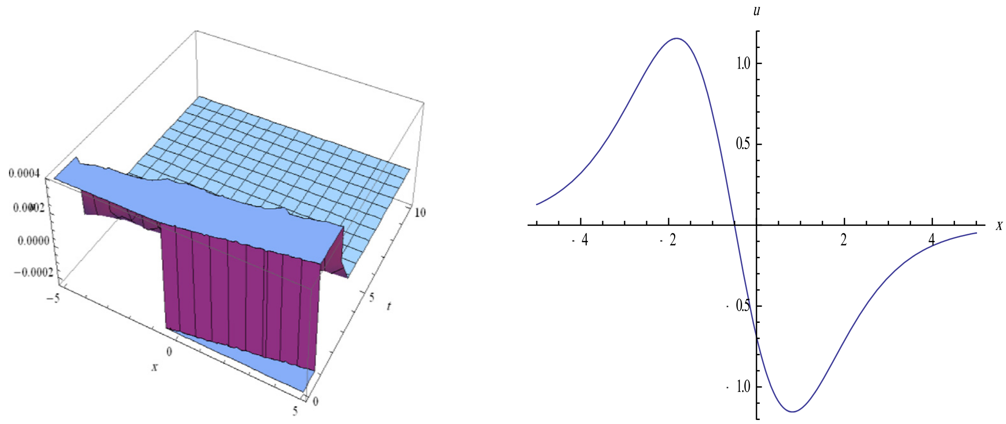

Fig. 3. The exact solution u_4 of Eq. (2) and its projection at $t = 0$ when $y = 0.5$.

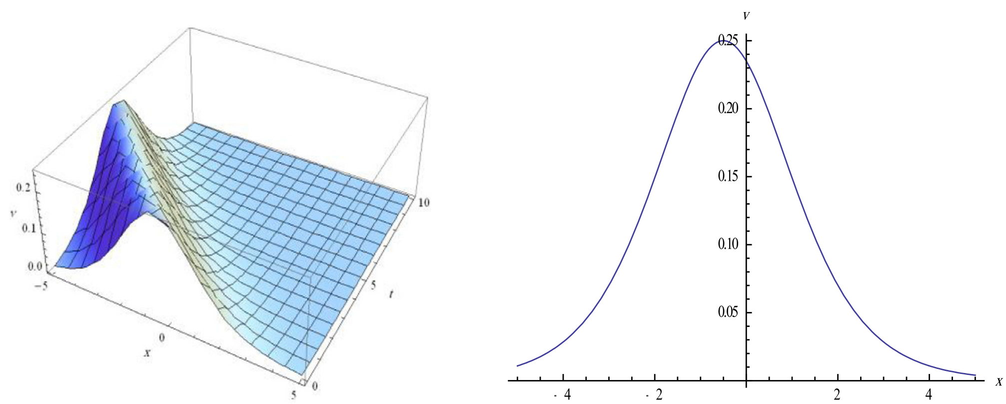

Fig. 4. The exact solution u_5 of Eq. (3) and its projection at $t = 0$ when $b_0 = 0.3,\ b_1 = 0.2,\ a_1 = 0.3, y = 0.5$.

3.4. Numerical solutions for the exact solution to $(2+1)$-dimensional integro-differential Sawada–Kotera equation

In this part, we illustrate the behavior of the obtained solution for $(2+1)$-dimensional integro-differential Sawada–Kotera equation in Section 3.3 when the parameters take some of special chooses (Fig. 3).

3.5. The generalized Kudryashov method for the first equation of two members of integro-differential KP hierarchy equations

In this subsection, we will study the traveling wave solutions of the first integro-differential KP hierarchy equations (3) by the generalized Kudryashov method. The Kadomtsev–Petviashvili equations are nonlinear partial differential equations in two spatial coordinates and one temporal coordinate, which describes the evolution of nonlinear, long waves of small amplitude with slow dependence on the transverse, coordinate. We use the following transformation

$$u(x, y, t) = v_{xx}(x, y, t). \tag{34}$$

Eq. (34) transforms Eq. (3) to the following nonlinear PDE

$$v_{xxt} = \frac{1}{2} v_{xxxxy} + \frac{1}{2} v_{yyy} + 2 v_{xxx} v_{xy} + 4 v_{xx} v_{xxy}. \tag{35}$$

The traveling wave transformation (26) permits us to convert the nonlinear PDE (35) to the following ODE:

$$\left(k + \frac{1}{2}\right) V'' + \frac{1}{2} V^{(4)} + 3\left(V''\right)^2 + C_3 = 0, \tag{36}$$

where C_3 is the integration constant. If we take the transformation $V'' = W$, Eq. (36) can be written in the following form:

$$\left(k + \frac{1}{2}\right) W + \frac{1}{2} W'' + 3W^2 + C_3 = 0. \tag{37}$$

Considering the homogeneous balance between the highest order derivative W'' and nonlinear term W^2 in (37), we get:

$$N = M + 2, \tag{38}$$

In the special case when $M = 1$ Eq. (8) reduces to:

$$W(\eta) = \frac{a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3}{b_0 + b_1 Q} \tag{39}$$

Eq. (39) is a solution to (37) when the constants $a_i, i = 0, 1, 2$ and $b_i, i = 0, 1$ satisfy the following results

$$C_3 = \frac{1}{2} \frac{(a_1 - b_0)(6a_1 - 6b_0 + b_1)}{{b_1}^2}, \quad a_0 = \frac{b_0(a_1 - b_0)}{b_1},$$
$$a_2 = -b_0 + b_1, \ a_3 = -b_1, \quad k = -\frac{6a_1 - 6b_0 + b_1}{b_1}, \tag{40}$$

When we substitute (40) into (39), we get:

$$W(\eta) = \frac{-b_1 + b_1(1 \pm e^{\eta}) + (a_1 - b_0)(1 \pm e^{\eta})^2}{b_1 (1 \pm e^{\eta})^2}, \tag{41}$$

where $\eta = x + y + (\frac{6a_1 - 6b_0 + b_1}{b_1})t$. Since $V'' = W$, we have

$$V(\xi) = \iint \left[\frac{-b_1 + b_1(1 \pm e^{\eta}) + (a_1 - b_0)(1 \pm e^{\eta})^2}{b_1 (1 \pm e^{\eta})^2} \right] d\eta d\eta, \tag{42}$$

or

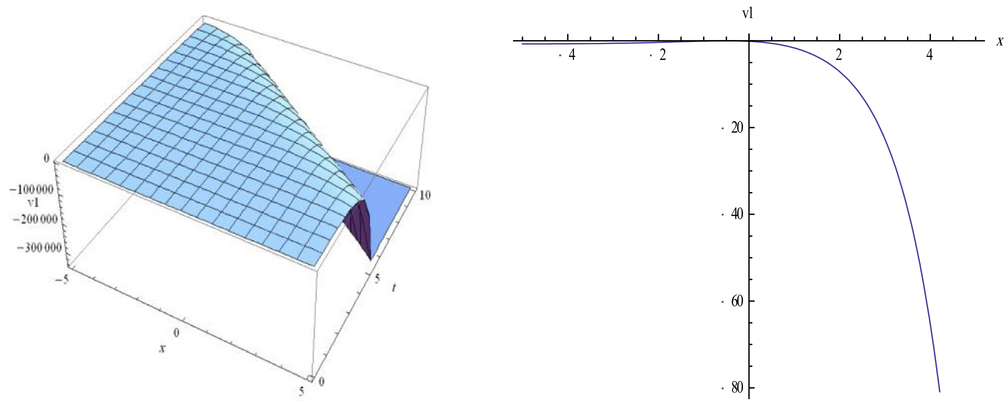

Fig. 5. The exact solution u_6 of Eq. (4) and its projection at $t = 0$ when $y = 0.5$.

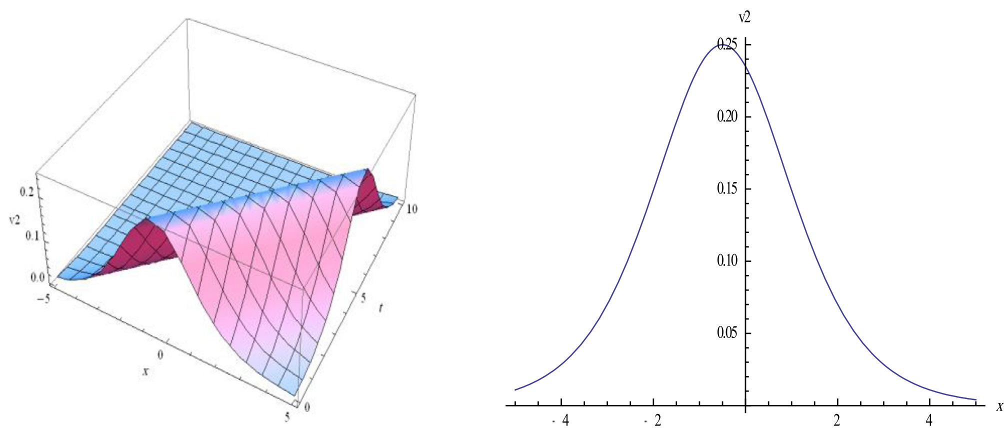

Fig. 6. The exact solution u_7 in Eq. (4) and its projection at $t = 0$ when $b_0 = 0.6,\ b_1 = 0.9,\ a_1 = 0.6, y = 0.5$.

$$u_5(x, y, t) = \frac{-b_1 + b_1\left(1 \pm e^{\left[x+y+\left(\frac{6a_1-6b_0+b_1}{b_1}\right)t\right]}\right) + (a_1 - b_0)\left(1 \pm e^{\left[x+y+\left(\frac{6a_1-6b_0+b_1}{b_1}\right)t\right]}\right)^2}{b_1\left(1 \pm e^{\left[x+y+\left(\frac{6a_1-6b_0+b_1}{b_1}\right)t\right]}\right)^2}. \tag{43}$$

Many other solutions are ommitted for convenience.

3.6. Numerical solutions for the exact solutions for the first equation of two members of integro-differential KP hierarchy equations

In this part, we illustrate the behavior of the obtained solution for the generalized Kudryashov method for the first equation of two members of integro-differential KP hierarchy equations which obtained in Section 3.5 when the parameter take some special choose (Fig. 4).

3.7. The generalized Kudryashov method for the second equation of two members of integro-differential KP hierarchy equations

In this subsection, we will discuss the exact traveling wave solutions of the second equation of integro-differential KP hierarchy equations (4) by the generalized Kudryashov method. Notice that the Eq. (4) contains integral operators of order one, two, and three. The traveling wave transformation (26) permits us to convert Eq. (4) to the following ODE:-

$$\left(\frac{5}{16} + k\right)V' + \frac{1}{16}V^{(5)} + \frac{30}{4}VV' + \frac{15}{2}V^2V' + \frac{5}{4}V'V'' + \frac{5}{4}\left(VV''\right)' + \frac{5}{8}V''' = 0. \tag{44}$$

By using the integration, we have

$$\left(\frac{5}{16} + k\right)V + \frac{1}{16}V^{(4)} + \frac{15}{4}V^2 + \frac{5}{2}V^3 + \frac{5}{8}V'^2 + \frac{5}{4}VV'' + \frac{5}{8}V'' + C_4 = 0, \tag{45}$$

where C_4 is the integration constant. Considering the homogeneous balance between the highest order derivative $V^{(4)}$ and nonlinear term V^3 in (45), we get:

$$N = M + 2. \tag{46}$$

Eq. (46) has infinity solutions, in the special case if $M = 1$ and $N = 3$, Eq. (8) reduces to

$$V(\xi) = \frac{a_0 + a_1Q + a_2Q^2 + a_3Q^3}{b_0 + b_1Q}. \tag{47}$$

Now substituting Eq. (47) into Eq. (45) and cleaning the denominator, we get a polynomial in $Q(\xi)$. Setting the coefficient of same power of $Q(\xi)$ to be zero, we get the system of nonlinear of algebraic equations. We solve the system of algebraic equation by using Maple to get the following results:

Case 1.

$$C_4 = \frac{3}{16}, \ \ a_0 = -\frac{3}{4}b_0, \ \ \ a_1 = 3b_0 - \frac{3}{4}b_1, \ \ a_2 = -3b_0 + 3b_1,$$
$$a_3 = -3\ b_1, \ \ k = \frac{43}{32}. \qquad (48)$$

When we substitute (49) into (47), we get:

$$V(\eta) = -\frac{3}{4}\left[\frac{4 - 4(1 \pm e^{\eta}) + (1 \pm e^{\eta})^2}{(1 \pm e^{\eta})^2}\right], \qquad (49)$$

where $\eta = x + y - \frac{43}{32}t$. Then the solution of the second equation of two members of integro-differential KP hierarchy equations takes the form:-

$$u_6(x,y,t) = -\frac{3}{4}\left[\frac{4 - 4\left(1 \pm e^{[x+y- \ \frac{43}{32}t]}\right) + \left(1 \pm e^{[x+y- \ \frac{43}{32}t]}\right)^2}{\left(1 \pm e^{[x+y- \ \frac{43}{32}t]}\right)^2}\right]. \qquad (50)$$

Case. 2

$$C_4 = \frac{1}{16}\frac{(\ a_1 - b_0)\left(80\ {a_1}^2 - 160a_1b_0 + 80a_1b_1 + 80{b_0}^2 - 80b_0b_1 + 11{b_1}^2\right)}{{b_1}^3},$$
$$a_0 = \frac{b_0(a_1 - b_0)}{b_1}, \ \ a_2 = -b_0 + b_1, \ \ a_3 = -b_1,$$
$$k = -\frac{1}{4}\frac{30\ {a_1}^2 - 60a_1b_0 + 35a_1b_1 + 30{b_0}^2 - 35b_0b_1 + 4{b_1}^2}{{b_1}^2}. \qquad (51)$$

Eqs. (51) and (47) lead to get:-

$$V(\xi) = \frac{-b_1 + b_1(1 \pm e^{\eta}) + (a_1 - b_0)(1 \pm e^{\eta})^2}{b_1(1 \pm e^{\eta})^2}, \qquad (52)$$

where

$$\xi = x + y + \left(\frac{1}{4}\frac{30\,{a_1}^2 - 60a_1b_0 + 35a_1b_1 + 30{b_0}^2 - 35b_0b_1 + 4{b_1}^2}{{b_1}^2}\right)t. \qquad (53)$$

Consequently the solution of the second equation of two members of integro-differential KP hierarchy equations takes the form:-

$$u_7(x,y,t) = \frac{-b_1 + b_1\left(1 \pm e^{[x+y-kt]}\right) + (a_1 - b_0)\left(1 \pm e^{[x+y-kt]}\right)^2}{b_1\left(1 \pm e^{[x+y-kt]}\right)^2}, \qquad (54)$$

where

$$k = -\frac{1}{4}\frac{30\ {a_1}^2 - 60a_1b_0 + 35a_1b_1 + 30{b_0}^2 - 35b_0b_1 + 4{b_1}^2}{{b_1}^2}. \qquad (55)$$

3.8. Numerical solutions for the exact solutions for the second equation of two members of integro-differential KP hierarchy equations

In this part, we illustrate the behavior of the obtained solutions for generalized Kudryashov method for the second equation of two members of integro-differential KP hierarchy equations in above Section 3.7 when the parameters take some of special chooses (Figs. 5 and 6).

4. Conclusion

The purpose of this study was to show that the exact solutions of some nonlinear PDEs obtained by the generalized Kudryashov method. We have obtained traveling wave solutions for some nonlinear integro-PDEs. The generalized Kudryashov method provides a powerful mathematical tool to obtain more exact solutions for the nonlinear integro PDEs in mathematical physics. The generalized Kudryashov method is effective, simple and convenient for nonlinear integro-PDEs. Moreover, we can say that GKM plays an important role to construct exact solutions of nonlinear integro-PDEs. Finally, we think that this method can be used to solve more complicated nonlinear partial differential equations.

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