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Short Communication

Note on “approximation space on novel granulations”

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ABSTRACT

In this note, we show that an alleged properties stated in [1] are invalid in general, by giving a counter-examples.

1. Introduction

Definition 1 [2]. Let U be a nonempty set and R be a binary relation on U. The *minimal neighbourhood* of $x \in U$ is defined as:

$$\langle x\rangle_R = \cap\{pR : x \in pR\}, \tag{1}$$

where $pR = \{q \in U : (p, q) \in R\}$.

Proposition 4 of [1] asserted that for any reflexive binary relation on a nonempty set U and $X\subseteq U$ the following pair of *lower and upper approximations*

$$\underline{apr}(X) = \bigcup_{\langle x\rangle_R\subseteq X} \langle x\rangle_R, \tag{2}$$

$$\overline{apr}(X) = \bigcap\{U - \langle x\rangle_R | X \subseteq U - \langle x\rangle_R\}. \tag{3}$$

satisfy the following properties:

(L_1) $\underline{apr}(X) = [\overline{apr}(X^c)]^c$,
(U_1) $\overline{apr}(X) = [\underline{apr}(X^c)]^c$,
(U_3) $\overline{apr}(X \cup Y) = \overline{apr}(X) \cup \overline{apr}(Y)$,
(U_4) $\overline{apr}(X \cap Y) \subseteq \overline{apr}(X) \cap \overline{apr}(Y)$,
(U_5) $X \subseteq Y \Rightarrow \overline{apr}(X) \subseteq \overline{apr}(Y)$,
(U_6) $\overline{apr}(U) = U$,
(U_7) $X \subseteq \overline{apr}(X)$.

These properties are wrong in general. Moreover, counter-examples are mentioned to prove our claim.

2. Counter-example

The following example shows that (L_1), (U_1), (U_3), (U_4), (U_5), (U_6) and (U_7) of [Proposition 4, 1] are wrong, in general.

Example 2. Let $U = \{a, b, c, d\}$ and $R = \Delta \cup \{(a, b), (a, d), (b, d), (c, d), (d, b)\}$, where Δ is the identity relation on U. Hence $\langle a\rangle_R = \{a, b, d\}$, $\langle b\rangle_R = \{b, d\}$, $\langle c\rangle_R = \{c, d\}$ and $\langle d\rangle_R = \{d\}$.

(L_1) Let $X = \{a, b\}$. Then $[\overline{apr}(X^c)]^c = [\overline{apr}(\{c, d\})]^c = \emptyset^c = U$ and $\underline{apr}(\{a, b\}) = \emptyset$. Hence $\underline{apr}(X) = [\overline{apr}(X^c)]^c$ is incorrect.
(U_1) Let $X = U$. Then $[\underline{apr}(X^c)]^c = U$, but $\overline{apr}(X) = \emptyset$. Thus $\overline{apr}(X) \neq [\underline{apr}(X^c)]^c$.
(U_3) Let $X = \{d\}$ and $Y = \{a, c\}$. This implies that $\overline{apr}(X) = \emptyset$, $\overline{apr}(Y) = \{a, c\}$. Hence $\overline{apr}(X \cup Y) = \emptyset$ and $\overline{apr}(X) \cup \overline{apr}(Y) = \{a, c\}$. As a result, $\overline{apr}(X \cup Y) \neq \overline{apr}(X) \cup \overline{apr}(Y)$.
(U_4) Let $X = \{c\}$ and $Y = \{c, d\}$, then $\overline{apr}(X) = \{c\}$, $\overline{apr}(Y) = \emptyset$ and $\overline{apr}(X \cap Y) = \{c\}$. Thus $\overline{apr}(X \cap Y) \nsubseteq \overline{apr}(X) \cap \overline{apr}(Y)$.
(U_5) Let $X = \{a\}$ and $Y = \{a, d\}$. Then $\overline{apr}(X) = \{a\}$ and $\overline{apr}(Y) = \emptyset$. Hence $X\subseteq Y$, but $\overline{apr}(X) \nsubseteq \overline{apr}(Y)$.
(U_6) $\overline{apr}(U) = \emptyset$. Therefore, $\overline{apr}(U) = U$ is invalid.
(U_7) Let $X = \{b, d\}$. Then $\overline{apr}(X) = \emptyset$. Thus $X \nsubseteq \overline{apr}(X)$. Consequently, $X \subseteq \overline{apr}(X)$ is wrong.

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