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Fixed point theorems for a generalized contraction mapping of rational type in symmetric spaces

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ABSTRACT

In this work, we establish some fixed point results for a contraction of rational type in symmetric spaces extending the fixed point theorems of Almeida, Roldan-Lopez-de-Hierro and Sadarangani [1].

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1. Introduction and preliminaries

It worth to mention that the use of triangle inequality in a metric space (X, d) is of extreme importance since it implies that (i) d is continuous, (ii) each open ball is an open set, (iii) a sequence may converge to a unique point, (iv) every convergent sequence is a Cauchy sequence and other things. One of the importance generalizations of metric spaces is symmetric spaces, where the triangle inequality is relaxed. It was not immediately observed that such spaces may fail to satisfy properties (i)–(iv). Hence, in some of last papers, the authors implicitly used some of conditions (i)–(iv), so that their results were inaccuracy. Various authors introduced many types, generalizations, and applications of generalized metric spaces until now (see, [2–5]).

On the other hand, In 2015, Almeida, Roldan-Lopez-de-Hierro and Sadarangani [1] proved that whenever f is a rational type contraction mapping from a complete metric space into itself, then f has a unique fixed point. In this paper, we introduce fixed point theorems for contraction mappings of rational type in symmetric spaces. Our results generalize the results due to Almeida, Roldan-Lopez-de-Hierro and Sadarangani [1].

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Next, we present some preliminaries and notations related to symmetric spaces and rational type contractions.

Definition 1.1. [6]. Suppose that X be a non-empty set and S: $X \times X \to [0, \infty)$ be a distance function such that:

(i) $S(x, y) = 0 \Leftrightarrow x = y$.
(ii) $S(x, y) = S(y, x)$,

for all $x, y \in X$.

We mean by a pair (X, S) with a symmetric space.

Definition 1.2. [6]. Let (X, S) be a symmetric space.

(a) A sequence $\{x_n\}$ in X is S-Cauchy sequence if $\lim_{n\to\infty} S(x_n, x_{n+r}) = 0,\ r \in N$(the set of all natural numbers).
(b) (X, S) is S-complete if for every S-Cauchy sequence $\{x_n\}$, there exists x in X with $\lim_{n\to\infty} S(x_n, x) = 0$.
(c) f: $X \to X$ is S-continuous if $\lim_{n\to\infty} S(x_n, x) = 0$ implies $\lim_{n\to\infty} S(fx_n, fx) = 0$.

We need the following properties in a symmetric space (X, S).

(W_3) [7] Given $\{x_n\}$, y and x in X ,$\lim_{n\to\infty} S(x_n, x) = 0$ and $\lim_{n\to\infty} S(x_n, y) = 0$ imply that $x = y$.

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(W_4) [7] Given $\{x_n\}$, $\{y_n\}$ and x in X ,$\lim_{n\to\infty} S(x_n, x) = 0$ and $\lim_{n\to\infty} S(x_n, y_n) = 0$ imply that $\lim_{n\to\infty} S(y_n, x) = 0$.

(1C) [8] A function S is 1-continuous if $\lim_{n\to\infty} S(x_n, x) = 0 \Longrightarrow \lim_{n\to\infty} S(x_n, y) = S(x, y)$.

Remark 1.1. [7]. $(W_4)\Longrightarrow(W_3)$.

Definition 1.3. [9]. Let f: $X \to X$ and β: $X \times X \to [0, \infty)$. The mapping f is $\beta-$admissible if, for all $x, y \in X$ such that $\beta(x, y) > 1$, we have $\beta(fx, fy) > 1$.

Definition 1.4. [9]. Let (X, S) be a symmetric space and β: $X \times X \to [0, \infty)$. X is $\beta-$regular if, for each sequence $\{x_n\}$ in X such that $\beta(x_n, x_{n+1}) > 1$ for all $n \in N$ and $\lim_{n\to\infty} x_n = x$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\beta(x_{n_k}, x) > 1$ $\forall$ $k \in N$.

In 2011, Haghi et al. [10] showed that some coincidence point and common fixed point generalizations in fixed point theory are not real generalizations. They gave the following lemma which show that the authors should take care in obtaining real generalizations in fixed point theory.

Lemma 1.1. *[10]. Let X be a nonempty set and f: X → X a function. Then there exists a subset E⊆X such that f(E) = f(X) and f: E → X is one-to-one.*

2. Main results

In this section we introduce some new fixed point results for a rational contraction self-mapping on symmetric spaces.

Theorem 2.1. *Suppose that (X, S) be a S-complete symmetric space satisfy (W_4) and (1C). Let f be a self-mapping on X, and the following condition holds:*

$$S(fx, fy) \le \phi(M(x, y)) + C\min\{S(x, fx), S(y, fy), S(x, fy), S(y, fx)\}\ \forall\, x, y \in X,\ C \ge 0, \tag{1}$$

where M(x, y) is defined by

$$M(x, y) = \max\left\{S(x, y), \frac{S(x, fx)(S(y, fy)+1)}{1+S(x, y)}, \frac{S(y, fy)(S(x, fx)+1)}{1+S(x, y)}\right\}.$$

and ϕ: $[0, \infty) \to [0, \infty)$ be a continuous, nondecreasing function and $\lim_{n\to\infty} \phi^n(t) = 0\ \forall\, t > 0$.

Then f have a unique fixed point.

Proof. Let $x_0 \in X$ be an arbitrary point and let $\{x_n\}$ be the sequence defined by $x_{n+1} = fx_n$ for all $n \in N$. If there exists $m \in N$ such that $x_m = x_{m+1}$, then $x_m = x_{m+1} = fx_m$, so x_m is a fixed point of f. In this case, the proof is finished. Suppose, on the contrary, that $x_{n+1} \ne x_n$ for all $n \in N$, that is $d(x_n, x_{n+1}) > 0$.

By (1), we have

$$\begin{aligned} S(fx_n, fx_{n+1}) &\le \phi(M(x_n, x_{n+1})) \\ &\quad + C\min\{S(x_n, fx_n), S(x_{n+1}, fx_{n+1}), \\ &\qquad S(x_n, fx_{n+1}), S(x_{n+1}, fx_n)\} \\ &= \phi(M(x_n, x_{n+1})) \end{aligned} \tag{2}$$

where

$$\begin{aligned} M(x_n, x_{n+1}) &= \max\left\{S(x_n, x_{n+1}), \frac{S(x_n, fx_n)(S(x_{n+1}, fx_{n+1})+1)}{1+S(x_n, x_{n+1})}, \frac{S(x_{n+1}, fx_{n+1})(S(x_n, fx_n)+1)}{1+S(x_n, x_{n+1})}\right\} \\ &= \max\left\{S(x_n, x_{n+1}), \frac{S(x_n, x_{n+1})(1+S(x_{n+2}, x_{n+1}))}{1+S(x_n, x_{n+1})}, S(x_{n+2}, x_{n+1})\right\}, \end{aligned}$$

we consider the following cases

- If $M(x_n, x_{n+1}) = S(x_n, x_{n+1})$ from (2) we have
$$S(x_{n+1}, x_{n+2}) \le \phi(S(x_n, x_{n+1})) < S(x_n, x_{n+1}) \tag{3}$$
- If $M(x_n, x_{n+1}) = \frac{S(x_n,x_{n+1})(1+S(x_{n+2},x_{n+1}))}{1+S(x_n,x_{n+1})}$ from (2) we obtain
$$\begin{aligned} S(x_{n+1}, x_{n+2}) &\le \phi\left(\frac{S(x_n, x_{n+1})(1+S(x_{n+2}, x_{n+1}))}{1+S(x_n, x_{n+1})}\right) \\ &< \frac{S(x_n, x_{n+1})(1+S(x_{n+2}, x_{n+1}))}{1+S(x_n, x_{n+1})}. \end{aligned}$$
Hence
$$S(x_{n+1}, x_{n+2}) < S(x_n, x_{n+1}),$$
that is (3) holds.
- If $M(x_n, x_{n+1}) = S(x_{n+2}, x_{n+1})$ from (2) we get
$$S(x_{n+2}, x_{n+1}) < S(x_{n+2}, x_{n+1}),$$

which is impossible.

In any case, we proved that (3) holds. Since $\{S(x_{n+1}, x_{n+2})\}$ is decreasing. Hence, it converges to a nonnegative number, $c \ge 0$. If $c > 0$, then letting $n \to +\infty$ in (2), we deduce

$$c \le \phi\left(\max\left\{c, \frac{c(1+c)}{1+c}, c\right\}\right) = \phi(c) < c,$$

which implies that $c = 0$, that is

$$\lim_{n\to\infty} S(x_{n+1}, x_{n+2}) = 0. \tag{4}$$

By using (W_4) and for any integer number r we have

$$\lim_{n\to\infty} S(x_n, x_{n+r}) = 0, \tag{5}$$

which implies that $\{x_n\}$ is S-Cauchy sequence. Since (X, S) is S-complete, there exists $u \in X$ such that $\lim_{n\to\infty} S(x_n, u) = 0$. From (W_4) we have

$$\lim_{n\to\infty} S(x_{n+1}, u) = 0.$$

Let $u \ne fu$. Applying (1) and using (1C) we get

$$\begin{aligned} S(fu, u) &= \lim_{n\to\infty} S(fu, x_{n+1}) = \lim_{n\to\infty} S(fu, fx_n) \\ &\le \lim_{n\to\infty} [\phi(M(u, x_n)) + C\min\{S(x_n, fx_n), \\ &\qquad S(u, fu), S(x_n, fu), S(u, fx_n)\}] \\ &= \lim_{n\to\infty} [\phi(M(u, x_n)) + C\min\{S(x_n, x_{n+1}), \\ &\qquad S(u, fu), S(x_n, fu), S(u, x_{n+1})\}] \\ &= \lim_{n\to\infty} [\phi(M(u, x_n))] < S(fu, u), \end{aligned} \tag{6}$$

where

$$\begin{aligned} M(u, x_n) &= \max\left\{S(u, x_n), \frac{S(u, fu)(S(x_n, fx_n)+1)}{1+S(u, x_n)}, \frac{S(x_n, fx_n)(S(u, fu)+1)}{1+S(u, x_n)}\right\} \\ &= \max\left\{S(u, x_n), \frac{S(u, fu)(S(x_n, x_{n+1})+1)}{1+S(u, x_n)},\right. \end{aligned}$$

$$\left.\frac{S(x_n, x_{n+1})(S(u, fu) + 1)}{1 + S(u, x_n)}\right\}$$
$$= S(u, fu) \text{ as } n \to \infty.$$

Which leads to a contradiction. Hence, $S(u, fu) = 0$, that is, $u = fu$ and so u is a fixed point for f.

Now, we prove that u is the unique fixed point of f. Let x and y be arbitrary fixed points of f such that $x = fx$ and $y = fy$. Using the condition (1), it follows that

$$\begin{aligned} S(x, y) = S(fx, fy) &\le \phi\Bigg(\max\Bigg\{S(x, y), \frac{S(x, fx)(S(y, fy) + 1)}{1 + S(x, y)}, \\ &\quad \frac{S(y, fy)(S(x, fx) + 1)}{1 + S(f_2u, f_2v)}\Bigg\}\Bigg) \\ &\quad + C\min\{S(x, fx), S(y, fy), S(x, fy), S(y, fx)\} \\ &= \phi(S(x, y)) < S(x, y), \end{aligned}$$

which implies that $S(x, y) = 0$. Thus, $x = y$ and f has a unique fixed point. □

Example 2.1. Suppose that $X = [0, 1]$ and $E = \{\frac{5}{6}, \frac{2}{3}, \frac{7}{12}, \frac{8}{15}\}$. Define S on $X \times X$ as follows:

$$S\left(\frac{5}{6}, \frac{2}{3}\right) = S\left(\frac{7}{12}, \frac{8}{15}\right) = \frac{4}{9}, \quad S\left(\frac{5}{6}, \frac{8}{15}\right) = S\left(\frac{2}{3}, \frac{7}{12}\right) = \frac{1}{3},$$
$$S\left(\frac{5}{6}, \frac{7}{12}\right) = S\left(\frac{2}{3}, \frac{8}{15}\right) = \frac{8}{9}, \quad S(x, y) = |x - y| \quad \text{otherwise}.$$

Then (X, S) is a symmetric space but not metric space. Let f: $X \to X$ and $\phi(t)$: $[0, \infty) \to [0, \infty)$ defined by $fx = \frac{1}{2}x$, and $\phi(t) = \frac{t}{2}$, $\forall t \in [0, \infty)$.

Then f and ϕ satisfy all the conditions of Theorem 2.1. Hence, 0 is the unique fixed point of f.

From Lemma 1.1, one can find that the following theorem is a consequence of Theorem 2.1.

Theorem 2.2. *Suppose that (X, S) be a symmetric space satisfy (W_4) and $(1C)$. Let f_1 and f_2 be self-mappings on X such that $f_1X \subset f_2X$. Suppose that (f_2X, S) is a S-complete symmetric space and the following condition holds:*

$$\begin{aligned} S(f_1x, f_1y) \le \phi(M(x, y)) + C\min\{&S(f_2x, f_1x), S(f_2y, f_1y), \\ &S(f_2x, f_1y), S(f_2y, f_1x)\}\ \forall x, y \in X,\ C \ge 0, \end{aligned} \tag{7}$$

where $M(x, y)$ is defined by

$$\begin{aligned} M(x, y) = \max\Bigg\{&S(f_2x, f_2y), \frac{S(f_2x, f_1x)(S(f_2y, f_1y) + 1)}{1 + S(f_2x, f_2y)}, \\ &\frac{S(f_2y, f_1y)(S(f_2x, f_1x) + 1)}{1 + S(f_2x, f_2y)}\Bigg\}. \end{aligned}$$

and ϕ: $[0, \infty) \to [0, \infty)$ be a continuous, nondecreasing function and $\phi(t) = 0 \iff t = 0$.

Then f_1 and f_2 have a unique point of coincidence in X. Moreover if f_1 and f_2 are weakly compatible, then f_1 and f_2 have a unique common fixed point.

Corollary 2.1. *Replacing the condition (1) in Theorem 2.1 with the following condition:*

$$\begin{aligned} S(fx, fy) &\le a_1S(x, y) + a_2\frac{S(x, fx)(S(y, fy) + 1)}{1 + S(x, y)} \\ &\quad + a_3\frac{S(y, fy)(S(x, fx) + 1)}{1 + S(x, y)} \\ &\quad + C\min\{S(x, fx), S(y, fy), S(x, fy), S(y, fx)\}, \end{aligned}$$

where $a_1, a_2, a_3, C \ge 0$, and $a_1 + a_2 + a_3 < 1$.

Then f has a unique fixed point in X.

Remark 2.1. [1, Theorem 7] is special case of Theorem 2.1.

Next, we introduce a fixed point theorem for a (α, ψ, ϕ)-contraction self-mapping of rational type in S-complete symmetric spaces.

Theorem 2.3. *Let (X, S) be a S-complete symmetric spaces satisfy (W_4) and $(1C)$. Let f be self-mapping satisfy the following condition:*

$$\phi(\beta(x, y)S(fx, fy)) \le \phi(M(x, y)) - \psi(M(x, y))\ \forall x, y \in X, \tag{8}$$

where $M(x, y)$ as in Theorem 2.1.

Consider also that the next conditions hold:

(i) *$\exists x_0 \in X$ such that $\beta(fx_0, x_0) \ge 1$,*
(ii) *f is $\beta-$admissible,*
(iii) *X is $\beta-$regular and $\beta(x_m, x_n) \ge 1$, $\forall m, n \in N$, $m \ne n$,*
(iv) *either $\beta(x, y) \ge 1$ or $\beta(y, x) \ge 1$,*
(iiv) *ϕ: $[0, \infty) \to [0, \infty)$ be a continuous, non-decreasing and $\phi(t) = 0 \iff t = 0$, and ψ: $[0, \infty) \to [0, \infty)$ be a lower semi-continuous function and $\psi(t) = 0 \iff t = 0$.*

Then f has a unique fixed point in X.

Proof. Suppose that $x_0 \in X$, $\beta(x_0, fx_0) \ge 1$. Define $\{x_n\}$ be a sequences in X such that $x_{n+1} = fx_n$. If $x_n = x_{n+1}$ which implies that x_{n+1} is a fixed point of f. Consequently, we can suppose that $x_n \ne x_{n+1}$ for all $n \in N$. From (i), we get that $\beta(x_0, fx_0) = \beta(x_0, x_1) \ge 1$. Also, by (ii) we have that $\beta(fx_0, fx_1) = \beta(x_1, x_2) \ge 1$, $\beta(fx_1, fx_2) = \beta(x_2, x_3) \ge 1$. Continuous with this process we obtain that $\beta(x_n, x_{n+1}) \ge 1$. Now, by using (8), we get

$$\begin{aligned} \phi(S(fx_n, fx_{n+1})) &\le \phi(\beta(x_n, x_{n+1})S(fx_n, fx_{n+1})) \\ &\le \phi(M(x_n, x_{n+1})) - \psi(M(x_n, x_{n+1})) \end{aligned} \tag{9}$$

where

$$\begin{aligned} M(x_n, x_{n+1}) &= \max\Bigg\{S(x_n, x_{n+1}), \frac{S(x_n, fx_n)(S(x_{n+1}, fx_{n+1}) + 1)}{1 + S(x_n, x_{n+1})}, \\ &\qquad \frac{S(x_{n+1}, fx_{n+1})(S(x_n, fx_n) + 1)}{1 + S(x_n, x_{n+1})}\Bigg\} \\ &= \max\Bigg\{S(x_n, x_{n+1}), \frac{S(x_n, x_{n+1})(1 + S(x_{n+1}, x_{n+2}))}{1 + S(x_n, x_{n+1})}, \\ &\qquad S(x_n, x_{n+2})\Bigg\}, \end{aligned}$$

we consider the following cases

- If $M(x_n, x_{n+1}) = S(x_n, x_{n+1})$ from (9) we have
$$\begin{aligned} \phi(S(x_{n+1}, x_{n+2})) &\le \phi(S(x_n, x_{n+1})) - \psi(S(x_n, x_{n+1})) \\ &< \phi(S(x_n, x_{n+1})), \end{aligned}$$
Since ϕ is nondecreasing we have
$$S(x_{n+1}, x_{n+2}) < S(x_n, x_{n+1}). \tag{10}$$
- If $M(x_n, x_{n+1}) = \frac{S(x_n, x_{n+1})(1+S(x_{n+1}, x_{n+2}))}{1+S(x_n, x_{n+1})}$ from (9) we obtain
$$\begin{aligned} \phi(S(x_{n+1}, x_{n+2})) &\le \phi\left(\frac{S(x_n, x_{n+1})(1 + S(x_{n+1}, x_{n+2}))}{1 + S(x_n, x_{n+1})}\right) \\ &\quad -\psi\left(\frac{S(x_n, x_{n+1})(1 + S(x_{n+1}, x_{n+2}))}{1 + S(x_n, x_{n+1})}\right) \\ &< \phi\left(\frac{S(x_n, x_{n+1})(1 + S(x_{n+1}, x_{n+2}))}{1 + S(x_n, x_{n+1})}\right). \end{aligned}$$

The nondecreasing property of ϕ implies that

$$\begin{aligned} S(x_{n+1}, x_{n+2}) &< \frac{S(x_n, x_{n+1})(1 + S(x_{n+1}, x_{n+2}))}{1 + S(x_n, x_{n+1})} \\ &\Longrightarrow S(x_{n+1}, x_{n+2}) + S(x_{n+1}, x_{n+2})S(x_n, x_{n+1}) \\ &< S(x_n, x_{n+1}) + S(x_{n+1}, x_{n+2})S(x_n, x_{n+1}) \\ &\Longrightarrow S(x_{n+1}, x_{n+2}) < S(x_n, x_{n+1}). \end{aligned} \tag{11}$$

Hence, (10) is obtained.

- If $M(x_n, x_{n+1}) = S(x_{n+1}, x_{n+2}))$. By (9) we obtain

$$\begin{aligned} \phi(S(x_{n+1}, x_{n+2})) &\le \phi(S(x_{n+1}, x_{n+2})) - \psi(S(x_{n+1}, x_{n+2})) \\ &< \phi(S(x_{n+1}, x_{n+2})), \end{aligned}$$

this is a contradiction.

In any case, we proved that (10) holds. Since $\{S(x_{n+1}, x_{n+2})\}$ is decreasing. Hence, it converges to 0, that is

$$\lim_{n\to\infty} S(x_{n+1}, x_{n+2}) = 0. \tag{12}$$

By (W_4) we get that $\{x_n\}$ is a S-Cauchy sequence. Since (X, S) is S-complete, there exists $u \in X$ such that $\lim_{n\to\infty} x_n = u$. From (W_4) $\lim_{n\to\infty} x_{n+1} = u$. If $u \neq fu$. Applying (8) and using $(1C)$ we obtain that

$$\begin{aligned} \phi(S(fu, u)) &= \lim_{k\to\infty} \phi(S(fu, x_{n+1})) \\ &\le \lim_{n\to\infty} [\phi(M(u, x_n)) - \phi(M(u, x_n))], \end{aligned} \tag{13}$$

where $M(u, x_n)$ as in (6)

We get from (13) that

$$\phi(S(u, fu)) < \phi(S(u, fu)),$$

which implies a contradiction, then $S(u, fu) = 0$, that is, $u = fu$ and so u is a fixed point for f.

Now, we prove that u is the unique fixed point of f. Let x and y be arbitrary fixed points of f such that $x = fx$ and $y = fy$. Using the condition (8), it follows that

$$\phi(S(x, y)) = \phi(S(fx, fy)) \le \phi(S(x, y)) - \psi(S(x, y)) < \phi(S(x, y)),$$

which implies that $S(x, y) = 0$. Thus, $x = y$ and f has a unique fixed point. □

Remark 2.2. [1, Theorem 16] is special case of Theorem 2.3.

In 2000, Branciari [3] introduced a new concept of generalized metric space as follows:

Definition 2.1. [3]. Suppose that X be a nonempty set and d: $X \times X \to [0, \infty)$ be a distance function such that for all $w, a, b, c \in X$ and $w \neq a \neq b \neq c$,

(i) $d(w, a) = 0 \Leftrightarrow w = a$,
(ii) $d(w, a) = d(a, w)$,
(iii) $d(w, a) \le d(a, b) + d(b, c) + d(c, w)$ (quadrilateral inequality).

Then we say that (X, d) generalized metric spaces (G.M.S, for short).

Proposition 2.1. *Let (X, d) be the G.M.S. Then (W_4) and $(1C)$ are satisfied.*

Definition 2.2. Assume that X be a non-empty set and S: $X \times X \to [0, \infty)$ be a distance function satisfy the conditions (i) and (ii) in Definition 2.1. Then (X, S) is called symmetric generalized metric spaces (S.G.M.S, for short).

Remark 2.3. Theorems 2.1 and 2.3 are correct in S.G.M.S.

Remark 2.4. It will be interesting to establish Theorems 2.1 and 2.3 for $n-$tupled fixed points as in M. Imdad et al. [11], Soliman [12–14] and Soliman et al.[15].

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