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New exact solution of coupled general equal width wave equation using sine-cosine function method

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ABSTRACT

In the present paper, we established a traveling wave solution by using sine-cosine-function algorithm for nonlinear partial differential equations. The method is used to obtain the exact solutions for different types of nonlinear partial differential equations such as, coupled general equal width waveequation (CGEWE) and special cases of thisequation CEW and CMEW equations, which are the important soliton equations. We plot the exact solutions for these equations at different time levels.

1. Introduction

In the last 50 years there are many studies for the nonlinear partial differential equations and these studies extended at last years to studying the systems of partial differential equations (coupled systems of partial differential equations) from both theoretical and practical points of view. With advances in computer science and the emergence some partial differential equations resulting from the engineering and scientific applications, which were difficult to resolve previously by normal methods. Now, with progress also in the science of numerical analysis and found some methods to solve these problems, which were difficult to resolve in the past, such as finite difference method [1,2]. Also, there exist some mathematical models that describe some of the complex phenomena in physics, chemistry and other various scientific fields had to be and there are many methods to solve these models and get the exact results of these models non-linear waves' equations, which plays an important role in the study of many phenomena physical. There are many numerical methods that were used in solving such phenomena such as finite element methods [3–10], Hirota's method [11], extended tanh-function method [12,13], sine-cosine method [14–16], Variational iterative method [17] and so on. The aim of this Letter is extended the sine-cosine function method to find the new exact solutions of some important nonlinear partial differential equations such as the coupled general equal width wave (CGEW) equation and special cases of thisequation CEWE and CMEWE. The coupled equal width wave equation given by Ali et al. [18] solved by EL-Sayed et al. using classical method [19].

2. The sine-cosine function method

Consider the system of nonlinear partial differential equations in the form

$$H(w, w_t, w_x, w_{xx}, w_{xxt}, g, g_t, g_x, g_{xx}, g_{xxt}, \dots) = 0, \tag{1}$$

Where $w(x,\ t)$and$g(x,\ t)$ are solutions for the system of nonlinear partial differential equations (1). We use the transformations,

$$\begin{aligned} w(x,t) &= h(\zeta), \\ g(x,t) &= l(\zeta), \end{aligned} \tag{2}$$

where $\zeta = x - st$ and s represents the constant velocity of a wave traveling in the positive direction of x-axis. Then, from Eq. (2), we get

$$\begin{aligned} &w_t = -sh'(\zeta), w_x = h'(\zeta), w_{xx} = h''(\zeta), w_{xxt} = -sh'''(\zeta), \ \dots \\ &g_t = -sl'(\zeta), g_x = l'(\zeta), g_{xx} = l''(\zeta), g_{xxt} = -sl'''(\zeta), \ \dots. \end{aligned} \tag{3}$$

Using Eq. (3) to transfer the system of nonlinear partial differential equations (1) to nonlinear ordinary differential equations

$$G(h', h'', h''', l', l'', l''', \dots) = 0, \tag{4}$$

In the Sine-Cosine technique, the solutions of (4) may be set in the form

$$\begin{aligned} h(\zeta) &= a\sin^m(\eta_1\zeta), |\zeta| \le \frac{\pi}{2\eta_1}, \\ l(\zeta) &= b\sin^n(\eta_2\zeta), |\zeta| \le \frac{\pi}{2\eta_2}, \end{aligned} \tag{5}$$

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or

$$h(\zeta) = a\cos^m(\eta_1\zeta), |\zeta| \le \frac{\pi}{2\eta_1},$$
$$l(\zeta) = b\cos^n(\eta_2\zeta), |\zeta| \le \frac{\pi}{2\eta_2}, \quad (6)$$

where a, η_1, η_2, b, n and $m$are parameters that will be determined. From Eq. (5), we get the following derivatives with respect to ζ:

$$\begin{aligned} &h(\xi) = a\sin^m(\eta_1\zeta),\\ &h'(\xi) = ma\eta_1\sin^{m-1}(\eta_1\zeta)\cos(\eta_1\zeta),\\ &h''(\xi) = -m^2a\eta_1^2\sin^m(\eta_1\zeta) + m(m-1)a\eta_1^2\sin^{m-2}(\eta_1\zeta),\\ &l(\xi) = b\sin^n(\eta_2\zeta),\\ &l'(\xi) = nb\eta_2\sin^{n-1}(\eta_2\zeta)\cos(\eta_2\zeta),\\ &l''(\xi) = -n^2b\eta_2^2\sin^n(\eta_2\zeta) + n(n-1)b\eta_2^2\sin^{n-2}(\eta_2\zeta). \end{aligned} \quad (7)$$

Similarly, from Eq. (6), we get

$$\begin{aligned} &h(\xi) = a\cos^m(\eta_1\zeta),\\ &h'(\xi) = -ma\eta_1\cos^{m-1}(\eta_1\zeta)\sin(\eta_1\zeta),\\ &h''(\xi) = -m^2a\eta_1^2\cos^m(\eta_1\zeta) + m(m-1)a\eta_1^2\cos^{m-2}(\eta_1\zeta),\\ &l(\xi) = b\cos^n(\eta_2\zeta),\\ &l'(\xi) = -nb\eta_2\cos^{n-1}(\eta_2\zeta)\sin(\eta_2\zeta),\\ &l''(\xi) = -n^2b\eta_2^2\cos^n(\eta_2\zeta) + n(n-1)b\eta_2^2\cos^{n-2}(\eta_2\zeta). \end{aligned} \quad (8)$$

Substituting (8) into the system of nonlinear ordinary differential equation (4) gives a trigonometric equation of $\cos^m(\eta_1\zeta)$and $\cos^n(\eta_2\zeta)$terms. To determine the parameters first balancing the exponents of each pair of cosine to determine$m$and$n$. Then we collect all terms with the same power in $\cos^m(\eta_1\zeta)$, $\cos^n(\eta_2\zeta)$and put to zero their coefficients to get a system of algebraic equations among the unknowns a, η_1, η_2, b, n and m. Now, the problem is reduced to a system of algebraic equations that can be solved to obtain the unknown parameters a, η_1, η_2, b, n and$m$. Similarly we can substituting (7) into the system of nonlinear ordinary differential equation (4) gives a trigonometric equation of $\sin^m(\eta_1\zeta)$and $\sin^n(\eta_2\zeta)$terms. We can obtain the unknown parameters a, η_1, η_2, b, n and$m$by the same way. Hence, the solutions considered in (5) and (6) are obtained.

3. Applications

3.1. Test problem

The coupled general equal width wave equation (CGEWE) Consider the following problem: Find functions $w(x, t)$and$g(x, t)$satisfying the coupled general equal width wave equation (CGEWE) [18] in the form,

$$\begin{aligned} &w_t + \varepsilon w^p w_x - \mu w_{xxt} + \varepsilon g^p g_x = 0,\\ &g_t + \varepsilon g^p g_x - \mu g_{xxt} = 0, \end{aligned} \quad (9)$$

where p is a positive integer.

By using (2) and (3) into (9) gives the following system of ordinary differential equations

$$\begin{aligned} &-sh' + \varepsilon h^p h' + \mu s h''' + \varepsilon l^p l' = 0,\\ &-sl' + \varepsilon l^p l' + \mu s l''' = 0. \end{aligned} \quad (10)$$

Integrating (10) once and taking the constant of integration to be zero, we find

$$\begin{aligned} &-sh + \varepsilon\frac{1}{p+1}h^{p+1} + \mu s h'' + \varepsilon\frac{1}{p+1}l^{p+1} = 0,\\ &-sl + \varepsilon\frac{1}{p+1}l^{p+1} + \mu s l'' = 0. \end{aligned} \quad (11)$$

Substituting Eq. (8) into (11) gives:

$$\begin{aligned} &-sa\cos^m(\eta_1\zeta) + \frac{\varepsilon}{(p+1)}a^{(p+1)}\cos^{(p+1)m}(\eta_1\zeta)\\ &\quad - s\mu m^2a\eta_1^2\cos^m(\eta_1\zeta) + s\mu m(m-1)a\eta_1^2\cos^{m-2}(\eta_1\zeta)\\ &\quad + \frac{\varepsilon}{(p+1)}b^{(p+1)}\cos^{(p+1)n}(\eta_2\zeta) = 0,\\ &-sb\cos^n(\eta_2\zeta) + \frac{\varepsilon}{(p+1)}b^{(p+1)}\cos^{(p+1)n}(\eta_2\zeta)\\ &\quad - s\mu n^2b\eta_2^2\cos^n(\eta_2\zeta) + s\mu n(n-1)b\eta_2^2\cos^{n-2}(\eta_2\zeta) = 0, \end{aligned} \quad (12)$$

The last system is satisfied only if the following system of algebraic equations holds:

$$\begin{aligned} &m-1 \ne 0,\\ &m-2 = (p+1)m = (p+1)n,\\ &\frac{\varepsilon}{(p+1)}a^{p+1} + \frac{\varepsilon}{(p+1)}b^{p+1} = -s\mu m(m-1)a\eta_1^2,\\ &-sa = s\mu m^2a\eta_1^2,\\ &n-1 \ne 0,\\ &n-2 = (p+1)n,\\ &\frac{\varepsilon}{(p+1)}b^{p+1} = -s\mu n(n-1)b\eta_2^2,\\ &-sb = s\mu n^2b\eta_2^2. \end{aligned} \quad (13)$$

We can get the same system (12) by substituting Eq. (7) into (11). To solve this system we must take different values of p for example if we take $p = 1$we get the exact solution of CEWE, if we take $p = 2$ we get the exact solution of CMEWE and if we take $p = 3$ we get the exact solution of CGEWE.

Now we study these cases case by case.

3.1.1. Case one (the exact solution of CEWE) at $p = 1$

Using MATHEMATICA package software for solving the system (12) at$p = 1$ we obtain:

$$m = n = -2, \quad \eta_1 = \eta_2 = \pm\frac{1}{2}\sqrt{\frac{-1}{\mu}},$$
$$b = \frac{3s}{\varepsilon}, \quad a_1 = \frac{3s}{2\varepsilon}(1 + i\sqrt{3}), \quad a_2 = \frac{3s}{2\varepsilon}(1 - i\sqrt{3}). \quad (14)$$

By back substitution (14) into (5) and (6) with (2) we obtain the exact solution of the CEWE in the form

$$g(x,t) = \frac{3s}{\varepsilon}\sec h^2\left(\frac{1}{2}\sqrt{\frac{1}{\mu}}(x - st)\right),$$
$$g(x,t) = -\frac{3s}{\varepsilon}\csc h^2\left(\frac{1}{2}\sqrt{\frac{1}{\mu}}(x - st)\right),$$
$$w_1(x,t) = \frac{3s}{2\varepsilon}(1 + i\sqrt{3})\sec h^2\left(\frac{1}{2}\sqrt{\frac{1}{\mu}}(x - st)\right),$$
$$w_1(x,t) = -\frac{3s}{2\varepsilon}(1 + i\sqrt{3})\csc h^2\left(\frac{1}{2}\sqrt{\frac{1}{\mu}}(x - st)\right),$$
$$w_2(x,t) = \frac{3s}{2\varepsilon}(1 - i\sqrt{3})\sec h^2\left(\frac{1}{2}\sqrt{\frac{1}{\mu}}(x - st)\right),$$
$$w_2(x,t) = -\frac{3s}{2\varepsilon}(1 - i\sqrt{3})\csc h^2\left(\frac{1}{2}\sqrt{\frac{1}{\mu}}(x - st)\right).$$

Now we can plot these solutions at different time levels and we can show the motion of solitary waves at Fig. 1.

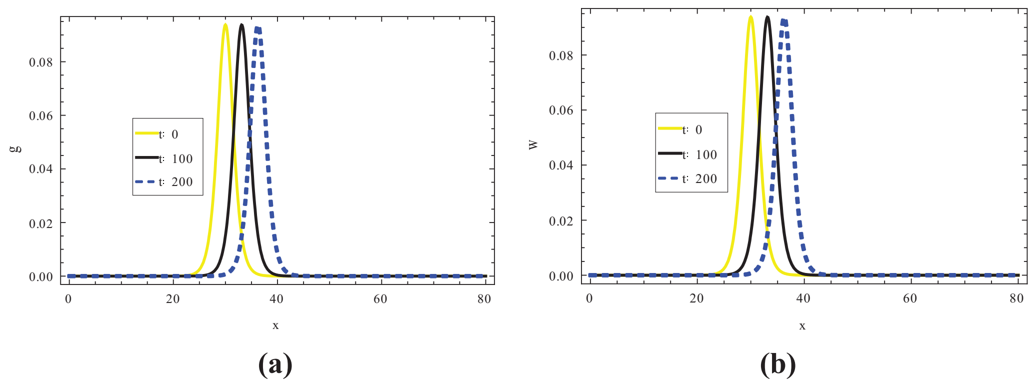

Fig. 1. Exact solutions for the CEWE at different time levels.

3.2. *Case two (the exact solution of CMEWE) at $p = 2$*

Using MATHEMATICA package software for solving the system (13) at $p = 2$ we obtain:

$$m = n = -1,$$

$$\eta_1 = \eta_2 = \pm\sqrt{\frac{-1}{\mu}},\ b = \sqrt{\frac{6s}{\varepsilon}},$$

$$a_1 = \frac{2s\varepsilon + (\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}\left(\frac{s}{\varepsilon}\right)^{\frac{3}{2}}\varepsilon^3)^{\frac{2}{3}}}{\varepsilon(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{1}{3}}},$$

$$a_2 = \frac{-2(1+i\sqrt{3})s\varepsilon - (1-i\sqrt{3})(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{2}{3}}}{2\varepsilon(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{1}{3}}},$$

$$a_3 = \frac{-2(1-i\sqrt{3})s\varepsilon - (1+i\sqrt{3})(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{2}{3}}}{2\varepsilon(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{1}{3}}}. \tag{15}$$

By back substitution (15) into (5) and (6) with (2) we obtain the exact solution of the CMEWE in the form

$$g(x,t) = \sqrt{\frac{6s}{\varepsilon}}\ \mathrm{sec}\,h\left(\sqrt{\frac{1}{\mu}}(x-st)\right),$$

$$g(x,t) = (-i)\sqrt{\frac{6s}{\varepsilon}}\ \mathrm{csc}\,h(\sqrt{\frac{1}{\mu}}(x-st)),$$

$$w_1(x,t) = \frac{2s\varepsilon + (\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{2}{3}}}{\varepsilon(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{1}{3}}}\ \mathrm{sec}\,h\left(\sqrt{\frac{1}{\mu}}(x-st)\right),$$

$$w_1(x,t) = (-i)\frac{2s\varepsilon + (\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{2}{3}}}{\varepsilon(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{1}{3}}} \times \mathrm{csc}\,h\left(\sqrt{\frac{1}{\mu}}(x-st)\right),$$

$$w_2(x,t) = \frac{-2(1+i\sqrt{3})s\varepsilon - (1-i\sqrt{3})(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{2}{3}}}{2\varepsilon(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}\left(\frac{s}{\varepsilon}\right)^{\frac{3}{2}}\varepsilon^3)^{\frac{1}{3}}} \times \mathrm{sec}\,h\left(\sqrt{\frac{1}{\mu}}(x-st)\right),$$

$$w_2(x,t) = (-i)\frac{-2(1+i\sqrt{3})s\varepsilon - (1-i\sqrt{3})(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{2}{3}}}{2\varepsilon(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{1}{3}}} \times \mathrm{csc}\,h\left(\sqrt{\frac{1}{\mu}}(x-st)\right),$$

$$w_3(x,t) = \frac{-2(1-i\sqrt{3})s\varepsilon - (1+i\sqrt{3})(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{2}{3}}}{2\varepsilon(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{1}{3}}} \times \mathrm{sec}\,h\left(\sqrt{\frac{1}{\mu}}(x-st)\right),$$

$$w_3(x,t) = (-i)\frac{-2(1-i\sqrt{3})s\varepsilon - (1+i\sqrt{3})(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{2}{3}}}{2\varepsilon(\sqrt{46}s^{\frac{3}{2}}\varepsilon^{\frac{3}{2}} - 3\sqrt{6}(\frac{s}{\varepsilon})^{\frac{3}{2}}\varepsilon^3)^{\frac{1}{3}}} \times \mathrm{csc}\,h\left(\sqrt{\frac{1}{\mu}}(x-st)\right).$$

Now we can plot these solutions at different time levels and we can show the motion of solitary waves at Fig. 2.

3.2.1. *Case three (the exact solution of CGEWE) at $p = 3$*

Using MATHEMATICA package software for solving the system (13) at $p = 3$ we obtain:

$$m = n = -\frac{2}{3},$$

$$\eta_1 = \eta_2 = \pm\frac{3}{2}\sqrt{\frac{-1}{\mu}},\quad b = \sqrt[3]{\frac{10s}{\varepsilon}},$$

$$a_1 = -\frac{\sqrt{\dfrac{8\ 2^{\frac{1}{6}}3^{\frac{2}{3}}5^{\frac{5}{6}}\left(\frac{s}{\varepsilon}\right)^{\frac{3}{2}}\varepsilon^4 + 6^{\frac{1}{3}}\left(45s^2\varepsilon^4 + 5\sqrt{3}\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac{2}{3}}}{\varepsilon^2\left(9s^2\varepsilon^4 + \sqrt{3}\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac{1}{3}}}}}{2\sqrt{3}}$$

$$-\frac{1}{2}\sqrt{-\frac{\left(\frac{5}{3}\right)^{\frac{2}{3}}\left(18s^2\varepsilon^4 + 2\sqrt{3}\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac{1}{3}}}{\varepsilon^2} - \frac{8\ 2^{\frac{1}{6}}5^{\frac{5}{6}}s\sqrt{\frac{s}{\varepsilon}}\varepsilon}{\left(27s^2\varepsilon^4 + 3\sqrt{3}\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac{1}{3}}} - \frac{20\sqrt{3}s}{\varepsilon\sqrt{\dfrac{8\ 2^{\frac{1}{6}}3^{\frac{2}{3}}5^{\frac{5}{6}}\left(\frac{s}{\varepsilon}\right)^{\frac{3}{2}}\varepsilon^4 + 6^{\frac{1}{3}}\left(45s^2\varepsilon^4 + 5\sqrt{3}\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac{2}{3}}}{\varepsilon^2\left(9s^2\varepsilon^4 + \sqrt{3}\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac{1}{3}}}}}},$$

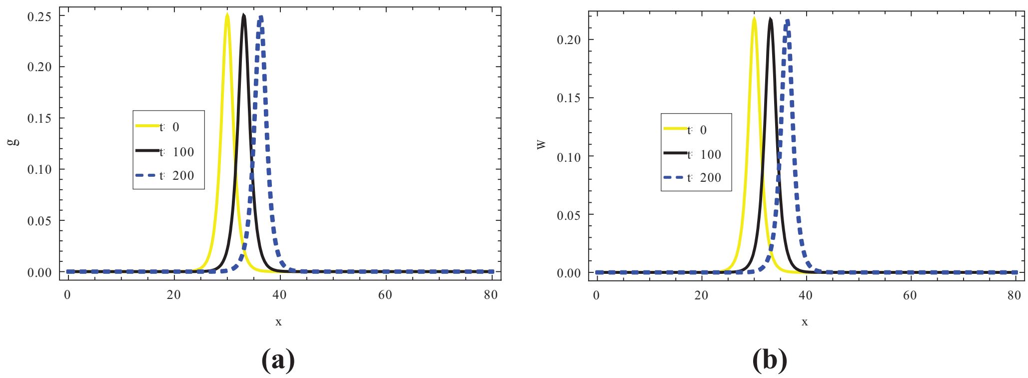

Fig. 2. Exact solutions for the CMEWE at different time levels.

$$a_2=-\frac{\sqrt{\frac{8\,2^{\frac16}3^{\frac23}5^{\frac56}\left(\frac{s}{\varepsilon}\right)^{\frac32}\varepsilon^4+6^{\frac13}\left(45s^2\varepsilon^4+5\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac23}}{\varepsilon^2\left(9s^2\varepsilon^4+\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}}}{2\sqrt3}+\frac12\sqrt{-\frac{\left(\frac53\right)^{\frac23}\left(18s^2\varepsilon^4+2\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}{\varepsilon^2}-\frac{8\,2^{\frac16}5^{\frac56}s\sqrt{\frac{s}{\varepsilon}}\varepsilon}{\left(27s^2\varepsilon^4+3\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}-\frac{20\sqrt3 s}{\varepsilon\sqrt{\frac{8\,2^{\frac16}3^{\frac23}5^{\frac56}\left(\frac{s}{\varepsilon}\right)^{\frac32}\varepsilon^4+6^{\frac13}\left(45s^2\varepsilon^4+5\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac23}}{\varepsilon^2\left(9s^2\varepsilon^4+\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}}}}$$

$$a_3=\frac{\sqrt{\frac{8\,2^{\frac16}3^{\frac23}5^{\frac56}\left(\frac{s}{\varepsilon}\right)^{\frac32}\varepsilon^4+6^{\frac13}\left(45s^2\varepsilon^4+5\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac23}}{\varepsilon^2\left(9s^2\varepsilon^4+\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}}}{2\sqrt3}-\frac12\sqrt{-\frac{\left(\frac53\right)^{\frac23}\left(18s^2\varepsilon^4+2\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}{\varepsilon^2}-\frac{8\,2^{\frac16}5^{\frac56}s\sqrt{\frac{s}{\varepsilon}}\varepsilon}{\left(27s^2\varepsilon^4+3\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}+\frac{20\sqrt3 s}{\varepsilon\sqrt{\frac{8\,2^{\frac16}3^{\frac23}5^{\frac56}\left(\frac{s}{\varepsilon}\right)^{\frac32}\varepsilon^4+6^{\frac13}\left(45s^2\varepsilon^4+5\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac23}}{\varepsilon^2\left(9s^2\varepsilon^4+\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}}}},$$

$$a_4=\frac{\sqrt{\frac{8\,2^{\frac16}3^{\frac23}5^{\frac56}\left(\frac{s}{\varepsilon}\right)^{\frac32}\varepsilon^4+6^{\frac13}\left(45s^2\varepsilon^4+5\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac23}}{\varepsilon^2\left(9s^2\varepsilon^4+\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}}}{2\sqrt3}+\frac12\sqrt{-\frac{\left(\frac53\right)^{\frac23}\left(18s^2\varepsilon^4+2\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}{\varepsilon^2}-\frac{8\,2^{\frac16}5^{\frac56}s\sqrt{\frac{s}{\varepsilon}}\varepsilon}{\left(27s^2\varepsilon^4+3\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}+\frac{20\sqrt3 s}{\varepsilon\sqrt{\frac{8\,2^{\frac16}3^{\frac23}5^{\frac56}\left(\frac{s}{\varepsilon}\right)^{\frac32}\varepsilon^4+6^{\frac13}\left(45s^2\varepsilon^4+5\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac23}}{\varepsilon^2\left(9s^2\varepsilon^4+\sqrt3\sqrt{-s^4\left(-27+256\sqrt{10}\sqrt{\frac{s}{\varepsilon}}\right)\varepsilon^8}\right)^{\frac13}}}}} \tag{16}$$

By back substitution (16) into (5) and (6) with (2) we obtain the exact solution of the CGEWE in the form

$$g(x,t)=\sqrt[3]{\frac{10s}{\varepsilon}\sec h^2\left[\pm\frac32\sqrt{\frac{1}{\mu}}(x-st)\right]},$$

$$g(x,t)=(-i)^{\frac23}\sqrt[3]{\frac{10s}{\varepsilon}\csc h^2\left[\pm\frac32\sqrt{\frac{1}{\mu}}(x-st)\right]},$$

$$w_1(x,t)=a_1\sqrt[3]{\sec h^2\left(\frac32\sqrt{\frac{1}{\mu}}(x-st)\right)},$$

$$w_1(x,t)=(-i)^{\frac23}a_1\sqrt[3]{\csc h^2\left(\frac32\sqrt{\frac{1}{\mu}}(x-st)\right)},$$

$$w_2(x,t)=a_2\sqrt[3]{\sec h^2\left(\frac32\sqrt{\frac{1}{\mu}}(x-st)\right)},$$

$$w_2(x,t)=(-i)^{\frac23}a_2\sqrt[3]{\csc h^2\left(\frac32\sqrt{\frac{1}{\mu}}(x-st)\right)},$$

$$w_3(x,t)=a_3\sqrt[3]{\sec h^2\left(\frac32\sqrt{\frac{1}{\mu}}(x-st)\right)},$$

$$w_3(x,t)=(-i)^{\frac23}a_3\sqrt[3]{\csc h^2\left(\frac32\sqrt{\frac{1}{\mu}}(x-st)\right)},$$

$$w_4(x,t)=a_4\sqrt[3]{\sec h^2\left(\frac32\sqrt{\frac{1}{\mu}}(x-st)\right)},$$

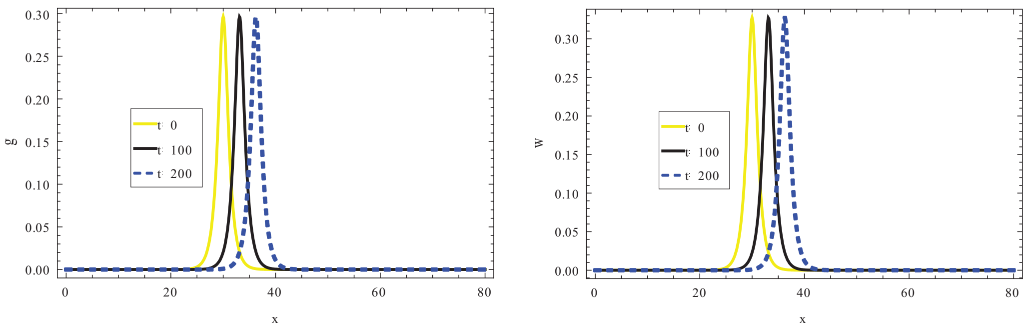

Fig. 3. Exact solutions for the CGEWE at different time levels.

$$w_4(x,t) = (-i)^{\frac{2}{3}} a_4 \sqrt[3]{\csc h^2\left(\frac{3}{2}\sqrt{\frac{1}{\mu}}(x-st)\right)}.$$

Now we can plot these solutions at different time levels and we can show the motion of solitary waves at Fig. 3.

4. Conclusion

In this Letter, the sine-cosine function method has been successfully applied to find the exact solution for different nonlinear partial differential equations such as CEW, CMEW and CGEW equations. The sine-cosine function method is used to find a new exact solution. Thus, we can say that the proposed method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas.

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