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Fuzzy Differential Subordinations for Univalent Functions Involving Rafid-Operator

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Abstract: In this paper, using the fuzzy differential subordination technique, we obtain several fuzzy subordination results associated with the Rafid integral operator.

Keywords: Analytic function; convex function; univalent function; fuzzy differential subordination; fuzzy best dominant; Rafid-operator; Rafid integral operator.

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1 Introduction and Preliminaries

In [1–6], authors used the general theory of fuzzy differential subordination in some earlier results on connections between various subclasses of analytic and univalent functions. In this paper, we use the fuzzy differential subordination method to obtain several fuzzy differential subordination results associated with the integral operator $R_{\mu}^{\theta}\xi(z)$.

Let $\mathscr{C}$ be the family of complex numbers, $D \subset \mathscr{C}$, $U = \{z \in \mathscr{C} : |z| < 1\}$ be the open unit disc, and let $H(D)$ denotes the class of analytic functions on D. For $a \in \mathscr{C}$ and $n \in \mathscr{N} = \{1, 2, \ldots\}$, assume that

$$H[a,n] = \left\{ \xi \in H(U) : \xi(z) = a + \sum_{k=n}^{\infty} a_k z^k,\ z \in U \right\}, \tag{1}$$

and

$$A_n = \left\{ \xi \in H(U) : \xi(z) = z + \sum_{k=n+1}^{\infty} a_k z^k,\ z \in U \right\}. \tag{2}$$

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Let $A_1 = A$. The subclass of A consisting of univalent functions is denoted by S. A function $\xi \in S$ is said to be convex if $\xi(U)$ is a convex domain. The function ξ is proved to be convex if and only if

$$R\left(1+\frac{z\xi''(z)}{\xi'(z)}\right) > 0,\ z \in U. \tag{3}$$

The class of convex functions, denoted by $\mathscr{K}$.

Definition 1. *[7, 8] Let $\xi, \varsigma \in H(U)$, we say that ξ is subordinate to ς, written $\xi \prec \varsigma$ $(z \in U)$, if there exists a Schwarz function $\omega \in U$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ $(z \in U)$, such that $\xi(z) = \varsigma(\omega(z))$. If $\varsigma(z)$ is univalent in U, then $\xi(z) \prec \varsigma(z)$ if and only if*

$$\xi(0) = \varsigma(0),\ \xi(U) \subset \varsigma(U). \tag{4}$$

To use the concept of fuzzy differential subordination, we need the following definitions and propositions:

Definition 2. *[9] If $F_B : X \to [0,1]$ and*

$$B = \{x \in X;\ 0 < F_B(x) \leq 1\}, \tag{5}$$

then the pair (B, F_B) is called a fuzzy subset of X and the set B is called the support of the fuzzy set (B, F_B), while the function F_B is called the membership function of the fuzzy set (B, F_B). We denote $B =$supp(B, F_B).

Proposition 1. *[10] Let $B =$supp(B, F_B) and $N =$supp(N, F_N).*
(i) If $(B, F_B) = (N, F_N)$, then $B = N$.
(ii) If $(B, F_B) \subseteq (N, F_N)$, then $B \subseteq N$.

Suppose that $\xi, \varsigma \in H(D)$. Set

$$\xi(D) = \{\xi(z) : 0 < F_{\xi(D)}\xi(z) \leq 1,\ z \in D\} = supp(\xi(D), F_{\xi(D)}) \tag{6}$$

and

$$\varsigma(D) = \{\varsigma(z) : 0 < F_{\varsigma(D)}\varsigma(z) \leq 1,\ z \in D\} = supp(\varsigma(D), F_{\varsigma(D)}). \tag{7}$$

Definition 3. *[10]) Let z_0 be a fixed point in D and $\xi, \varsigma \in H(D)$. Then ξ is said to be fuzzy subordinate to ς, written $\xi \prec_F \varsigma$ or $\xi(z) \prec_F \varsigma(z)$, if there exists a function $F : \mathscr{C} \to [0,1]$, such that $\xi(z_0) = \varsigma(z_0)$, and $F_{\xi(D)}(\xi(z)) \leq F_{\varsigma(D)}(\varsigma(z))$ for all $z \in D$.*

Proposition 2. *[10] Let $\xi, \varsigma \in H(D)$. If $\xi(z) \prec_F \varsigma(z)$, then*
(i) $\xi(z_0) = \varsigma(z_0)$ (z_0 is a fixed point in D)
(ii) $\xi(D) \subseteq \varsigma(D)$, $F_{\xi(D)}\xi(z) \leq F_{\varsigma(D)}\varsigma(z)$ for all $z \in D$ where $\xi(D)$ and $\varsigma(D)$ are given by (6) and (7).

Definition 4. *[11] Let $\eta \in S$ and $\varphi(m, s, j; z) : \mathscr{C}^3 \times U \to \mathscr{C}$ with $\varphi(a, 0, 0; 0) = \eta(0)$, and let σ is analytic in U with $\sigma(0) = a$ $(a \in \mathscr{C})$. If σ satisfies the (second order) fuzzy differential subordination*

$$F_{\varphi(\mathscr{C}^3 \times U)}\varphi(\sigma(z), z\sigma'(z), z^2\sigma''(z); z) \leq F_{\eta(U)}\eta(z), \tag{8}$$

or equivalently to

$$\varphi(\sigma(z), z\sigma'(z), z^2\sigma''(z); z) \prec_F \eta(z),\ z \in U, \tag{9}$$

then σ is called a fuzzy solution of the fuzzy differential subordination. If the univalent function χ satisfies $\sigma(0) = \chi(0)$ and

$$F_{\sigma(U)}\sigma(z) \leq F_{\chi(U)}\chi(z), \tag{10}$$

or equivalently to

$$\sigma(z) \prec_F \chi(z),\ z \in U\ , \tag{11}$$

then χ is called a fuzzy dominant of the fuzzy solutions of the fuzzy differential subordination for all σ satisfying (9). Let $F : \mathscr{C} \to [0,1]$, a fuzzy dominant $\widetilde{\chi}$ that satisfies $\widetilde{\chi}(0) = \chi(0)$ and

$$F_{\widetilde{\chi}(U)}\widetilde{\chi}(z) \leq F_{\chi(U)}\chi(z), \tag{12}$$

which is equivalent to

$$\widetilde{\chi}(z) \prec_F \chi(z),\ z \in U, \tag{13}$$

for all fuzzy dominants χ of (9) is called the fuzzy best dominant of (9).

For $0 \leq \mu < 1, 0 \leq \theta \leq 1$ and $\xi \in A$, Atshan and Buti [12] introduced the Rafid operator $R_\mu^\theta : A \to A$ as follows:

$$\begin{aligned} R_\mu^\theta \xi(z) &= \frac{1}{(1-\mu)^{\theta+1}\Gamma(\theta+1)} \int_0^\infty t^{\theta-1}\, e^{-\left(\frac{t}{1-\mu}\right)}\ \xi(zt)dt \\ &= z + \sum_{k=2}^\infty \frac{(1-\mu)^{k-1}\ \Gamma(\theta+k)}{\Gamma(\theta+1)}\ a_k z^k. \end{aligned} \tag{14}$$

Using (14), it is easily verified that

$$z\left(R_\mu^\theta \xi(z)\right)' = (\theta+1)R_\mu^{\theta+1}\xi(z) - \theta R_\mu^\theta \xi(z) \tag{15}$$

By using the integral operator $R_\mu^\theta \xi(z)$ defined by (14), we will derive several fuzzy differential subordinations for this class.

Definition 5. *A function $\xi \in A$ belongs to the class $M_\delta^F(\theta,\mu)$ for all $\delta \in [0,1)$, $0 \leq \mu < 1$ and $0 \leq \theta \leq 1$ if it satisfies the inequality*

$$F_{(R_\mu^\theta \xi)'(U)}(R_\mu^\theta \xi(z))' > \delta,\ \ z \in U. \tag{16}$$

In our investigation, we will need to consider the following lemmas.

Lemma 1. *[8] Let $\psi \in A$ satisfies*

$$\Re(1 + z\psi''(z)/\psi'(z)) > -\frac{1}{2},\ \ z \in U. \tag{17}$$

If the function $\Lambda(z)$ of the form

$$\Lambda(z) = \frac{1}{z}\int_0^z \psi(t)dt,\ \ z \in U \tag{18}$$

then $\Lambda \in \mathscr{K}$.

Lemma 2. *[13] Assume that $\upsilon \in \mathscr{C}^* = \mathscr{C} \backslash \{0\}$ with $\Re(\upsilon) \geq 0$, η is a convex function with $\eta(0) = a$ and $\varphi : \mathscr{C}^2 \times U \to \mathscr{C}$. If $\sigma \in H[a,n]$ satisfies*

$$\sigma(0) = a,\ \varphi(\sigma(z),\ z\sigma'(z); z) = \sigma(z) + \upsilon^{-1} z\sigma'(z) \tag{19}$$

and

$$F_{\varphi(\mathscr{C}^2 \times U)} \left(\sigma(z) + \frac{1}{\upsilon} z\sigma'(z) \right) \leq F_{\eta(U)} \eta(z), \tag{20}$$

which is equivalent to

$$\sigma(z) + \frac{1}{\upsilon} z\sigma'(z) \prec_F \eta(z), z \in U, \tag{21}$$

then

$$F_{\sigma(U)} \sigma(z) \leq F_{\chi(U)} \chi(z) \leq F_{\eta(U)} \eta(z), \tag{22}$$

or equivalently to

$$\sigma(z) \prec_F \chi(z) \prec_F \eta(z),\ z \in U, \tag{23}$$

where

$$\chi(z) = \frac{\upsilon}{n z^{\upsilon/n}} \int_0^z \eta(t) t^{-1+\upsilon/n} dt,\ \ z \in U. \tag{24}$$

χ is a convex function and, it is the fuzzy best dominant.

Lemma 3. *[13] Let ς be a convex function in U and let*

$$\psi(z) = \varsigma(z) + n\gamma z\varsigma'(z)\ \ (\gamma > 0, n \in \mathscr{N}, z \in U). \tag{25}$$

If $\sigma(z) = \varsigma(0) + \sum_{k=n}^{\infty} \sigma_k z^k$ is analytic in U and

$$F_{\sigma(U)} \left(\sigma(z) + \gamma z\sigma'(z) \right) \leq F_{\psi(U)} \psi(z), \tag{26}$$

which is equivalent to

$$\sigma(z) + \gamma z\sigma'(z) \prec_F \psi(z)\ (z \in U). \tag{27}$$

Then

$$F_{\sigma(U)}(\sigma(z)) \leq F_{\varsigma(U)} \varsigma(z), \tag{28}$$

or equivalently to

$$\sigma(z) \prec_F \varsigma(z)\ (z \in U). \tag{29}$$

The result is sharp.

2 Main results

Through this paper, we assume that $\delta \in [0,1)$, $0 \leq \mu < 1$, $0 \leq \theta \leq 1$ and $z \in U$.

Theorem 1. *Suppose that $\lambda > 0$, r be a convex function in U and $\eta(z) = r(z) + zr'(z)/(\lambda+2)$. And let $\xi \in A$ and*

$$G(z) = I^{\lambda}\xi(z) = \frac{\lambda+2}{z^{\lambda+1}}\int_0^z t^{\lambda}\xi(t)dt. \tag{30}$$

If

$$F_{(R_\mu^\theta \xi)'(U)}(R_\mu^\theta \xi(z))' \leq F_{\eta(U)}\eta(z), \tag{31}$$

which is equivalent to

$$(R_\mu^\theta \xi(z))' \prec_F \eta(z), \tag{32}$$

then

$$F_{(R_\mu^\theta G)'(U)}(R_\mu^\theta G(z))' \leq F_{r(U)}r(z), \ \ i.e.\ \ (R_\mu^\theta G(z))' \prec_F r(z). \tag{33}$$

The result is sharp.

Proof . Since

$$z^{\lambda+1}G(z) = (\lambda+2)\int_0^z t^{\lambda}\xi(t)dt. \tag{34}$$

Differentiating (34), we get

$$(\lambda+1)G(z) + zG'(z) = (\lambda+2)\xi(z) \tag{35}$$

and

$$(\lambda+1)R_\mu^\theta G(z) + z(R_\mu^\theta G(z))' = (\lambda+2)R_\mu^\theta \xi(z). \tag{36}$$

Differentiating (36) we obtain

$$(R_\mu^\theta G(z))' + \frac{1}{(\lambda+2)}z(R_\mu^\theta G(z))'' = (R_\mu^\theta \xi(z))'. \tag{37}$$

By using (37), the fuzzy differential subordination (32) becomes

$$F_{R_\mu^\theta G(U)}\left((R_\mu^\theta G(z))' + \frac{1}{(\lambda+2)}z(R_\mu^\theta G(z))''\right) \leq F_{r(U)}\left(r(z) + \frac{1}{(\lambda+2)}zr'(z)\right). \tag{38}$$

Let

$$\chi(z) = (R_\mu^\theta G(z))',\ z \in U, \tag{39}$$

then $\chi \in H[1,1]$. Replacing (39) in (38) we have

$$F_{\chi(U)}\left(\chi(z) + \frac{1}{(\lambda+2)}z\chi'(z)\right) \leq F_{r(U)}\left(r(z) + \frac{1}{(\lambda+2)}zr'(z)\right). \tag{40}$$

Applying Lemma 3, we get

$$F_{\chi(U)}\chi(z) \leq F_{r(U)}r(z), \tag{41}$$

or

$$F_{(R_\mu^\theta G(z))'(U)}(R_\mu^\theta G(z))' \leq F_{r(U)} r(z), \tag{42}$$

which is equivalent to

$$(R_\mu^\theta G(z))' \prec_F r(z), \tag{43}$$

and r is the fuzzy best dominant.

Theorem 2. *If $\eta(z) = (1+(2\alpha-1)z)/(1+z)$, $\alpha \in [0,1)$, $\lambda > 0$ and I^λ is given by (30), then*

$$I^\lambda(M_\alpha^F(\theta,\mu)) \subset M_\zeta^F(\theta,\mu) \tag{44}$$

where

$$\zeta = 2\alpha - 1 + 2(\lambda+2)(1-\alpha)\int_0^1 \frac{t^{\lambda+1}}{t+1}dt. \tag{45}$$

Proof . Since η is convex function in U. By using the same technique as in the proof of Theorem 1, we get

$$F_{\chi(U)}\left(\chi(z) + \frac{1}{(\lambda+2)} z\chi'(z)\right) \leq F_{\eta(U)}(\eta(z)), \tag{46}$$

where $\chi(z)$ is defined in (39). An application of Lemma 2 gives

$$F_{\chi(U)}\chi(z) \leq F_{r(U)} r(z) \leq F_{\eta(U)}\eta(z) \tag{47}$$

or equivalently to

$$F_{(R_\mu^\theta G)'(U)}(R_\mu^\theta G(z))' \leq F_{r(U)} r(z) \leq F_{\eta(U)}\eta(z) \tag{48}$$

where

$$r(z) = \frac{\lambda+2}{z^{\lambda+2}}\int_0^z t^{\lambda+1}\frac{1+(2\alpha-1)t}{1+t}dt = (2\alpha-1) + \frac{2(\lambda+2)(1-\alpha)}{z^{\lambda+2}}\int_0^z \frac{t^{\lambda+1}}{t+1}dt. \tag{49}$$

Since r is convex and $r(U)$ is symmetric with respect to the real axis, we conclude

$$F_{(R_\mu^\theta G)'(U)}(R_\mu^\theta G(z))' \geq \min_{|z|=1} F_{r(U)} r(z) = F_{r(U)} r(1) \tag{50}$$

and $\zeta = r(1) = 2\alpha - 1 + 2(\lambda+2)(1-\alpha)\int_0^1 \frac{t^{\lambda+1}}{t+1}dt$, which ends the proof of the theorem.

Theorem 3. *Let r is a convex function in U with $r(0) = 1$, and let*

$$\eta(z) = r(z) + zr'(z). \tag{51}$$

If $\xi \in A$ satisfies the following fuzzy differential subordination

$$F_{(R_\mu^\theta \xi)'(U)}(R_\mu^\theta \xi(z))' \leq F_{\eta(U)}\eta(z), \tag{52}$$

which is equivalent to

$$(R_\mu^\theta \xi(z))' \prec_F \eta(z). \tag{53}$$

Then

$$F_{R_\mu^\theta \xi(U)} \frac{R_\mu^\theta \xi(z)}{z} \leq F_{r(U)} r(z), \tag{54}$$

or equivalently to

$$\frac{R_\mu^\theta \xi(z)}{z} \prec_F r(z). \tag{55}$$

The result is sharp.

Proof . By denoting

$$\chi(z) = \frac{1}{z} R_\mu^\theta \xi(z) = 1 + \sum_{k=2}^{\infty} \frac{(1-\mu)^{k-1}\, \Gamma(\theta+k)}{\Gamma(\theta+1)}\, a_k z^{k-1} \tag{56}$$

we obtain that

$$\chi(z) + z\chi'(z) = (R_\mu^\theta \xi(z))'. \tag{57}$$

From (53), we get

$$F_{\chi(U)}(\chi(z) + z\chi'(z)) \leq F_{\eta(U)} \eta(z) = F_{r(U)}(r(z) + zr'(z)). \tag{58}$$

An application of Lemma 3 gives

$$F_{\chi(U)} \chi(z) \leq F_{r(U)} r(z) \Rightarrow F_{R_\mu^\theta \xi(U)} \frac{R_\mu^\theta \xi(z)}{z} \leq F_{r(U)} r(z), \tag{59}$$

which completes the proof of the theorem.

Theorem 4. *Let* $\eta \in H(U)$ *with* $\eta(0) = 1$ *and* $\Re\left(\frac{1+z\eta''(z)}{\eta'(z)}\right) > -\frac{1}{2}$. *If* $\xi \in A$ *and satisfies the following fuzzy differential subordination*

$$F_{(R_\mu^\theta \xi)'(U)} (R_\mu^\theta \xi(z))' \leq F_{\eta(U)} \eta(z), \quad i.e.\ (R_\mu^\theta \xi(z))' \prec_F \eta(z), \tag{60}$$

then

$$F_{R_\mu^\theta \xi(U)} \frac{R_\mu^\theta \xi(z)}{z} \leq F_{r(U)} r(z), \tag{61}$$

which is equivalent to

$$\frac{R_\mu^\theta \xi(z)}{z} \prec_F r(z), \tag{62}$$

where

$$r(z) = \frac{1}{z} \int_0^z \eta(t) dt, \tag{63}$$

is a convex function and it is the fuzzy best dominant.

Proof . Since $\Re\left(\frac{1+z\eta''(z)}{\eta'(z)}\right) > -\frac{1}{2}$, an application Lemma 1 gives

$$r(z) = \frac{1}{z}\int_0^z \eta(t)dt \tag{64}$$

is a convex function, and

$$r(z) + zr'(z) = \eta(z). \tag{65}$$

Let

$$\chi(z) = \frac{R_\mu^\theta \xi(z)}{z} = 1 + \sum_{k=2}^{\infty} \frac{(1-\mu)^{k-1}\,\Gamma(\theta+k)}{\Gamma(\theta+1)}\, a_k z^{k-1}, \quad \chi \in H[1,1], \tag{66}$$

then, we have $\chi(z) + z\chi'(z) = (R_\mu^\theta \xi(z))'$, then (60) is equivalent to

$$F_{\chi(U)}(\chi(z) + z\chi'(z)) \le F_{\eta(U)}\eta(z). \tag{67}$$

An application of Lemma 3 gives

$$F_{\chi(U)}\chi(z) \le F_{r(U)}r(z) \Rightarrow F_{R_\mu^\theta \xi(U)}\frac{R_\mu^\theta \xi(z)}{z} \le F_{r(U)}r(z), \tag{68}$$

which is equivalent to

$$\frac{R_\mu^\theta \xi(z)}{z} \prec_F r(z). \tag{69}$$

Putting $\eta(z) = \frac{1+(2\beta-1)z}{1+z}$ in Theorem 4, we obtain the following corollary:

Corollary 1. *Let* $\eta(z) = \frac{1+(2\beta-1)z}{1+z}$, $0 \le \beta < 1$. *If* $\xi \in A$ *satisfies the fuzzy differential subordination*

$$F_{(R_\mu^\theta \xi)'(U)}(R_\mu^\theta \xi(z))' \le F_{\eta(U)}\eta(z), \quad i.e.\ (R_\mu^\theta \xi(z))' \prec_F \eta(z), \tag{70}$$

then

$$F_{R_\mu^\theta \xi(U)}\frac{R_\mu^\theta \xi(z)}{z} \le F_{r(U)}r(z), \tag{71}$$

or equivalently to

$$\frac{R_\mu^\theta \xi(z)}{z} \prec_F r(z), \tag{72}$$

where r is given by

$$r(z) = 2\beta - 1 + \frac{2(1-\beta)}{z}\ln(1+z) \tag{73}$$

is a convex function in U and it is the fuzzy best dominant.

Theorem 5. *Let r be a convex function in U with* $r(0) = 1$, *and let* $\eta(z) = r(z) + zr'(z)$. *If* $\xi \in A$ *satisfies the following fuzzy differential subordination*

$$F_{R_\mu^\theta \xi(U)}\left[(\theta+2)\frac{R_\mu^{\theta+2}\xi(z)}{R_\mu^\theta \xi(z)} - (\theta+1)\left(\frac{R_\mu^{\theta+1}\xi(z)}{R_\mu^\theta \xi(z)}\right)^2\right] \le F_{\eta(U)}\eta(z), \tag{74}$$

which is equivalent to

$$\left[(\theta+2)\frac{R_\mu^{\theta+2}\xi(z)}{R_\mu^{\theta}\xi(z)}-(\theta+1)\left(\frac{R_\mu^{\theta+1}\xi(z)}{R_\mu^{\theta}\xi(z)}\right)^2\right]\prec_F \eta(z), \tag{75}$$

then

$$F_{R_\mu^\theta\xi(U)}\frac{R_\mu^{\theta+1}\xi(z)}{R_\mu^{\theta}\xi(z)}\leq F_{r(U)}r(z), \tag{76}$$

or equivalently to

$$\frac{R_\mu^{\theta+1}\xi(z)}{R_\mu^{\theta}\xi(z)}\prec_F r(z). \tag{77}$$

The result is sharp.

Proof . Let

$$\chi(z)=\frac{R_\mu^{\theta+1}\xi(z)}{R_\mu^{\theta}\xi(z)} \tag{78}$$

By differentiating (78) and using the identity (15), we have

$$\begin{aligned}\frac{z\chi'(z)}{\chi(z)}&=(\theta+2)\frac{R_\mu^{\theta+2}\xi(z)}{R_\mu^{\theta+1}\xi(z)}-(\theta+1)\frac{R_\mu^{\theta+1}\xi(z)}{R_\mu^{\theta}\xi(z)}-1\\&=(\theta+2)\frac{R_\mu^{\theta+2}\xi(z)}{R_\mu^{\theta+1}\xi(z)}-(\theta+1)\chi(z)-1.\end{aligned}$$

It follows that

$$\chi(z)+z\chi'(z)=(\theta+2)\frac{R_\mu^{\theta+2}\xi(z)}{R_\mu^{\theta}\xi(z)}-(\theta+1)\left(\frac{R_\mu^{\theta+1}\xi(z)}{R_\mu^{\theta}\xi(z)}\right)^2. \tag{79}$$

Hence the fuzzy differential subordination (74) is equivalent to

$$F_{\chi(U)}(\chi(z)+z\chi'(z))\leq F_{\eta(U)}\eta(z)=F_{r(U)}(r(z)+zr'(z)). \tag{80}$$

By applying Lemma 3, we get

$$F_{\chi(U)}\chi(z)\leq F_{r(U)}r(z)\Rightarrow F_{R_\mu^\theta\xi(U)}\frac{R_\mu^{\theta+1}\xi(z)}{R_\mu^{\theta}\xi(z)}\leq F_{r(U)}r(z). \tag{81}$$

It ends the proof of Theorem 5.

3 Conclusion

In recent years, the concept of fuzzy differential subordination, which generalizes differential subordination, has been applied to some classes of analytic and univalent functions. This paper presents several fuzzy subordination results associated with the Rafid integral operator.

Availability of data and material

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Conflict of interest

The author declares that he has no competing interests.

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Authors' contributions

I completed the manuscript without anyone's contribution. The author read and approved the final manuscript.

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