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Grill Nano topological spaces with grill Nano generalized closed sets

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ABSTRACT

The grill Nano generalized closed sets is an expansion of Nano generalized closed sets in grill Nano topological spaces, which is the frame of reference measurement, inference and reasoning in many applications such as computer science and information systems. So, this paper aims to study and analyze this expansion through the topological structure via the concept of grill. Some important characteristics and main properties which are related with these sets are obtained.

1. Introduction

The concept of grill topological spaces, which is grounded on two operators, is Φ and Ψ. Choquet [1] was the first introduced this concept. It has been found out that there is some of similarity between Choquat concept and that ideals, nets and filters. A number of theories and features has been handled in [2–5]. It helps to expand the topological structure which is used to measure the description rather than quantity, such as love, intelligence, beauty, quality of education and etc. Also, it expands the topological structure by using the concept of grill changes in lower approximation, upper approximation and boundary region, which opens up new horizons in Nano topological spaces. In 1970, Levine [6] was the first author to introduce the idea of expansion closed sets. The idea of Nano topological structure is grounded on lower, upper and boundary approximations of a subset of an universe set with an equivalence relation on it. Also, it helps to introduce definition of closed, interior and closure set via concept of Nano . In 2013, Lellis [7] was the first to develop this idea. This research objectives are to insert grill in Nano generalized closed between Nano topology on the space. Some of important relations are obtained.

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2. Preliminaries

Definition 2.1. A nonempty subcollection G of a space X which carries topology τ is named grill [1] on this space if the following conditions are true:

(1) $\phi \notin G$,
(2) $A \in G$ and $A \subseteq B \subseteq X \Rightarrow B \in G$,
(3) if $A \cup B \in G$ for $A, B \subseteq X$, then $A \in G$ or $B \in G$.

Since the grill depends on the two mappings Φ and Ψ which is generated a unique grill topological space finer than τ on space X denoted by τ_G on X have been discussion in [3,5].

Definition 2.2 (see [6]). A subset B from a topology τ on the space X is said to be generalized closed set (shortly g-closed) if $Cl(B) \subseteq U$ for $B \subseteq U$ and U is open in (X, τ). A set B of topological space (X, τ) is called g-open if $X \backslash B$ is g-closed.

Definition 2.3 (see [8]). Suppose X is a miss-null finite set of objects nominated the universe and the equivalence relation R on X named as indiscernibility relation form a knowledge base (X, R). Then, X contains elements belonging to the same equivalence class which is said to be indiscernible with different elements. If $A \subseteq X$, the set of all elements of X which with certain assorted as elements of A to R is denoted by $\underline{R}\, A$ called lower approximation of a set A which is defined as $\underline{R}A = \{x \in X : R_x \subseteq A\}$ where R_x denotes the equivalence class containing $x \in X$.

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The upper approximation of a set A to R is the set of all objects, which possibly assorted as A to R and is denoted by $\overline{R}A$ which is defined as $\overline{R}A = \{x \in X : R_x \cap A \neq \phi\}$.

The boundary region of A to R is the set of all objects which can be assorted neither as A nor as the complement of A to R and is denoted by $B_R(A)$ which is defined as $B_R(A) = \overline{R}A - \underline{R}A$.

Property 2.4 (see [9]). If (X, R) is a knowledge base and $A, B \subseteq X$, then from the definition of approximation space we can get the following properties:

(1) $\underline{R}A \subseteq A \subseteq \overline{R}A$.
(2) $\underline{R}\phi = \overline{R}\phi = \phi$.
(3) $\underline{R}X = \overline{R}X = X$.
(4) $\overline{R}(A \cup B) = \overline{R}A \cup \overline{R}B$.
(5) $\underline{R}(A \cap B) = \overline{R}B \cap \underline{R}B$.
(6) If $A \subseteq B$, then $\overline{R}A \subseteq \overline{R}B, \overline{R}B \subseteq \underline{R}B$.
(7) $\underline{R}\ (A \cup B) \supseteq \underline{R}\ A \cup\ \underline{R}\ B$.
(8) $\overline{R}(A \cap B) \subseteq \overline{R}A \cap \overline{R}B$.
(9) $\underline{R}A^c = [\overline{R}A]^c$ and $\overline{R}A^c = [\underline{R}A]^c$.
(10) $\underline{R}\underline{R}A = \overline{R}\underline{R}A = \underline{R}A$.
(11) $\overline{R}\overline{R}A = \underline{R}\overline{R}A = \overline{R}A$.

Definition 2.5 (see [7]). Let X be cosmos set, R be an equivalence relation on X and $\tau_R(A) = \{X, \phi, \underline{R}A, \overline{R}A, B_R(A)\}$ where $A \subseteq X$. Then by above property, $\tau_R(A)$ satisfies the condition of topology on X which is called Nano topology on X with respect to A. $(X, \tau_R(A))$ is invited the Nano topological space. Elements of Nano topological are Nano open sets in X. Elements of $[\tau_R(A)]^c$ are called Nano closed sets with $[\tau_R(A)]^c$ being called dual Nano topology of $\tau_R(A)$.

Remark 2.6 (see [7]). Let $\tau_R(A)$ be Nano topology on X to A, then the set $B = \{X, \underline{R}A, B_R(A)\}$ is the base for $\tau_R(A)$.

Definition 2.7 (see [7]). If $(X, \tau_R(A))$ is Nano topological space with respect to A where $A \subseteq X$, if $B \subseteq X$, then

(1) The Nano interior of the set B defined as the union of all Nano open subsets contained in B and is defined by $NInt(B)$. That is $NInt(B)$ is the greatest Nano open subset of B.
(2) The Nano closure of the set B defined as the intersection of all Nano closed containing B and is denoted by $NCl(B)$. $NCl(B)$ is the smallest Nano closed set containing B.

Definition 2.8 (See [8]). A subset B of $(X, \tau_R(A))$ is called Nano generalized closed sets (shortly Ng-closed) if $NCl(B) \subseteq U$ for $B \subseteq U$ and U is Nano open in $(X, \tau_R(A))$.

3. Grill Nano Generalized Closed in Grill Nano Topological Space

Here, we introduce and study grill Nano generalized closed between grill Nano topological space.

Definition 3.1. Suppose that $(X, \tau_R(A))$ is Nano topology on the space X. Also, a set G satisfied conditions of the grill on X. A subset B of grill Nano topological space $(X, \tau_R(A), G)$ is $G - Ng$-closed if $\Phi(B) \subseteq U$ for $B \subseteq U$ for all U is Nano open. A subset B of X is said to be $G - Ng$-open if $X \backslash B$ is $G - Ng$-closed.

Remark 3.2.

(1) Each N-closed set is Ng-closed.
(2) Each Ng-closed set is $G - Ng$-closed but not vice versa as Example 3.3.

Example 3.3. When $X = \{1, 2, 3, 4\}$ with $X/R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $A = \{1, 2\}$. Then $\tau_R(A) = \{X, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$ which are Nano open sets. The Nano closed sets $= \{\phi, X, \{3\}, \{1, 3\}, \{2, 3, 4\}\}$ with $G = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, X\}$. This shows that the set $\{2\} \subseteq \{2, 3, 4\}$ is $G - Ng-$ closed but not $Ng-$ closed and the set $\{1, 2, 3\} \subseteq \{1, 2, 3, 4\}$ is Ng-closed but not N-closed.

Theorem 3.4. *A subset B of $(X, \tau_R(A), G)$ is $G - Ng$-closed if $\Phi(B) \backslash B$ contains nonempty $G - Ng$-closed set.*

Proof. Let B is $G - Ng$-closed set. Then $\Phi(B) \subseteq U$ for $B \subseteq U$ and U is Nano open. Let Y be a grill Nano closed subset of $\Phi(B) \backslash B \subseteq U$. Then $B \subseteq Y^c$ and Y^c is grill Nano generalized open. Since B is $G - Ng$-closed set, $\Phi(B) \subseteq Y^c$ or $Y \subseteq [\Phi(B)]^c$. That is $Y \subseteq \Phi(B)$ and $Y \subseteq [\Phi(B)]^c$ implies that $Y \subseteq \phi$. So Y is empty. □

Theorem 3.5. *If B and C are $G - Ng$-closed sets, then $B \cup C$ is $G - Ng$-closed.*

Proof. Let B and C are $G - Ng$-closed sets. Then $\Phi(B) \subseteq U$ whenever $B \subseteq U$ and U is Nano open and $\Phi(C) \subseteq U$ for $C \subseteq U$ and U is Nano open. Since B and C are subsets of U, $B \cup C \subseteq U$ and U is Nano open. Then $\Phi(B \cup C) = \Phi(B) \cup \Phi(C) \subseteq U$ which leads to $B \cup C$ is $G - Ng$-closed set. □

Remark 3.6. Presumably B and C are $G - Ng$-closed sets, then the intersection of them is $G - Ng$-closed set.

Example 3.7. Let $X = \{1, 2, 3\}$ with $X/R = \{\{1\}, \{2, 3\}\}$ and $A = \{1, 3\}$. Then the Nano topology $\tau_R(A) = \{X, \phi, \{1\}, \{2, 3\}\}$. The grill $G = \{\{1\}, \{1, 2\}, \{1, 3\}, X\}$, then the grill Nano generalized closed are $P(X)$. That is intersection of any two $G - Ng$-closed sets is $G - Ng$-closed set.

Theorem 3.8. *Let B be $G - Ng$-closed set and $B \subseteq C \subseteq \Phi(B)$, then C is $G - Ng$-closed set.*

Proof. Let $C \subseteq U$ whenever U is Nano open in $\tau_R(A)$ with grill G. Then $B \subseteq C$ implies $B \subseteq U$. Since B is $G - Ng$-closed, $\Phi(B) \subseteq U$. Also, $C \subseteq \Phi(B)$ implies $\Phi(C) \subseteq \Phi(B)$. Thus $\Phi(C) \subseteq U$ and so C is $G - Ng$-closed. □

Theorem 3.9. *Each Nano closed is grill Nano generalized closed.*

Proof. Let $(X, \tau_R(A), G)$ be a grill Nano topological space, $B \subseteq U$ and U is Nano open in $\tau_R(A)$. Since B is Nano closed then , it is $N - g-$closed that is $NCl(B) = B$ and $\Phi(B) \subseteq B$ which leads to $\Phi(B) \subseteq NCl(B)$. Thus $\Phi(B) \subseteq B \subseteq U$. Hence B is grill Nano generalized closed set. □

Example 3.10. When $X = \{a, b, c, d\}$ with $X/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $A = \{a, c\}$. Then $\tau_R(A) = \{X, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$ which are Nano open sets. The Nano closed sets $= \{\phi, X, \{b, c, d\}, \{b\}, \{a, b\}\}$ with $G = \{\{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, X\}$. Let $B = \{a, b, c\}$ and $B \subseteq X$, $\Phi(B) \subseteq X$ which implies that B is $G - Ng$-closed while B is not Nano closed.

This model demonstrates that the converse of Theorem 3.9 is not true in general.

Theorem 3.11. *An $G - Ng$-closed set B is Nano closed if and only if $\Phi(B) \backslash B$ Nano closed.*

Proof. (Necessity) Presumably that B is Nano closed. Then $\Phi(B) = B$ and so $\Phi(B) \backslash B = \phi$ which is Nano closed.

(sufficiency) Suppose $\Phi(B) \backslash B$ is Nano closed. Then $\Phi(B) \backslash B = \phi$ since B is Nano closed. That is $\Phi(B) = B$ or B is Nano closed. □

Theorem 3.12. *Suppose that $C \subseteq B \subseteq X$, C is an $G - Ng$-closed set relative to B and B is an $G - Ng$-closed subset of X. We conclude that C is $G - Ng$-closed relative to X.*

Proof. Let $C \subseteq U$ and suppose that U is Nano open. Then $C \subseteq B \cap U$. Therefore $\Phi B(C) \subseteq B \cap U$. It follows then that $B \cap \Phi(C) \subseteq B \cap U$ and $B \subseteq U \cup \Phi(C)$. Since B is $G - Ng$-closed in X, we have $\Phi(B) \subseteq U \cup \Phi(C)$. Therefore $\Phi(C) \subseteq \Phi(B) \subseteq U \cup \Phi(C)$, so $\Phi(C) \subseteq U$. Then C is $G - Ng$-closed relative to X □

Corollary 3.13. *Let B be a $G-Ng$-closed set and suppose that F is Nano closed set. Then $B\cap F$ is a $G-Ng$-closed set which is given in the following example.*

Example 3.14. Let $X=\{1,2,3,4\}$, $A=\{1,2\}$ and $X/R=\{\{1\},\{3\},\{2,4\}\}$. Then $\tau_R(A)=\{X,\phi,\{1\},\{1,2,4\},\{2,4\}\}$ which are Nano open sets. Nano closed sets $=\{\phi,X,\{2,3,4\},\{3\},\{1,3\}\}$ with $G=\{\{1,3\},\{1,2,3\},\{1,3,4\},X\}$. Let $B=\{1,2,3\}$ and $F=\{1,3\}$. Then $B\cap F=\{1,3\}$ is an $G-Ng$-closed set.

Theorem 3.15. *For each $a\in X$ in a grill Nano topological space $(X, \tau_R(A), G)$, either $\{a\}$ is Nano closed or $\{a\}^c$ is grill Nano generalized closed in $\tau_R(A)$.*

Proof. Suppose $\{a\}$ is not Nano closed in X. Then $\{a\}^c$ is not Nano open and the only Nano open set containing $\{a\}^c$ is $U\subseteq X$. This means that $\{a\}^c\subseteq X$ and therefore $\Phi(\{a\}^c)\subseteq X$. This leads to $\{a\}^c$ is grill Nano generalized closed set in $\tau_R(A)$. □

4. Conclusion

Because of the topological space is stripped of the geometric form and it is used to measure things that are difficult to measure, such as intelligence, beauty and goodness. So, we used the concept of grill to expand this space to help us measure the things that are difficult to measure. In this paper, properties of $G-Ng$-closed sets between Nano topological spaces in terms of grill are obtained and the expansion of Nano generalized closed sets by grill is introduced.

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References

[1] G. Choquet, Sur les notions de filter et. grill, Completes Rendus Acad. Sci. Paris 224 (1947) 171–173.
[2] A. Al-Omari, T. Noiri, Decomposition of continuity via grilles, Jordan J. Math. Stat. 4 (1) (2011) 33–46.
[3] E. Hater, S. Jafari, On some new classes of sets and a new decomposition of continuity via grills, J. Adv. Math. Studies. 3 (1) (2010) 33–40.
[4] A.A. Nasef, A.A. Azzam, On slighlitly g-precontinuous functions, in: 5th Int. Conference on Math. and Infor. Science(ICMIS2016), 2016, pp. 11–13. Zewail City of scince and technology
[5] B. Roy, M.N. Mukherjee, On a typical topology induced by a grill, Soochow J. Math. 33 (4) (2007) 771–786.
[6] N. Levine, Generalized closed sets in topological spaces, Rend Circ. Mat. Palemo 19 (1970) 89–96.
[7] M.L. Thivagar, C. Richard, On Nano forms of weakly open sets, Int. J. Math. Stat. Invention 1 (1) (2013) 31–37.
[8] Z. pawlak, Rough sets, Int. J. Comput. Inf. Sci. 11 (5) (1982) 341–356.
[9] Z. pawlak, Rough Sets, Theoretical Aspects of Reasoning about Data, 9, Kluwer Academic publishers, Dordrecht, 1991.