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Correlation measure for fuzzy multisets

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ABSTRACT

Fuzzy and multi set theories are recent approaches for information system and data mining. In this paper, we aim to construct correlation measure connected to these theories. We define fuzzy multi correlation measure and many of its properties are investigated, and some examples and applications are given. Also, we present new view of reduction for fuzzy multi information system.

1. Introduction

Fuzzy sets are sets whose elements represent by degrees of membership. L.A.Zadeh [1] introduced the concepts of fuzzy sets. Since then, many new approaches and theories have been proposed. Moreover the concept of Intuitiontic Fuzzy set (*IFS*) was proposed by K.T.Atanassor [2,3].

In ordinary set, a set is a well-defined collection of distinct objects and permit us to have almost one occurrence of each element, multisets or bag permit us to have multiple occurrence of the elements. Multisets have various applications, such as some chemical terminology form by multiset as chemical soup of molecules [4]. Multiset is used in graph theory and in DNA computing [5], multisets have become an important tool in data base, and in computer science. Yager introduced the concepts of fuzzy multisets [6].

An element of (FMS) can be repeated more than once with the same or different membership values. Then T.K Shinoj and Sunil Jacob John introduce the concept of Intuitionistic Fuzzy Multi sets (*IFMS*) [7]. The definition of correlation measure of fuzzy multi set is an extension of the correlation measure of (*IFS*) and (*IFMS*) [8–10].

Pawlak introduce the theory of rough set as extension of the set theory [11,12] by information inadequate and incomplete. Information system is a term which has wide area of applications [11,13]. Mathematically it represents data tables and expressed using objects and attributes. One of the fundamental aims in information system is to reduce the number of attributes to get redacts without affecting the accuracy of decisions [14,15]. There are many ways to reduce the condition or objects from decision information, such as classification of data, indiscernibility matrix and function.

In this paper, we begin with the introduction of multiset, fuzzy set, fuzzy multi set and correlation measure of *IFS* and *IFMS* in Section 2. In Sections 3 and 4, we define new measure of correlation *FMS* and apply this measure on medical diagnosis and selecting specialization. We also attempt to introduce a new view of reduction for fuzzy multi information system and the definition of lower and upper fuzzy multi set by using threshold indiscernibility matrix in Sections 5 and 6. Besides, several examples are given and we close the paper with some concluding remarks (Tables 1–8).

2. Preliminaries

Definition 2.1. [2] A fuzzy set can is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$ and X be a nonempty set, $\mu_A(x) \in [0.1]$ is the membership of $x \in X$ in A.

Definition 2.2. [3] We can define an Intuitionistic fuzzy set (*IFS*) A in X as $A = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle : x \in X\}$ where μ_A: $X \to [0,\ 1]$ and ϑ_A: $X \to [0,\ 1]$ such that $0 \le \mu_A(x) + \vartheta_A(x) \le 1, \forall x \in X$. And $\mu_A(x)\, and\, \vartheta_A(x) \in [0.1]$ are the membership and non-membership functions of the fuzzy set A.

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The complement set A^c of A is defined as

$$A^c = \{\langle x, \vartheta_A(x), \ \mu_A(x)\rangle : x \in X\}$$

Definition 2.3. [6] An mset M drawn from the set X is represented by s function count M or C_M, defined as C_M: $X \to N$, where N represents the set of nonnegative integers.

In Definition 2.3, $C_M(x)$ is the number of occurrences of the element x in the mset M however those elements which are not included in the mset M have zero count.

Definition 2.4. Yager [6] first discussed fuzzy multisets, although he uses the term of fuzzy bag, an element of X can be repeated more than once in the same or different membership values. Therefore an FMS A is given by $A = \{(x_i, (\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^p(x)), x \in X\}$ of

$X = \{x_1, x_2, \dots, x_n\}$ where $\left(\mu_A^1(x) \geq \mu_A^2(x) \geq \dots \geq \mu_A^p(x)\right)$

An Intuitionistic fuzzy multi set (*IFMS*) A defined as

$$A = \left\{(x_i, \left(\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^p(x), \vartheta_A^1(x), \vartheta_A^2(x), \dots, \vartheta_A^p(x)\right) x \in X\right\}$$

where

$$\left(\mu_A^1(x) \geq \mu_A^2(x) \geq \dots \geq \mu_A^p(x)\right)$$

Definition 2.5. [6] The cardinality membership function $M_C(x)$ and the non-membership $NM_C(x)$ is the length of an element x in an *IFMS* A can be defined as $\eta = |M_C(x)| = |NM_C(x)|$. If A, B and C are the *IFMS* defined on X, then the cardinality $\eta = Max\{\eta(A), \eta(B), \eta(C)\}$.

Definition 2.6. (fuzzy correlation measure) [16, 6]:

Let $A = \{\langle x_i, \mu_A(x_i)\rangle, x_i \in x\}$ and $B = \{\langle x_i, \mu_B(x_i)\rangle, x_i \in x\}$ be two *FSs* on a finite set $X = \{x_1, x_2, \dots, x_n\}$, then the correlation coefficient of A and B is

$$\rho_{FS}(A, B) = \frac{C_{FS}(A, B)}{\sqrt{C_{FS}(A, A) * C_{FS}(B, B)}}$$

where

$$C_{FS}(A, B) = \sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i)$$

and

$$C_{FS}(A, A) = \sum_{i=1}^{n} \mu_A(x_i)\mu_A(x_i)$$

Definition 2.7. (Intuitionistic fuzzy correlation measure) [16, 6]:

Let $A = \{\langle x_i, \mu_A(x_i), \vartheta_A(x_i)\rangle, x_i \in x\}$ and $B = \{\langle x_i, \mu_B(x_i), \vartheta_B(x_i)\rangle, x_i \in x\}$ be two *IFSs* on the finite set $X = \{x_1, x_2, \dots, x_n\}$, then the correlation coefficient of A and B is

$$\rho_{IFS}(A, B) = \frac{C_{IFS}(A, B)}{\sqrt{C_{IFS}(A, A) * C_{IFS}(B, B)}}$$

where $C_{IFS}(A, B) = \sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i) + \vartheta_A(x_i)\vartheta_B(x_i)$ and $C_{IFS}(A, A) = \sum_{i=1}^{n} \mu_A(x_i)\mu_A(x_i) + \vartheta_A(x_i)\vartheta_A(x_i)$

Definition 2.8. (Intuitionistic fuzzy multi correlation measure) [17]:

Let $X = \{x_1, x_2, \dots, x_n\}$ be the finite set and $A = \{\langle x_i, \mu_A^j(x_i), \vartheta_A^j(x_i)\rangle, x_i \in x\}$, $B = \{\langle x_i, \mu_B^j(x_i), \vartheta_B^j(x_i)\rangle, x_i \in x\}$ be two *IFMS*, then the correlation coefficient of A and B

$$\rho_{IFMS}(A, B) = \frac{C_{IFMS}(A, B)}{\sqrt{C_{IFMS}(A, A) * C_{IFMS}(B, B)}}$$

Where $C_{IFMS}(A, B) = \frac{1}{\eta} \sum_{i=1}^{n} (\mu_A(x_i)\mu_B(x_i) + \vartheta_A^j(x_i)\vartheta_B^j(x_i))$

$$C_{IFMS}(A, A) = \frac{1}{\eta} \sum_{i=1}^{n} (\mu_A(x_i)\mu_A(x_i) + \vartheta_A^j(x_i)\vartheta_A^j(x_i))$$

In the next section, we will define correlation measure of fuzzy multiset (*FMS*) and its properties and apply this measure on medical diagnosis.

3. Correlation measure for fuzzy multisets (*FMS*)

Definition 3.1. Let $A = \{(x_i, (\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^p(x)), x \in X\}$ and $B = \{(x_i, (\mu_B^1(x), \mu_B^2(x), \dots, \mu_B^k(x)), x \in X\}$ be two fuzzy multiset on a finite set $X = \{x_1, x_2, \dots, x_n\}$ then the fuzzy multi correlation measure of A and B is

$$\rho_{FMS}(A, B) = \frac{C_{FMS}(A, B)}{\sqrt{C_{FMS}(A, A) * C_{FMS}(B, B)}}$$

Where

$$C_{FMS}(A, B) = \frac{1}{\eta} \sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i)$$

And

$$C_{FMS}(A, A) = \frac{1}{\eta} \sum_{i=1}^{n} \mu_A(x_i)\mu_A(x_i)$$

Proposition 3.1. The defined measure $\rho_{FMS}(A, \ B)$ between *FMS* A and B satisfies the following properties

i. $0 \leq \rho_{FMS}(A, B) \leq 1$
ii. $\rho_{FMS}(A, B) = 1$ iff $A = B$
iii. $\rho_{FMS}(A, B) = \rho_{FMS}(B, A)$

Proof.

i. $\rho_{FMS}(A, \ B)$ lies between 0 and 1beacuse, the membership functions of the *FMSs* lies between 0 and 1.
ii. Let the two *FMS* A and B be equal .i.e. $(A = B)$ hence for any $\mu_A^j(x_i) = \mu_B^j(x_i)$ then $C_{FMS}(A, A) = C_{FMS}(B, B) = \frac{1}{\eta} \sum_{i=1}^{n} \mu_A(x_i)\mu_A(x_i)$ And $C_{FMS}(A, B) = \frac{1}{\eta} \sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i) = C_{FMS}(A, \ A)$ Hence $\rho_{FMS}(A, B) = 1$, let the $\rho_{FMS}(A, B) = 1$ .i.e. $1 = \frac{C_{FMS}(A,B)}{\sqrt{C_{FMS}(A,A) * C_{FMS}(B,B)}}$ this refers that $\mu_A^j(x_i) = \mu_B^j(x_i)$ for all values i, j hence $A = B$.
iii. $\rho_{FMS}(A, B) = \rho_{FMS}(B, A)$. It is obvious that $\rho_{FMS}(A, B) = \frac{C_{FMS}(A,B)}{\sqrt{C_{FMS}(A,A) * C_{FMS}(B,B)}} = \rho_{FMS}(B, A)$ as $C_{FMS}(A, B) = \frac{1}{\eta} \sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i) = \frac{1}{\eta} \sum_{i=1}^{n} \mu_B(x_i)\mu_A(x_i) = C_{FMS}(B, A)$.

Example 3.1. Let $A = \{(0.3, 0.2, 0.1)/x, (1, 0.5, 0.5)/y\}$ and $B = \{(0.7, 0.5, 0.4)/w, (0.5, 0.4, 0.4)/z\}$ here the cardinality $\eta = 2$

$$C_{FMS}(A, B) = \frac{1}{2}[(0.3)(0.7) + (1)(0.5) + (0.2)(0.5) + (0.5)(0.4) + (0.1)(0.4) + (0.5)(0.4)] = 0.625$$

And the correlation *FMS*measure is $\rho_{FMS}(A, B) = 0.805$

Proposition 3.2. Let A_1, A_2, B_1, B_2 are *FMS* such that

$A_1 \subseteq A_2$, $B_1 \subseteq B_2$ then

i. $C_{FMS}(A_1, B_1) \leq C_{FMS}(A_2, B_2)$
ii. $\rho_{FMS}(A_1, B_1) \leq \rho_{FMS}(A_2, B_2)$

Table 1
FMS P: The relation between patient and symptoms.

p	Temperature	Cough	Throat pain	Headache	Body pain
p_1	(0.6,0.7,0.5)	(0.4,0.3,0.4)	(0.1,0.2,0)	(0.5,0.6,0.7)	(0.2,0.3,0.4)
p_2	(0.4,0.3,0.5)	(0.7,0.6,0.8)	(0.6,0.5,0.4)	(0.3,0.6,0.2)	(0.8,0.7,0.5)
p_3	(0.1,0.2,0.1)	(0.3,0.2,0.1)	(0.8,0.7,0.8)	(0.3,0.2,0.2)	(0.4,0.3,0.2)
p_4	(0.3,0.4,0.2)	(0.4,0.3,0.1)	(0.2,0.1,0)	(0.5,0.6,0.3)	(0.4,0.5,0.4)

Table 2
FMSs R: the relation among symptoms and diseases.

R	Viral fever	Tuberculosis	Typhoid	Throat disease
Temperature	(0.8,0.9,0.85)	(0.2,0.3,0.25)	(0.5,0.7,0.6)	(0.1,0.3,0.2)
Cough	(0.2,0.3,0.25)	(0.9,1,0.95)	(0.3,0.5,0.4)	(0.3,0.4,0.35)
Throat pain	(0.3,0.5,0.4)	(0.7,0.8,0.75)	(0.2,0.3,0.25)	(0.8,0.9,0.85)
Headache	(0.5,0.7,0.6)	(0.6,0.7,0.65)	(0.2,0.4,0.3)	(0.1,0.2,0.15)
Body pain	(0.5,0.6,0.55)	(0.7,0.8,0.75)	(0.4,0.6,0.5)	(0.1,0.2,0.15)

Proof.

i. $C_{FMS}(A_1, B_1) = \frac{1}{\eta}\sum_{i=1}^{n}\mu_{A_1}(x_i)\mu_{B_1}(x_i)$ and $C_{FMS}(A_2, B_2) = \frac{1}{\eta}\sum_{i=1}^{n}\mu_{A_2}(x_i)\mu_{B_2}(x_i)$, $\mu_{A_1}^{j}(x) \le \mu_{A_2}^{j}(x)$ $(A_1 \subseteq A_2)$, for all values of j, $\mu_{B_1}^{j}(x) \le \mu_{B_2}^{j}(x)$ then $C_{FMS}(A_1, B_1) \le C_{FMS}(A_2, B_2)$
ii. It is obvious from (i)

We will present an application of *FMS* correlation measure in medical diagnosis in the following example**.** As medical diagnosis contains a large amount of uncertainties and increased volume of information available to physicians from new updated technologies, the process of classifying different set of symptoms under a single name of a disease.

Example 3.2. Let $p = \{p_1, p_2, p_3, p_4\}$ be a set of patients, $D = \{$ Fever, Tuberculosis, Typhoid, Throat disease$\}$ be the set of diseases and $S = \{$ Temperature, Cough, Throat pain, Headache, Body pain$\}$ be the set of symptoms.

Our solution is to examine the patient at different time intervals (three times a day).

$$P_1 = \left\{\begin{matrix} T/(0.6, 0.7, 0.5), C/(0.4, 0.3, 0.4), Th/(0.1, 0.2, 0), \\ H/(0.5, 0.6, 0.7), B/(0.2, 0.3, 0.4) \end{matrix}\right\},$$
$$P_2 = \left\{\begin{matrix} T/(0.4, 0.3, 0.5), C/(0.7, 0.6, 0.8), Th/(0.6, 0.5, 0.4), \\ H/(0.3, 0.6, 0.2), B/(0.8, 0.7, 0.5) \end{matrix}\right\}$$
$$P_3 = \left\{\begin{matrix} T/(0.1, 0.2, 0.1), C/(0.3, 0.2, 0.1), Th/(0.8, 0.7, 0.8), \\ H/(0.3, 0.2, 0.2), B/(0.4, 0.3, 0.2) \end{matrix}\right\}$$
$$P_4 = \left\{\begin{matrix} T/(0.3, 0.2, 0.2), C/(0.4, 0.3, 0.1), Th/(0.2, 0.1, 0), \\ H/(0.5, 0.6, 0.3), B/(0.4, 0.5, 0.4) \end{matrix}\right\}$$

Fever
$$= \left\{\begin{matrix} T/(0.8, 0.9, 0.85), C/(0.2, 0.3, 0.25), Th/(0.3, 0.5, 0.4), \\ H/(0.5, 0.7, 0.6), B/(0.5, 0.6, 0.55) \end{matrix}\right\}$$

Tuberculosis
$$= \left\{\begin{matrix} T/(0.2, 0.3, 0.25), C/(0.9, 1, 0.95), Th/(0.7, 0.8, 0.75), \\ H/(0.6, 0.7, 0.65), B/(0.7, 0.8, 0.75) \end{matrix}\right\}$$

Typhoid
$$= \left\{\begin{matrix} T/(0.5, 0.7, 0.6), C/(0.3, 0.5, 0.4), Th/(0.2, 0.3, 0.25), \\ H/(0.2, 0.4, 0.3), B/(0.4, 0.6, 0.5) \end{matrix}\right\}$$

Throat disease
$$= \left\{\begin{matrix} T/(0.1, 0.3, 0.2), C/(0.3, 0.4, 0.35), Th/(0.8, 0.9, 0.85), \\ H/(0.1, 0.2, 0.15), B/(0.1, 0.2, 0.15) \end{matrix}\right\}$$

Table 3
The correlation measure between FMSs P and R.

Correlation measure	Viral fever	Tuberculosis	Typhoid	Throat disease
p_1	**0.9292**	0.7579	0.8997	0.4032
p_2	0.8227	**0.9514**	0.8970	0.747
p_3	0.6448	0.8729	0.6419	**0.9437**
p_4	0.8557	0.835	**0.8951**	0.480

From the table above, the following decision are made on, the highest measure gives that patient P_1 suffers from Viral Fever, patient P_2 suffers from Tuberculosis, patient P_2 suffers from Throat disease and the patient P_4 suffers from Typhoid.

Also we will present another application of *FMS* correlation measure in selecting specialization. A case study for engineering students will explain in details in the following example.

4. Application of *FMS* correlation measure in selecting specialization

In Egypt for example, we suffer from lack of specialization and how choosing the department or the faculty for each student. In Egypt each student chooses the department or faculty according to its fame and ignores the fact that whether this student is suitable to this department or not. So we need choosing the suitable department to each student by its degree. In faculty of engineering for example each department needs student to be excellent in specific objects. We use the correlation measure of FMSs as tool since it incorporates the membership degree (i.e. the marks of three exam to each student, written exam, oral exam and summer training)

Table 4
Student vs. subjects.

S	Logic	Math	Drawing	Physic	Mechanic
S_1	(0.9,0.7,0.8)	(0.9,0.7,0.8)	(0.6,0.4,0.3)	(0.9,0.8,0.8)	(0.5,0.5,0.4)
S_2	(0.5,0.4,0.5)	(0.6,0.4,0.5)	(0.5,0.5,0.4)	(0.4,0.3,0.4)	(0.8,0.9,0.7)
S_3	(0.7,0.6,0.7)	(0.5,0.4,0.4)	(0.9,0.7,0.8)	(0.5,0.4,0.3)	(0.4,0.5,0.3)
S_4	(0.7,0.6,0.7)	(0.8,0.7,0.7)	(0.8,0.7,0.7)	(0.6,0.5,0.6)	(0.5,0.4,0.5)

Table 5
Department vs. subjects.

R	Logic	Math	Drawing	Physics	Mechanic
Architecture	(0.8,0.7,0.9)	(0.5,0.4,0.4)	(0.9,0.8,0.9)	(0.5,0.4,0.4)	(0.5,0.3,0.4)
Electrical	(0.8,0.7,0.8)	(0.9,0.8,0.7)	(0.5,0.4,0.3)	(0.8,0.8,0.9)	(0.5,0.5,0.4)
Civil	(0.8,0.8,0.9)	(0.7,0.6,0.6)	(0.9,0.8,0.9)	(0.5,0.4,0.3)	(0.6,0.7,0.6)
Mechanical	(0.6,0.6,0.5)	(0.5,0.5,0.4)	(0.8,0.7,0.7)	(0.5,0.5,0.4)	(0.9,0.8,0.9)

Table 6
The correlation measure between FMSs S and R.

Correlation measure	Architecture	Electrical	Civil	Mechanical
S_1	0.8858	**0.995**	0.904	0.872
S_2	0.8699	0.8832	0.936	**0.974**
S_3	**0.9902**	0.8812	0.986	0.934
S_4	0.9614	0.953	**0.973**	0.935

Table 7
Student vs. subjects.

S	Math	Drawing	Physic	Mechanic
S_1	(0.9,0.7,0.8)	(0.6,0.4,0.3)	(0.9,0.8,0.8)	(0.5,0.5,0.4)
S_2	(0.6,0.4,0.5)	(0.5,0.5,0.4)	(0.4,0.3,0.4)	(0.8,0.9,0.7)
S_3	(0.5,0.4,0.4)	(0.9,0.7,0.8)	(0.5,0.4,0.3)	(0.4,0.5,0.3)
S_4	(0.8,0.7,0.7)	(0.8,0.7,0.7)	(0.6,0.5,0.6)	(0.5,0.4,0.5)

Table 8
Department vs. subjects.

R	Math	Drawing	Physics	Mechanic
Architecture	(0.5,0.4,0.4)	(0.9,0.8,0.9)	(0.5,0.4,0.4)	(0.5,0.3,0.4)
Electrical	(0.9,0.8,0.7)	(0.5,0.4,0.3)	(0.8,0.8,0.9)	(0.5,0.5,0.4)
Civil	(0.7,0.6,0.6)	(0.9,0.8,0.9)	(0.5,0.4,0.3)	(0.6,0.7,0.6)
Mechanical	(0.5,0.5,0.4)	(0.8,0.7,0.7)	(0.5,0.5,0.4)	(0.9,0.8,0.9)

Example 4.1.

$$S_1 = \left\{ \begin{matrix} L/(0.9, 0.7, 0.8), Math/(0.9, 0.7, 0.8), D/(0.6, 0.4, 0.3), \\ Ph/(0.9, 0.8, 0.8), Me/(0.5, 0.5, 0.4) \end{matrix} \right\}$$

$$S_2 = \left\{ \begin{matrix} L/(0.5, 0.4, 0.5), Math/(0.6, 0.4, 0.5), D/(0.5, 0.5, 0.4), \\ Ph/(0.4, 0.3, 0.4), Me/(0.8, 0.9, 0.7) \end{matrix} \right\}$$

$$S_3 = \left\{ \begin{matrix} L/(0.7, 0.6, 0.7), Math/(0.5, 0.4, 0.4), D/(0.9, 0.7, 0.8), \\ Ph/(0.5, 0.4, 0.3), Me/(0.4, 0.5, 0.3) \end{matrix} \right\}$$

$$S_4 = \left\{ \begin{matrix} L/(0.7, 0.6, 0.7), Math/(0.8, 0.7, 0.7), D/(0.8, 0.7, 0.7), \\ Ph/(0.6, 0.5, 0.6), Me/(0.5, 0.4, 0.5) \end{matrix} \right\}$$

$$\text{Architecture} = \left\{ \begin{matrix} L/(0.8, 0.7, 0.9), Math/(0.5, 0.4, 0.4), D/(0.9, 0.8, 0.9), \\ Ph/(0.5, 0.4, 0.4), Me/(0.5, 0.3, 0.4) \end{matrix} \right\}$$

$$\text{Electrical} = \left\{ \begin{matrix} L/(0.8, 0.7, 0.8), Math/(0.9, 0.8, 0.7), D/(0.5, 0.4, 0.3), \\ Ph/(0.8, 0.8, 0.9), Me/(0.5, 0.5, 0.4) \end{matrix} \right\}$$

$$\text{civil} = \left\{ \begin{matrix} L/(0.8, 0.8, 0.9), Math/(0.7, 0.6, 0.6), D/(0.9, 0.8, 0.9), \\ Ph/(0.5, 0.4, 0.3), Me/(0.6, 0.7, 0.6) \end{matrix} \right\}$$

$$\text{Mechanical} = \left\{ \begin{matrix} L/(0.6, 0.6, 0.5), Math/(0.5, 0.5, 0.4), D/(0.8, 0.7, 0.7), \\ Ph/(0.5, 0.5, 0.4), Me/(0.9, 0.8, 0.9) \end{matrix} \right\}$$

5. New view of reduction for fuzzy multi information system

In this section, we attempt to apply reduction by using discernibility matrix and discernibility function from the correlation measure of fuzzy multi between students and college department by form threshold matrix at choice value $\propto$, which convert the correlation attributes to binary information system.

Example 5.1. from Example 4.1, we find that, some subjects do not affect the choice of departments because they are important for all departments, or it will not be a condition in the selection of sections so it can be ignored for simplicity.

We reduce the column of logic and it found that it does not affect the outcome.

Definition 5.1. (discernibility matrix) [14, 15]: An information system S defines a matrix M_A called discernibility matrices. Each entry $M_A(x, y) \subseteq A$ consists of a set of attributes that can be used to discern between objects $x, y \in U$:

Table 9
The correlation measure between FMSs S and R.

Correlation measure	Architecture	Electrical	Civil	Mechanical
S_1	0.8424	**0.995**	0.8702	0.8514
S_2	0.8594	0.864	0.9355	**0.972**
S_3	**0.9892**	0.838	0.982	0.931
S_4	0.959	0.940	**0.9696**	0.926

Table 10
Threshold indiscernibility matrix.

Correlation measure	Architecture	Electrical	Civil	Mechanical
S_1	0	1	0	0
S_2	0	0	1	1
S_3	1	0	1	1
S_4	1	1	1	1

Table 11
Discernibility matrix.

	S_2	S_3	S_4
S_1	*Ele, Ci, Me*	*Ar, Ele, Ci, Me*	*Ar, Ci, Me*
S_2		*Ar*	*Ar, Ele*
S_3			*Ele*

$M_A(x, y) = \{a \in A : a(x) \neq a(y)\}$ M_A is an $|U| \times |U|$ matrix, in the discernibility matrix has the form: $M_{ij} = \{a \in A : a(x_i) \neq a(x_j)\}, i, j \in [1, n], \ n = |U|$

Definition 5.2. (discernibility function) [14, 15]: The discernibility function of a discernibility matrix is defined by: $f(M) = \wedge\{\vee(M(x, y)) : \forall x, y \in U, M(x, y) \neq \emptyset\}$ the expression $\vee(M(x, y))$ is the disjunction of all attributes in $M(x, y)$ indicating that the object pair (x, y) can be distinguished by any attribute in $M(x, y)$. The expression $\wedge\{\vee(M(x, y))\}$ is the conjunction of all $\vee(M(x, y))$, indicating that the family of discernible object pairs can be distinguished by a set of attributes satisfying $\wedge\{\vee(M(x, y))\}$. The discernibility function can be used to state an important result regarding the set of reduces of an information table.

Definition 5.3. (threshold indiscernibility matrix): let ρ_{FMS} be a correlation measure of fuzzy multi set and FMIS be a fuzzy multi information system, to formed a crisp information system dependent of the value of ρ_{FMS} as: if $\rho_{FMS} \geq \propto$ the coefficient value will be 1 and if $\rho_{FMS} < \propto$ the coefficient value will be zero as in Example 5.2

Note: (reduction of attributes) let $B \subseteq A$, $a \in B$, and then a is superfluous in B if: $U/IND(B) = U/IND(B - \{a\})$.

Example 5.2. Continued from Example 5.1 we can use Table 9 of correlation measure to create the discernibility matrix as following:

$$if\ \rho_{FMS}(S, R) \ \geq \ 0.9$$

The discernibility matrix of Table 10 is shown in Table 11
Reduction of Attributes:

$U/IND(Ar) = \{\{S_1, S_2\}, \{S_3, S_4\}\}, U/IND(Ele) = \{\{S_1, S_4\}, \{S_2, S_3\}\},$
$U/IND(Ci) = \{\{S_1\}, \{S_2, S_3, S_4\}\}, U/IND(Me) = \{\{S_1\}, \{S_2, S_3, S_4\}\}$
$U/IND(A) = \{\{S_1\}, \{S_2\}, \{S_3\}, \{S_4\}\}$
$U/IND(A) \neq U/IND(A - Ar), U/IND(A) \neq U/IND(A - Ele)$
$U/IND(A) = U/IND(A - Ci), U/IND(A) = U/IND(A - Me)$
$U/IND(A) = U/IND(A - \{Ci, Me\})$

By discernibility function:

$$f(Ar, Ele, Ci, Me) = \{Ele + Ci + Me\} \cdot \{Ar + Ele + Ci + Me\} \cdot \{Ar + Ci + Me\} \cdot \{Ar\} \cdot \{Ar + Ele\} \cdot \{Ele\} = Ar \cdot Ele$$

From the above example we notice that the reduction process is not the meaning of the deleted department's college but, facilitates the process of taking the decision and also, collects similar students in one case because if we study all states for all students and discuss all possible values we find that the number of all possible states is 2^4 state. And we can look for the Part of the deleted in which cases may require it.

6. Upper and lower for fuzzy multi information system

At this section, we present the definition of lower and upper fuzzy multi information system by using threshold indiscernibility matrix.

Definition 6.1. let ρ_{FMS} be a fuzzy multi correlation measure of set and O_i, A_i the sets of objects and attributes respectively in fuzzy multi information system $(FMIS)$ then the upper and the lower of a threshold set A is $\overline{A_\alpha} = \cup\{O_i : \sum A_i \geq \sum A_j\}$, $\underline{A_\propto} = A \cap \{O_i : \sum A_i \geq \sum A_j\}$ where A_i the value of attributes of object O_i in threshold indiscernibility matrix using the correlation measure ρ_{FMS} at optional value α and A_j the value of attributes of each member of the set A and $i \neq j$.

Example 6.1. from Example 5.2 we can use Table 10 of threshold indiscernibility matrix as following:

If $A = \{S_1, S_3\}$ then $\overline{A_\propto} = \cup\{\{S_1, S_2, S_3, S_4\}, \{S_3, S_4\}\} = \{S_1, S_2, S_3, S_4\}$

$\underline{A_\propto} = A \cap \{S_1, S_2, S_3, S_4\} \cap \{S_3, S_4\} = \{S_3\}$

We can calculate the upper and the lower of a set A at some attributes as following: If $A = \{S_1, S_3\}$ then $\overline{A_{\propto_{Ar,El}}} = \cup\{\{S_1, S_3, S_4\}, \{S_1, S_3, S_4\}\} = \{S_1, S_3, S_4\}$

$\underline{A_{\propto_{Ar,El}}} = A \cap \{S_1, S_3, S_4\} \cap \{S_1, S_3, S_4\} = \{S_1, S_3\}$

Definition 6.2.

i. The object x_i surely belongs to threshold set $A_\propto$ if $x_i \in \underline{A_\propto}$ and denoted by $x \,\underline{\in}\, A_\propto$.
ii. The object x_i is possibly belong to threshold set $A_\propto$ if $x_i \in \overline{A_\propto}$ and denoted by $x \,\bar{\in}\, A_\propto$.

Proposition 6.1. Let $A_\propto$ and $B_\propto$ be thresholds sets of fuzzy multi information system of objects then the following properties are satisfied:

i. $A_\propto \subseteq B_\propto$ *implies that* $x_i \,\underline{\in}\, A_\propto$ *so* $x_i \,\underline{\in}\, B_\propto$ and all $x_j \bar{\in} A_\propto$ implies that $x_j \bar{\in} B_\propto$.
ii. $x_i \bar{\in} (A_\propto \cup B_\propto)$ if $x_i \bar{\in} A_\propto$ or $x_i \bar{\in} B_\propto$.
iii. $x_i \bar{\in} (A_\propto \cap B_\propto)$ iff $x_i \bar{\in} A_\propto$ and $x_i \bar{\in} B_\propto$
iv. $x_i \,\underline{\in}\, A_\propto$ or $x_i \,\underline{\in}\, B_\propto$ implies that $x_i \,\underline{\in}\, A_\propto \cup B_\propto$.
v. $x_i \,\underline{\in}\, A_\propto \cap B_\propto$ implies that $x_i \,\underline{\in}\, A_\propto$ and $x_i \,\underline{\in}\, B_\propto$.

Proof.

i. Let $x \,\underline{\in}\, A_\propto$ *then* $x \in A$ *with* $\rho \geq \propto$ *then* $x \in B$ *with* $\rho \geq \propto$, since $\underline{A} \subseteq \underline{B}$ and $\rho \geq \propto$ so $\underline{A_\propto} \subseteq \underline{B_\propto}$ therefore $x \,\underline{\in}\, B_\propto$. Similarly for possibly belongs.

We can prove (ii) to (v) from the above definitions.

7. Conclusion

In this paper, we defined the FMS correlation measure and proved some of its properties and applied this measure on medical diagnosis and selecting specialization. Also, we introduced a new view of reduction for fuzzy multi information system. Finally, we introduced the definition of lower and upper fuzzy multi set by using threshold indiscernibility matrix.

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