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Original Article

A note on “On generalizing covering approximation space”

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ABSTRACT

We show that [Lemma 3.3, p. 538] which was introduced in [1] is incorrect in general, by giving counter examples. Consequently, [Proposition 3.2, p. 539] is also incorrect. Moreover, the correction form of the incorrect results in [1] is presented.

1. Introduction

The original rough set theory introduced by Pawlak was based on an equivalence relation on a finite universe X. For practice use, there have been some extensions on Pawlak's original concept. One extension is to replace the equivalence relation by an arbitrary binary relation [2,3]. The other direction is to study rough set via topological method [4–6]. A covering of a universe is a generalization of the concept of partition of the universe. Rough sets based on coverings instead of partitions have been studied in several papers [7–10].

2. Preliminaries

The aim of this section is to present the basic concepts and properties of rough sets.

Definition 2.1 [1]**.** Let $U \neq \phi$ be a finite set and R be a binary relation on U. Two different coverings for U induced from the binary relation R are defined as follows:

1. Right covering (briefly, r-cover): $\mathcal{C}_r = \{xR : \forall x \in U \text{ and } U = \cup_{x\in U} xR\}$.
2. Left covering (briefly, l-cover): $\mathcal{C}_l = \{Rx : \forall x \in U \text{ and } U = \cup_{x\in U} Rx\}$.

Definition 2.2 [1]**.** Let $U \neq \phi$ be a finite set, R be a binary relation on U and $\mathcal{C}_n$ be $n-$cover of U associated to R, where $n \in \{r, l\}$. Then, the triple $\langle U, R, \mathcal{C}_n\rangle$ is called a $\mathcal{G}_n-$covering approximation space (briefly, $\mathcal{G}_n - CAS$).

Definition 2.3 [1]**.** Let the triple $\langle U, R, \mathcal{C}_n\rangle$ be a $\mathcal{G}_n - CAS$. For every element $x \in U$, four different neighborhoods $N_j(x)$, where $j \in \{r, l, i, u\}$ are defined as follows:

1. r-neighborhood: $N_r(x) = \cap\{K \in \mathcal{C}_r : x \in K\}$.
2. l-neighborhood: $N_l(x) = \cap\{K \in \mathcal{C}_l : x \in K\}$.
3. i-neighborhood: $N_i(x) = N_r(x) \cap N_l(x)$.
4. u-neighborhood: $N_u(x) = N_r(x) \cup N_l(x)$.

Definition 2.4 [1]**.** Let the triple $\langle U, R, \mathcal{C}_n\rangle$ be a $\mathcal{G}_n - CAS$ and $A \subseteq U$. For each $j \in \{r, l, i, u\}$, the $j-$lower and the $j-$upper approximations of A are defined respectively as follows:

$$\underline{R}_j(A) = \{x \in A : N_j(x) \subseteq A\}, \tag{1}$$

$$\overline{R}_j(A) = \{x \in U : N_j(x) \cap A \neq \phi\}. \tag{2}$$

Proposition 2.1 [1]**.** *Let the triple $\langle U, R, \mathcal{C}_n\rangle$ be a $\mathcal{G}_n - CAS$ and $A \subseteq U$. For each $j \in \{r, l, i, u\}$, and A, $B \subseteq U$, the following statements hold:*

1. $\underline{R}_j(A) \subseteq A \subseteq \overline{R}_j(A)$.
2. *If $A \subseteq B$, then $\underline{R}_j(A) \subseteq \underline{R}_j(B)$ and $\overline{R}_j(A) \subseteq \overline{R}_j(B)$.*

3. Counter-example

In this section, we point out where the errors occur in [1] and then give counter examples to confirm our claim. Finally, the correction form of these errors is presented.

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Firstly, in [1, Lemma 3.3, p. 538], the authors proved that for the triple $\langle U, R, \mathcal{C}_n\rangle$ is a $\mathcal{G}_n - CAS$ and for each $j \in \{r, l, i, u\}$, if $x \in N_j(y)$, then $N_j(x) \subseteq N_j(y)$.

The following example shows that if $x \in N_u(y)$, then $N_u(x) \nsubseteq N_u(y)$.

Example 3.1. Let $U = \{a, b, c, d\}$ be a universe set, $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ be an ideal on U and $R = \{(a, a), (a, b), (a, d), (b, a), (b, d), (c, a), (c, c), (c, d), (d, a), (d, d)\}$ be a binary relation on U. Then, $aR = \{a, b, d\}$, $bR = \{a, d\}$, $cR = \{a, c, d\}$ and $dR = \{a, d\}$. Thus, $N_r(a) = \{a, d\}$, $N_r(b) = \{a, b, d\}$, $N_r(c) = \{a, c, d\}$ and $N_r(d) = \{a, d\}$. Similarly, $Ra = Rd = U$, $Rb = \{a\}$, and $Rc = \{c\}$. Thus, $N_l(a) = \{a\}$, $N_l(b) = N_l(d) = U$, and $N_l(c) = \{c\}$. Hence, $N_u(a) = \{a, d\}$, $N_u(b) = N_u(d) = U$, and $N_u(c) = \{a, c, d\}$. It's clear that $d \in N_u(a)$, but $N_u(d) \nsubseteq N_u(a)$. Additionally, $d \in N_u(c)$, but $N_u(d) \nsubseteq N_u(c)$.

The following lemma is the correction form of [Lemma 3.3, p. 538] in [1].

Lemma 3.1. *Let the triple $\langle U, R, \mathcal{C}_n\rangle$ be a $\mathcal{G}_n - CAS$. Thus, for each $j \in \{r, l, i\}$ if $x \in N_j(y)$, then $N_j(x) \subseteq N_j(y)$.*

Proof. The proof is similar as [Lemma 3.3, p. 538] in [1].

Secondly, in [1, Proposition 3.2, p. 539], the authors proved that for the triple $\langle U, R, \mathcal{C}_n\rangle$ is a $\mathcal{G}_n - CAS$ and for each $A \subseteq U$, $j \in \{r, l, i, u\}$. Then, $\underline{R}_j(\underline{R}_j(A)) = \underline{R}_j(A)$, and $\overline{R}_j(\overline{R}_j(A)) = \overline{R}_j(A)$. □

Example 3.1 shows that

1. $\underline{R}_u(\underline{R}_u(A)) \neq \underline{R}_u(A)$. Take $A = \{a, c, d\}$, then $\underline{R}_u(A) = \{a, c\}$, and $\underline{R}_u(\underline{R}_u(A)) = \phi$.
2. $\overline{R}_u(\overline{R}_u(A)) \neq \overline{R}_u(A)$. Take $A = \{b, c\}$, then $\overline{R}_u(A) = \{b, c, d\}$, and $\overline{R}_u(\overline{R}_u(A)) = U$.

The following proposition is the correction form of [Proposition 3.2, p. 539] in [1].

Proposition 3.1. *Let the triple $\langle U, R, \mathcal{C}_n\rangle$ be a $\mathcal{G}_n - CAS$ and $A \subseteq U$. Then,*

1. $\underline{R}_j(\underline{R}_j(A)) = \underline{R}_j(A)$, *and* $\overline{R}_j(\overline{R}_j(A)) = \overline{R}_j(A)$ *for each* $j \in \{r, l, i\}$.
2. $\underline{R}_u(\underline{R}_u(A)) \subseteq \underline{R}_u(A)$, *and* $\overline{R}_u(\overline{R}_u(A)) \supseteq \overline{R}_u(A)$.

Proof.

1. The proof is similar as [Proposition 3.2, p. 539] in [1].
2. Since, $\underline{R}_u(A) \subseteq A$ by No. 1 in Proposition 2.1. Then, $\underline{R}_u(\underline{R}_u(A)) \subseteq \underline{R}_u(A)$ by No. 2 in Proposition 2.1. Similarly, $\overline{R}_u(\overline{R}_u(A)) \supseteq \overline{R}_u(A)$.

□

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