# Topological approach to retrieve missing values in incomplete information systems

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## ARTICLE INFO

## ABSTRACT

We used some new results related to topology to retrieve the original values of the missing values in the incomplete information systems.

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## 1. Introduction

The theory that Pawlak discovered in the early 1980s [1], that called "rough set theory" is an extension of the classical set theory for the study of incomplete information systems. Many researchers have been made proposals for the generalizations and interpretations of rough sets [2–20]. The Pawlak theory is considered as a new mathematical tool to computer applications using also fuzzy set theory [21,22]. Nowdays, a lot of researchers are interested to generalize Pawlak theory in other fields of applications [23–27].

A lot of attractive generalizations to equivalence relation have been proposed such as tolerance relations [28], similarity relations [29], topological bases and subbases [30–34] and others [35–38]. Many exertions from researchers to use other approaches of the universe of discourse for starting a new generalizations of rough sets by coverings [39]. Others [40–45] tried to combine fuzzy sets with rough sets in a fruitful way by defining rough fuzzy sets and fuzzy rough sets. Furthermore, another group of researchers tried

to characterize a measure of roughness of a fuzzy set making use of the concept of rough fuzzy sets [46,47]. Some results of these generalizations are obtained about rough sets and fuzzy sets in [48–55]. Pawlak rough set theory generate a special type of topological space called the clopen space (the space have each open set is closed) and this topology is used in many fields of real life applications. The concept of topological rough set given by Wiweger [33] in 1989 is the first topological generalization of rough sets. In 1983 Abd El-Monsef et al. [56] introduced the concept of $\beta$-open sets that can used widely in the topological generalizations of rough sets. In 2006 Hatir and Noiri [57] introduced the concept of $\delta\beta$-open sets that we used in this paper to introduce a solution of the problem of the missing values in information systems.

Rokach in [58] considered the idea of ensemble methodology to build a predictive model by integrating multiple models. In [59] Hamid Parvin and others have been provided a good way to have a near-optimal classifying system for any problem depend on a technique called ensemble learning. This technique became one of the most challenging problems in classifier ensemble that introducing a suitable ensemble of base classifiers. Parvin with others in [60] have been proposed an ensemble based approach for feature selection. They aimed at overcoming the problem of pa-

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rameter sensitivity of feature selection approaches. The effectiveness of non-weighing-based sampling technique, comparing the efficacy of sampling with and without replacement, in conjunction with several consensus algorithms have been invested in [61]. In addition, experimental results have shown improved stability and accuracy for clustering structures obtained via bootstrapping, subsampling, and boosting techniques. In [62] Ahmad and others have been presented a clustering algorithm based on $k$-mean paradigm that works well for data with mixed numeric and categorical features. They have been proposed new cost function and distance measure based on co-occurrence of values.

In this paper, we introduced a topological method to retrieve missing values in the incomplete informations. Short illustration on missing values are given in Section 2. In Section 3 we give the properties of rough set theory and their approximations. Section 4 gives the basic concepts of incomplete information systems. In Section 5 we introduce the basics of near topological approximations. The main aim of Section 6 is that how we make use of near topological approximations to retrieve missing values in incomplete information system. Section 7 shows the phsolophy that retrieve missing values. Conclusions of the work are given in Section 8.

## 2. Basics of missing values

In incomplete information systems, absent attribute values befall when no information value is stored for some characteristics. Lost data are a mutual occurrence and can have the noteworthy result of the assumptions that drained from the information.

For treatment, absent attribute values, we may be using one of the following:

1. Erasing cases with missing attribute values.
2. Replacing a lost attribute value of the greatest common value of that attribute.
3. Transmission all likely values for the missing attribute value.
4. Relieving a lost attribute value by the mean for numerical attributes.
5. Transfer to a lost attribute value the corresponding value taken from the closest case.
6. Relieving a lost attribute value by a new value computed from a new data set.
7. Seeing the original attribute as a decision.

We need some assumptions to find out the missing values some of them are:

- All result attribute values are well defined.
- The elements of the universe that have lost values have the same accidental in our classification.
- The field of every disorder attribute is completely definite.
- The dataset is reliable.

Noises of incomplete decision table represented objects $U$ and columns represent attributes $At$. Qualities are dividing to main parts condition $C$ and choice attributes. On this table, a function that maps the direct product of $U$ and $C$ into the set of all attribute values is called an information function and denoted by $f$.

Classical rough set theory used an equivalence relation as a tool to deal with the uncertain data given in an incomplete information system. This relation for any subset of the conditional attributes is defined as follows:

$$R = \{(x, y) \in U \times U : \forall a \in X \ f(x, a) = f(y, a)\}, X \subseteq C.$$

Rough elementary sets are generated using the partition $U/R$.

Two rough approximations are defined using the relation $R$ for any subset of objects $A \subseteq U$ as follows: The first one is the lower approximation $\underline{R}(A) = \cup \{G \in R/U : G \subseteq A\}$. The second one is the upper approximation $\overline{R}(A) = \cup \{G \in R/U : G \cap A \neq \varphi\}$.

The boundary region of $A$, $A^b = \overline{R}(A) - \underline{R}(A)$ is the set of fundamentals for which there is a hesitation about their transmission to $A$. Currently, by means of new topological strategies the boundary region rotten into many districts that well known and smaller.

Reductions of information system are all subsets $X$ of attributes such that if it is a minimal subset $X' \subset X \subseteq C$, then it must keep the quality of classification unmoved.

We make use of the decision rules generated from the incomplete information system to construct a knowledge base for this system. A decision rule is defined by the equation:

$$\underset{a \in X}{\wedge}(f(a(x), v)) \to (D(x), k) \ \text{where} \ v \in V_a \ \text{and} \ k \in V_D$$

The straightforward condition is usually resembled to as an attribute value pair $(a(x), v)$ and decision value pair $(D(x), k)$. Each decision rule $r$ has two main parts: $r_s$ and $r_t$ the condition and decision parts congruently.

## 3. Topological spaces and their approximations

Usually a topological space is defined to be a set $U$ and a family $\tau$ of subsets of $U$ satisfying the following conditions:

(1) $\varphi, U \in \tau$
(2) Random unions of open subsets of $U$ is being a member in $\tau$.
(3) Limited intersections of open subsets of $U$ is being a member in $\tau$.

$cl_\tau(A) = \{F \subseteq U : A \subseteq F, F^c \in \tau\}$ is the closure of a subset $A \subset U$. $int_\tau(A) = \bigcup\{G \subseteq X : G \subseteq A, G \in \tau\}$ is the interior of a subset $A \subseteq U$. $b(A) = cl_\tau(A) - int_\tau(A)$ is the boundary of a subset $A \subseteq U$.

Now we will make use of the relation $R$ to define a topology $\tau_R$ on the universe of dicourse. $\tau_R$ has the neighborhood $R_x = \{y : (x, y) \in R, x, y \in U\}$. We define two topological notion as follows:

$cl_{\tau_R}(A) = \{x \in U : R_x \cap A \neq \varphi\}$ and $int_{\tau_R}(A) = \{x \in U : R_x \subseteq A\}$, called the closure approximation and interior approximation of $A \subseteq U$ respectively. $P_{\tau_R}(A) = int_{\tau_R}(A)$ the positive region, $N_{\tau_R}(A) = U - cl_{\tau_R}(A)$ the negative region and $b_{\tau_R}(A) = cl_{\tau_R}(A) - int_{\tau_R}(A)$ the boundary region of $A$, respectively.

The degree of wholeness can also be characterized by the accurateness measure, in which $|R|$ represents the cardinality of a subset $A \subseteq U$ as follows:

$$\alpha_{\tau_R}(A) = \frac{|int_{\tau_R}(A)|}{|cl_{\tau_R}(A)|}, \ A \neq \varphi.$$

## 4. Generalizations of approximations

The $\delta$-closure of a subset $A \subseteq U$ of the topology $\tau$, is defined by $cl_\tau^\delta(A) = \{x \in X : A \cap int_\tau(cl_\tau(G)) \neq \varphi, G \in \tau, x \in G\}$. $A$ is $\delta$-closed set if $A = cl_\tau^\delta(A)$. The complement of a $\delta$-closed set is $\delta$-open, such that $int_\tau^\delta(A) = U \setminus cl_\tau^\delta(U \setminus A)$.

A subset $A$ is $\Omega$-open if $A \subseteq cl_\tau(int_\tau(cl_\tau^\delta(A)))$. The family of all $\Omega$-open sets of $U$ is denoted by $\Omega O(U)$. The complement of $\Omega$-open set is called $\Omega$-closed set. The family of $\Omega$-closed sets is denoted by $\Omega C(U)$.

The pair $(U, R_\Omega)$ is called a $\Omega$-approximation space where $R_\Omega$ is a general relation used to get a subbase for a topology $\tau_{R_\Omega}$ on $U$ which generates the class $\Omega O(U)$. $\Omega$-interior and $\Omega$-closure of any subset $A \subseteq U$ is defined as:

$$int_{R_\Omega}(A) = \cup\{G \in \Omega O(U) : G \subseteq A\}, \ cl_{R_\Omega}(A) = \cap\{F \in \Omega C(U) : F \supseteq A\}.$$

We call $P_{\tau_{R_\Omega}}(A) = int_{\tau_{R_\Omega}}(A)$ the confident region of $A$, $N_{\tau_{R_\Omega}}(A) = U - cl_{\tau_{R_\Omega}}(A)$ denote the undesirable region of $A$ and the set $b_{\tau_{R_\Omega}}(A) = cl_{\tau_{R_\Omega}}(A) - int_{\tau_{R_\Omega}}(A)$ denote the frontier region of $A$.

The grade of wholeness using $\Omega$-accuracy amount is branded as follows:

$$\alpha_{R_\Omega}(A) = \frac{|\mathrm{int}_{R_\Omega}(A)|}{|cl_{R_\Omega}(A)|}, \ A \neq \varphi$$

## 5. Properties of $\Omega$–approximations

In this section, we introduce some propositions to discuss the main possessions of the organization regions $P_{\tau_{R_\Omega}}(A)$, $N_{\tau_{R_\Omega}}(A)$ and $b_{\tau_{R_\Omega}}(A)$.

**Theorem 5.1.** *For $A \subset U$, we have,*

(1) $b_{\tau_{R_\Omega}}(A) \cap P_{\tau_{R_\Omega}}(A) = \varphi$,
(2) $b_{\tau_{R_\Omega}}(A) \cap N_{\tau_{R_\Omega}}(A) = \varphi$,
(3) $cl_{\tau_{R_\Omega}}(A) = b_{\tau_{R_\Omega}}(A) \cup P_{\tau_{R_\Omega}}(A)$
(4) $P_{\tau_R}(A)$, $N_{\tau_R}(A)$ and $b_{\tau_R}(A)$ form a partition of U,
(5) $P_{\tau_{R_\Omega}}(A)$, $P_{\tau_R} - P_{\tau_{R_\Omega}}(A)$, $A - P_{\tau_R}(A)$, $cl_{\tau_R} - A$, $cl_{\tau_{R_\Omega}}(A) - cl_{\tau_R}$ and $U - cl_{\tau_{R_\Omega}}(A)$ form modification partition of U.

**Proof.**

(1) $b_{\tau_{R_\Omega}}(A) \cap P_{\tau_{R_\Omega}}(A) = (cl_{\tau_{R_\Omega}}(A) - \mathrm{int}_{\tau_{R_\Omega}}(A)) \cap \mathrm{int}_{\tau_{R_\Omega}}(A) = \varphi$
(2) $b_{\tau_{R_\Omega}}(A) \cap N_{\tau_{R_\Omega}}(A) = (cl_{\tau_{R_\Omega}}(A) - \mathrm{int}_{\tau_{R_\Omega}}(A)) \cap (U - cl_{\tau_{R_\Omega}}(A)) = \varphi$
(3) $b_{\tau_{R_\Omega}}(A) \cup P_{\tau_{R_\Omega}}(A) = (cl_{\tau_{R_\Omega}}(A) - \mathrm{int}_{\tau_{R_\Omega}}(A)) \cup \mathrm{int}_{\tau_{R_\Omega}}(A) = cl_{\tau_{R_\Omega}}(A)$
(4) and (5) are straight, since these regions are disjoint.

**Theorem 5.2.** *For any two subsets A, $B \subseteq U$, the boundary region has the following properties:*

(1) $b_{\tau_{R_\Omega}}(U) = \varphi$,
(2) $b_{\tau_{R_\Omega}}(A) = b_{\tau_{R_\Omega}}(U - A)$,
(3) $b_{\tau_{R_\Omega}}(b_{\tau_{R_\Omega}}(A)) \subset b_{\tau_{R_\Omega}}(A)$,
(4) $b_{\tau_{R_\Omega}}(A \cap B) \subset b_{\tau_{R_\Omega}}(A) \cup b_{\tau_{R_\Omega}}(B)$

**Proof.**

(1) & (2) are obvious.
(3) $b_{\tau_{R_\Omega}}(b_{\tau_{R_\Omega}}(A)) = b_{\tau_{R_\Omega}}(cl_{\tau_{R_\Omega}}(A) \cap cl_{\tau_{R_\Omega}}(U - A))$
$= cl_{\tau_{R_\Omega}}(cl_{\tau_{R_\Omega}}(A) \cap cl_{\tau_{R_\Omega}}(U - A)) \cap (U - cl_{\tau_{R_\Omega}}(cl_{\tau_{R_\Omega}}(A) \cap cl_{\tau_{R_\Omega}}(U - A))$
$\subset cl_{\tau_{R_\Omega}}(A) \cap cl_{\tau_{R_\Omega}}(U - A) = b_{\tau_{R_\Omega}}(A)$
(4) $b_{\tau_{R_\Omega}}(A \cap B) = cl_{\tau_{R_\Omega}}(A \cap B) \cap cl_{\tau_{R_\Omega}}(U - (A \cap B))$
$= cl_{\tau_{R_\Omega}}(A \cap B) \cap (cl_{\tau_{R_\Omega}}(U - A) \cup cl_{\tau_{R_\Omega}}(U - B))$
$= (cl_{\tau_{R_\Omega}}(A \cap B) \cap (cl_{\tau_{R_{\delta\beta}}}(U - A)) \cup (cl_{\tau_{R_\Omega}}(A \cap B) \cap cl_{\tau_{R_\Omega}}(U - B))$
$\subset (cl_{\tau_{R_\Omega}}(A) \cap (cl_{\tau_{R_\Omega}}(U - A)) \cup (cl_{\tau_{R_\Omega}}(B) \cap cl_{\tau_{R_\Omega}}(U - B))$
$\subset b_{\tau_{R_\Omega}}(A) \cup b_{\tau_{R_\Omega}}(B)$

**Theorem 5.3.** *For any two subsets A, $B \subseteq U$, the negative region has the following properties:*

(1) $N_{\tau_{R_\Omega}}(\varphi) = U$,
(2) $N_{\tau_{R_\Omega}}(A) = P_{\tau_{R_\Omega}}(U - A)$,
(3) $N_{\tau_{R_\Omega}}(A) \cap A = \varphi$,
(4) $N_{\tau_{R_\Omega}}(U - N_{\tau_{R_\Omega}}(A)) = N_{\tau_{R_\Omega}}(A)$,
(5) $N_{\tau_{R_\Omega}}(A \cup B) \subset N_{\tau_{R_\Omega}}(A) \cup N_{\tau_{R_\Omega}}(B)$,
(6) $N_{\tau_{R_\Omega}}(A \cap B) \supset N_{\tau_{R_\Omega}}(A) \cap N_{\tau_{R_\Omega}}(B)$

**Proof.** (1)–(4) are obvious.

(5) $N_{\tau_{R_\Omega}}(A \cup B) = U - cl_{\tau_{R_\Omega}}(A \cup B)$
$\subset U - (cl_{\tau_{R_\Omega}}(A) \cup cl_{\tau_{R_\Omega}}(B))$
$\subset (U - cl_{\tau_{R_\Omega}}(A)) \cup (U - cl_{\tau_{R_\Omega}}(B))$
$\subset N_{\tau_{R_\Omega}}(A) \cup N_{\tau_{R_\Omega}}(B)$

(6) $N_{\tau_{R_\Omega}}(A) \cap N_{\tau_{R_\Omega}}(B) = (U - cl_{\tau_{R_\Omega}}(A)) \cap (U - cl_{\tau_{R_\Omega}}(B))$
$= U - (cl_{\tau_{R_\Omega}}(A) \cup cl_{\tau_{R_\Omega}}(B))$
$\subset U - cl_{\tau_{R_\Omega}}(A \cap B)$
$= N_{\tau_{R_\Omega}}(A \cap B)$

## 6. Phsolosophy that retrieve missing values

In this section, we used the $\Omega$-topological technique to generated reducts and decision rules from incomplete suggestion systems.

To describe a topology of the universe of the incomplete information system $IS = (U, At)$ first, we make use of the condition attributes to define the following three binary relations:

For any $b \in At$ and $\forall x, y \in U$:

$R_1 = \{(x, y) : f_b(x) = f_b(y), f_b(x) \neq *, f_b(y) \neq *\}$
$R_2 = \{(x, y) : f_b(x) = f_b(y) = *, f_D(x) = f_D(y)\}$
$R_3 = \{(x, y) : f_b(x) = f_b(y), f_b(x) \neq *, f_b(y) = *$,
  or $f_b(x) = *, f_b(y) \neq *, f_D(x) = f_D(y)\}$

Second, we make use of the blocks of the above relations to construct the class $S_b(x) = \{R_i(x) : x \in U, i = 1, 2, 3\}$, where $R_i(x) = \{y : xR_iy\}$. The arbitrary intersections of members of the class $S_b(x)$ is the elementary blocks of a topological base $\beta_b$. For a subset of condition attributes $B \subseteq At$ the class of all elementary sets of the topological base relation is denoted by $\beta_B$, where $\beta_B = \bigcap_{b \in B} \beta_b$.

The base $\beta_b$ creates a topology on $U$ by denoted $\tau_b$. For a subset of state attributes $B \subseteq At$ the base $\beta_B$ construct the topology $\tau_B$ of all intersections of topologies $\tau_b$, for all $b \in B$. The conclusion topology $\tau_D$ is the topology generated by the elementary sets $U/R_D$ as a base ($R_D$ is discernibility relation on the decision characteristic).

The elements of the class $\beta_B$ are the basis for redefining the concepts of lower and upper approximations of a set $X \subseteq U$ using the set of attributes $B \subseteq At$. $\Omega$-lower approximation and $\Omega$-upper approximation of any non-empty subset $A$ of $U$ are correspondingly:

$$\mathrm{int}_\Omega(A) = \cup\{G \in \Omega O(U) : G \subseteq A\}, \ cl_\Omega(A) = \cap\{F \in \Omega C(U) : F \supseteq A\},$$

We can divide the elements of the construction using the condition attribute $b \in At$ to 6 regions of any nonempty subset $A$ of $U$:

1. $P_{\tau_{R_\Omega}}(A)$,
2. $P_{\tau_R}(A) - P_{\tau_{R_\Omega}}(A)$,
3. $A - P_{\tau_R}(A)$,
4. $cl_{\tau_R}(A) - A$,
5. $cl_{\tau_{R_\Omega}}(A) - cl_{\tau_R}(A)$,
6. $U - cl_{\tau_{R_\Omega}}(A)$.

We call these six regions the compensation regions.

We will characterize the degree of wholeness in incomplete information system using $\Omega$-accuracy calculates that before defined by:

$$\alpha_{R_\Omega}(A) = \frac{|\underline{R}_\Omega(A)|}{|\overline{R}_\Omega(A)|}, \ \text{where} \ A \neq \varphi$$

The strength of mind line of attack of the reducts and the center of our system are as follows:

1. Apply the three relations $R_i, i = 1, 2, 3$ for each condition attribute $b \in At$ and for each subset of condition attributes $B \in At$.
2. Control the topology $\tau_b$ for each condition attribute $b \in At$ and the topology $\tau_B$ for each subset of condition attributes $B \in At$.

**Table 1**
Cars description.

| Car | Body | Style | Color | Decision (D) |
|---|---|---|---|---|
| $C_1$ | Good | * | Bad | Accepted (A) |
| $C_2$ | Very good | Accept | Good | Accepted (A) |
| $C_3$ | * | Reject | Bad | Rejected (R) |
| $C_4$ | Good | Accept | Good | Accepted (A) |
| $C_5$ | Good | * | Good | Rejected (R) |
| $C_6$ | * | Accept | Bad | Rejected (R) |
| $C_7$ | Fair | Reject | Good | Rejected (R) |
| $C_8$ | * | Accept | * | Accepted (A) |

3. Regulate the six separation regions of each constricted topology.
4. Subsets of condition attributes that have the same six separation regions are the reducts of the system.
5. The intersection of the reducts sets are the core of the system.

## 7. Illustration examples

We will illustrate the method of retrieve the lost values in incomplete tables by example. We suggested dividing the attribute values to positive and negative values. With "Car Table", Table 1, positive and negative values of "Body" attribute are $V_{Body}^+ = \{Good, very\ Good\}$ and $V_{Body}^- = \{Fair\}$. By the same way, positive and negative values of "Style" attribute are $V_{Style}^+ = \{Accept\}$ and $V_{Style}^- = \{Reject\}$. The attribute color has the two parts $V_{Color}^+ = \{Good\}$ and $V_{Color}^- = \{Bad\}$. Decision attribute is divided into two opinions either Accepted (A) or Rejected (R), then we can write $V_{Decision}^+ = \{Accepted\}$ and $V_{Decision}^- = \{Rejected\}$.

By returning to Table 1, we have:

- $f(C_3, Body) = *$, this car has a negative decision i.e., $f(C_3, D) = R$, if we get the residue information of that row of this car we will find that $f(C_3, Style) = R$ and $f(C_3, Color) = Bad$. Then the certain decision here with respect to the value of $C_3$ conforming to the attribute "Body" is "Fair" i.e., $f(C_3, Body) \in V_{Body}^-$ and $f(C_3, Body) = Fair$.

- $f(C_6, Body) = *$, this car has a negative decision i.e., $f(C_6, D) = R$, if we get the residue information of that row of this car we will find that $f(C_6, Style) = Accept$ and $f(C_6, Color) = Bad$. Then the certain decision here with respect to the value of $C_6$ conforming to the attribute "Body" is Fair i.e., $f(C_6, Body) \in V_{Body}^-$ and $f(C_6, Body) = Fair$, there is no possible choice.

- $f(C_1, Style) = *$, this car has a positive decision i.e., $f(C_1, D) = A$, if we get the residue information of that row of this car we will find that $f(C_1, Body) = Good$ and $f(C_1, Color) = Bad$. Then the certain decision here with respect to the value of $C_1$ conforming to the attribute "Style" is yes i.e., $f(C_1, Style) \in V_{Style}^+$ or $f(C_1, Style) = Accept$, and there is no possible choice.

- $f(C_5, Style) = *$, this car has a negative decision i.e., $f(C_5, D) = R$, if we get the residue information of that row of this car we will find that $f(C_5, Body) = Good$ and $f(C_5, Color) = Good$. Then the certain decision here with respect to the value of $C_5$ conforming to the attribute "Style" is no i.e., $f(C_5, Style) \in V_{Style}^-$ or $f(C_5, Style) = R$, and there is no possible choice.

Rendering to the above organization, we can obtain the lost values of the table.

## 8. Suppositions

Grounded on topological spaces, we have given an underlying philosophy to elucidate how collations achieved in incomplete information systems. In our classical, one may exhibit the correspondence relation by inquiring an information system by means of uncertain descriptions.

In this work, we make use of the near topological notations to define a new classification of the lost value of the incomplete information system. Using this technique, we classified the given values of each attributes into positive and negative values.

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## References

[1] Z. Pawlak, Rough sets, Int. J. Comput. Inform. Sci. 11 (1982) 341–356.
[2] H.M. Abu-Donia, Comparison between different kinds of approximations by using a family of binary relations, Knowl. Based Syst. 21 (2008) 911–919.
[3] H.M. Abu-Donia, A.S. Salama, $\beta$-approximation spaces, J. Hybrid Comput. Res. 1 (July(2)) (2008).
[4] H.M. Abu-Donia, A.A. Nasef, E.A. Marei, Finite information systems, Appl. Math. Inf. Sci. 1 (1) (2007) 13–21.
[5] M. Chuchro, On rongh sets in topological Boolean algebras, Rough Sets Knowl. Discov. Baff AB (1993) 157–160.
[6] J. James, F.James Alpigini, Peters, andrzej skowron and ning zhong, rough set elements, rough sets and current trends in compuring, in: 5th Int. conf. Malvern, PA, USA, Proc. Springer, 2002, pp. 12–16.
[7] G. Liu, Y. Sai, A comparison of two types of rough sets induced by coverings, Int. J. Approx. Reason. 50 (2009) 521–528.
[8] Y. Ouyang, Z. Wang, H. Zhang, On fuzzy rough sets based on tolerance relations, Inf. Sci. 180 (2010) 532–542.
[9] K. Qin, J. Yang, Z. Pei, Generalized rough sets based on reflexive and transitive relations, Inf. Sci. 178 (2008) 4138–4141.
[10] A.S. Salama, Topological solution of missing attribute values problem in incomplete information tables, Inf. Sci. 180 (2010) 631–639.
[11] A. Wasilewska, L. Vigneron, On generalized rough sets, in: Proc. of 5th Work Shop on Rough Sets and Soft of Compiding at the 3rd Joint Conf. On Information Sci., 1997, pp. 20–24.
[12] U. Wybraniec-Skardowska, On a generalization of approximation space, Bull. Polish Acad. Sci. 37 (1989) 51–61.
[13] Y.Y. Yao, T.Y. Lin, Generalization of rough sets using modal logic, Intell. Automat. Soft Comput. 2 (1996) 103–120.
[14] Y.Y. Yao, Two views of the theory of rough sets in finite Universes, Int. J. Approx. Reason. 15 (1997) 291–317.
[15] Y.Y. Yao, Generalized rough set models, in: Rough Sets in Knowledge Discovery, Physica-verlag, Hidelberg, 1998, pp. 286–318.
[16] Y.Y. Yao, On generalized rough set theory, rough sets, fuzzy sets, in: Data Mining and Granular computing" Proc. of the 9th Int. Conf, 1999, pp. 44–51.
[17] D. Yu, Q. Hu, C. Wu, Uncertainty measures for fuzzy relations and their applications, Appl. Soft Comput. 7 (2007) 1135–1143.
[18] H.P. Zhang, Y. Ouyang, Z. Wangc, Note on generalized rough sets based on reflexive and transitive relations, Inf. Sci. 179 (2009) 471–473.
[19] W. Zhu, Relationship between generalized rough sets based on binary relation and covering, Inf. Sci. 179 (2009) 210–225.
[20] W. Zhu, Generalized rough sets based on relations, Inf Sci 177 (2007) 4997–5011.
[21] X. Wang, EricC.C. Tsang, S. Zhao, D. Chen, D.S. Yeung, Learning fuzzy rules from fuzzy samples based on rough set technique, Inf. Sci. 177 (2007) 4493–4514.
[22] L.A. Zadeh, Fuzzy sets, Inf. Cont. 8 (1965) 338–353.
[23] Z. Bonikowski, Algebraic structures of rough sets, in: W. Ziarko (Ed.), Rough Sets, Fuzzy Sets and Knowledge Discovery, Springer, London, 1994, pp. 243–247.
[24] E. Bryniaski, A calculus of rough sets of the first order, Bull. Polish Acad. Sci. 16 (1998) 71–77.
[25] G. Cattaneo, Abstract approximation spaces for rough theories, in:, in: L. Polkowski, A. Skowron (Eds.), Rough Sets in Knowledge Discovery 1: Methodology and Applications, Physica-Verlag, Heidelberg, 1998, pp. 59–98.
[26] N.E. El-Tayar, R.S. Tsai, P.A. Carruptand, B. Testa, J. Chem. Soc. Perkin Trans. 2 (1992) 79–84.
[27] Y.Y. Yao, Constructive and algebraic methods of theory of rough sets, Inf. Sci. 109 (1998) 21–47.
[28] A. Skowron, J. Stepaniuk, Tolerance approximation spaces, Fund. Inf. 27 (1996) 245–253.
[29] R. Slowinski, D. Vanderpooten, A generalized definition of rough approximations based on similarity, IEEE Trans. Knowl. Data Eng. 12 (2000) 331–336.
[30] E.F. Lashin, A.M. Kozae, A.A. Abo Khadra, T. Medhat, Rough set theory for topological spaces, Int. J. Approx.Reason. 40 (2005) 35–43.
[31] K. Qin, Zheng Pei, On the topological properties of fuzzy rough sets, Fuzzy Sets Syst. 151 (2005) 601–613.
[32] A. Wasilewska, Topological rough algebras, in: T.Y. Lin, N. Cercone (Eds.), Rough Sets and Data Mining, Kluwer Academic Publishers, Boston, 1997, pp. 425–441.

[33] A. Wiweger, On topological rough sets, Bull. Pol. Acad. Math. 37 (1989) 89–93.
[34] W. Zhu, Topological approaches to covering rough sets, Inf. Sci. 177 (2007) 1499–1508.
[35] Z. Bonikowski, E. Bryniarski, U. Wybraniec, Extensions and intentions in the rough set theory, Inf. Sci. 107 (1998) 149–167.
[36] M. Kryszkiewicz, Rough set approach to incomplete information systems, Inf. Sci. 112 (1998) 39–49.
[37] J.A. Pomykala, Approximation operations in approximation space, Bull. Polish Acad. Sci. 9-10 (1987) 653–662.
[38] Y.Y. Yao, Relational interpretations of neighborhood operators and rough set approximation operators, Inf. Sci. 111 (1998) 239–259.
[39] T. Deng, Y. Chen, W. Xu, Q. Dai, A novel approach to fuzzy rough sets based on a fuzzy covering, Inf. Sci. 177 (2007) 2308–2326.
[40] M. Banerjee, S.K. Pal, Roughness of a fuzzy set, Inf. Sci 93 (1995) 235–246.
[41] R. Biswas, On rough sets and fuzzy rough sets, Bull. Polish Acad. Sci. Math. 42 (1992) 343–349.
[42] C. Degang, Y. Wenxia, L. Fachao, Measures of general fuzzy rough sets on a probabilistic space, Inf. Sci. 178 (2008) 3177–3187.
[43] G. Liu, Axiomatic systems for rough sets and fuzzy rough sets, Int. J. Approx. Reason 48 (2008) 857–867.
[44] A.M. Rolka, L. Rolka, Fuzzy rough approximations of process data, Int. J. Approx. Reason 49 (2008) 301–315.
[45] S. Zhao, E.C.C. Tsang, On fuzzy approximation operators in attribute reduction with fuzzy rough sets, Inf. Sci. 178 (2008) 3163–3176.
[46] R. Biswas, On rough fuzzy sets, Bull. Polish Acad. Sci. Math. 42 (1994) 352–355.
[47] D. Dubois, H. Prade, Rough fuzzy sets and fuzzy rough sets, Int. J. Gen. Syst. 17 (1990) 191–208.
[48] K. Chakrabarty, R. Biswas, S. Nanda, Fuzziness in rough sets, Fuzzy Sets Syst. 110 (2000) 247–251.
[49] X. Chen, Q. Li, Construction of rough approximations in fuzzy setting, Fuzzy Sets Syst. 158 (2007) 2641–2653.
[50] Z. Gong, B. Sun, D. Chen, Rough set theory for the interval-valued fuzzy information systems, Inf. Sci. 178 (2008) 1968–1985.
[51] T.J. Li, Y. Leung, W.X. Zhang, Generalized fuzzy rough approximation operators based on fuzzy coverings, Int. J. Approx. Reason. 48 (2008) 836–856.
[52] G. Liu, Generalized rough sets over fuzzy lattices, Inf. Sci. 178 (2008) 1651–1662.
[53] A. Nakamura, Fuzzy rough sets, Note Multiple Valued Logic Jpn 9 (1988) 1–8.
[54] S. Nanda, S. Majumda, Fuzzy rough sets, Fuzzy Sets Syst. 45 (1992) 157–160.
[55] Y. Yang, Chris Hinde, A new extension of fuzzy sets using rough sets: R-fuzzy sets, Inf. Sci. 180 (2010) 354–365.
[56] M.E. Abd El-Monsef, S.N. El-Deeb, R.A. Mahmoud, $\beta$-open sets and $\beta$-continuous mappings, Bull. Fac. Assiut Uni. 12 (1) (1983) 77–90.
[57] E. Hatir, T. Noiri, Decompositions of continuity and complete continuity, Acta Math. Hungary 11 (3) (2006) 281–287 4.
[58] L. Rokach, Ensemble-based classifiers, Artif. Intell. Rev. 33 (1) (2010) 1–39.
[59] H. Parvin, M. MirnabiBaboli, H. Alinejad-Rokny, Proposing a classifier ensemble framework based on classifier selection and decision tree, Eng. Appl. Artif. Intell. 37 (2015) 34–42.
[60] H. Parvin, H. Alinejad-Rokny, M. Asadi, An ensemble based approach for feature selection, Aust. J. Basic Appl. Sci. 5 (9) (2011) 1153–4463.
[61] H. Parvin, B. Minaei-Bidgoli, H. Alinejad-Rokny, W.F. Punch, Data weighing mechanisms for clustering ensembles, Comput. Electric.s Eng. 39 (5) (2013) 1433–1450.
[62] A. Ahmad, L. Dey, A k-mean clustering algorithm for mixed numeric and categorical data, Data Knowl. Eng. 63 (2) (2007) 503–527.