Journal of the Egyptian Mathematical Society 25 (2017) 158–163

Contents lists available at ScienceDirect

Journal of the Egyptian Mathematical Society

journal homepage: www.elsevier.com/locate/joems

Original Article

Almost contra $\beta\theta$-continuity in topological spaces☆

M. Caldas[a], M. Ganster[b], S. Jafari[c,*], T. Noiri[d], V. Popa[e]

[a] Departamento de Matematica Aplicada, Universidade Federal Fluminense, Rua Mario Santos Braga, s/n 24020-140, Niteroi, RJ, Brazil
[b] Department of Mathematics, Graz University of Technology, Steyrergasse 30, A-8010 Graz, Austria
[c] College of Vestsjaelland South, Herrestraede 11, 4200 Slagelse, Denmark
[d] 2949-1 Shiokita-cho, Hinagu, Yatsushiro-shi, Kumamoto-ken, 869-5142, Japan
[e] Department of Mathematics and Informatics, Faculty of Sciences "Vasile Alecsandri", University of Bacău, 157 Calea Mărăşeşti, Bacău, 600115, Romania

ARTICLE INFO

Article history:
Received 27 April 2016
Revised 16 August 2016
Accepted 20 August 2016
Available online 28 August 2016

MSC:
Primary 54C08
54C10
Secondary 54C05

keywords:
Topological space
$\beta\theta$-Open set
$\beta\theta$-Closed set
Almost contra-continuous function
Almost contra $\beta\theta$-continuous function

ABSTRACT

In this paper, we introduce and investigate the notion of almost contra $\beta\theta$-continuous functions by utilizing $\beta\theta$-closed sets. We obtain fundamental properties of almost contra $\beta\theta$-continuous functions and discuss the relationships between almost contra $\beta\theta$-continuity and other related functions.

1. Introduction and preliminaries

Recently, Baker (resp. Ekici, Noiri and Popa) introduced and investigated the notions of contra almost β-continuity [1] (resp. almost contra pre-continuity [2,3]) as a continuation of research done by Caldas and Jafari [4] (resp. Jafari and Noiri [5]) on the notion of contra-β-continuity (resp. contra pre-continuity). In this paper, new generalizations of contra $\beta\theta$-continuity [6] by using $\beta\theta$-closed sets called almost contra $\beta\theta$-continuity are presented. We obtain some characterizations of almost contra $\beta\theta$-continuous functions and investigate their properties and the relationships between almost contra $\beta\theta$-continuity and other related generalized forms of continuity.

Throughout this paper, by (X, τ) and (Y, σ) (or X and Y) we always mean topological spaces. Let A be a subset of X. We denote the interior, the closure and the complement of a set A by $Int(A)$, $Cl(A)$ and $X\backslash A$, respectively. A subset A of X is said to be regular open (resp. regular closed) if $A = Int(Cl(A))$ (resp. $A = Cl(Int(A))$). A subset A of a space X is called preopen [7] (resp. semi-open [8], β-open [9], α-open [10]) if $A \subset Int(Cl(A))$ (resp. $A \subset Cl(Int(A))$, $A \subset Cl(Int(Cl(A)))$, $A \subset Int(Cl(Int(A)))$). The complement of a preopen (resp. semi-open, β-open, α-open) set is said to be preclosed (resp. semi-closed, β-closed, α-closed). The collection of all open (resp. closed, regular open, preopen, semiopen, β-open) subsets of X will be denoted by $O(X)$ (resp. $C(X)$, $RO(X)$, $PO(X)$, $SO(X)$, $\beta O(X)$). We set $RO(X, x) = \{U : x \in U \in RO(X, \tau)\}$, $SO(X, x) = \{U : x \in U \in SO(X, \tau)\}$ and $\beta O(X, x) = \{U : x \in U \in \beta O(X, \tau)\}$. We denote the collection of all regular closed subsets of X by $RC(X)$. We set $RC(X, x) = \{U : x \in U \in RC(X, \tau)\}$. We denote the collection of all β-regular (i.e., if it is both β-open and β-closed) subsets of X by $\beta R(X)$. A point $x \in X$ is said to be a θ-semi-cluster point [11] of a subset S of X if $Cl(U) \cap A \neq \emptyset$ for every $U \in SO(X, x)$. The set of all θ-semi-cluster points of A is called the θ-semi-closure of A and is denoted by $\theta sCl(A)$. A subset A is called θ-semi-closed [11] if $A = \theta sCl(A)$. The complement of a θ-semi-closed set is called θ-semi-open.

The $\beta\theta$-closure of A [12], denoted by $\beta Cl_\theta(A)$, is defined to be the set of all $x \in X$ such that $\beta Cl(O) \cap A \neq \emptyset$ for every $O \in \beta O(X, \tau)$ with $x \in O$. The set $\{x \in X : \beta Cl_\theta(O) \subset A \text{ for some } O \in \beta O(X, x)\}$

☆ Dedicated to our friend and colleague the late Professor Mohamad Ezat Abd El-Monsef

* Corresponding author.
E-mail addresses: gmamccs@vm.uff.br (M. Caldas), ganster@weyl.math.tu-graz.ac.at (M. Ganster), jafaripersia@gmail.com (S. Jafari), t.noiri@nifty.com (T. Noiri), v.popa@ub.ro (V. Popa).

http://dx.doi.org/10.1016/j.joems.2016.08.002

is called the $\beta\theta$-interior of A and is denoted by $\beta Int_\theta(A)$. A subset A is said to be $\beta\theta$-closed [12] if $A = \beta Cl_\theta(A)$. The complement of a $\beta\theta$-closed set is said to be $\beta\theta$-open. The family of all $\beta\theta$-open (resp. $\beta\theta$-closed) subsets of X is denoted by $\beta\theta O(X, \tau)$ or $\beta\theta O(X)$ (resp. $\beta\theta C(X, \tau)$). We set $\beta\theta O(X, x) = \{U : x \in U \in \beta\theta O(X, \tau)\}$ and $\beta\theta C(X, x) = \{U : x \in U \in \beta\theta C(X, \tau)\}$.

We recall the following two lemmas which were obtained by Noiri [12].

Lemma 1.1 [12]. *Let A be a subset of a topological space* (X, τ).

(i) *If* $A \in \beta O(X, \tau)$, *then* $\beta Cl(A) \in \beta R(X)$.
(ii) $A \in \beta R(X)$ *if and only if* $A \in \beta\theta O(X) \cap \beta\theta C(X)$.

Lemma 1.2 [12]. *For the* $\beta\theta$*-closure of a subset A of a topological space* (X, τ), *the following properties are hold:*

(i) $A \subset \beta Cl(A) \subset \beta Cl_\theta(A)$ *and* $\beta Cl(A) = \beta Cl_\theta(A)$ *if* $A \in \beta O(X)$.
(ii) *If* $A \subset B$, *then* $\beta Cl_\theta(A) \subset \beta Cl_\theta(B)$.
(iii) *If* $A_\alpha \in \beta\theta C(X)$ *for each* $\alpha \in A$, *then* $\bigcap\{A_\alpha \mid \alpha \in A\} \in \beta\theta C(X)$.
(iv) *If* $A_\alpha \in \beta\theta O(X)$ *for each* $\alpha \in A$, *then* $\bigcup\{A_\alpha \mid \alpha \in A\} \in \beta\theta O(X)$.
(v) $\beta Cl_\theta(\beta Cl_\theta(A)) = \beta Cl_\theta(A)$ *and* $\beta Cl_\theta(A) \in \beta\theta C(X)$.

Definition 1. A function $f: X \to Y$ is said to be:

(1) $\beta\theta$-continuous [12] if $f^{-1}(V)$ is $\beta\theta$-closed for every closed set V in Y, equivalently if the inverse image of every open set V in Y is $\beta\theta$-open in X.
(2) Almost $\beta\theta$-continuous if $f^{-1}(V)$ is $\beta\theta$-closed in X for every regular closed set V in Y.
(3) Contra R-maps [13] (resp. contra-continuous [14], contra $\beta\theta$-continuous [6]) if $f^{-1}(V)$ is regular closed (resp. closed, $\beta\theta$-closed) in X for every regular open (resp. open, open) set V of Y.
(4) Almost contra pre-continuous [2] (resp. almost contra β-continuous [1], almost contra -continuous [1]) if $f^{-1}(V)$ is preclosed (resp. β-closed, closed) in X for every regular open set V of Y.
(5) Regular set-connected [15] if $f^{-1}(V)$ is clopen in X for every regular open set V in Y.

2. Characterizations

Definition 2. A function $f: X \to Y$ is said to be almost contra $\beta\theta$-continuous if $f^{-1}(V)$ is $\beta\theta$-closed in X for each regular open set V of Y.

Definition 3. Let A be a subset of a space (X, τ). The set $\bigcap\{U \in RO(X) : A \subset U\}$ is called the r-kernel of A [13] and is denoted by $rker(A)$.

Lemma 2.1 (Ekici [13]). *For subsets A and B of a space X, the following properties hold:*

(1) $x \in rker(A)$ *if and only if* $A \cap F \neq \emptyset$ *for any* $F \in RC(X, x)$.
(2) $A \subset rker(A)$ *and* $A = rker(A)$ *if A is regular open in X.*
(3) *If* $A \subset B$, *then* $rker(A) \subset rker(B)$.

Theorem 2.2. *For a function* $f: X \to Y$, *the following properties are equivalent:*

(1) *f is almost contra* $\beta\theta$*-continuous;*
(2) *The inverse image of each regular closed set in Y is* $\beta\theta$*-open in X;*
(3) *For each point x in X and each* $V \in RC(Y, f(x))$, *there is a* $U \in \beta\theta O(X, x)$ *such that* $f(U) \subset V$;
(4) *For each point x in X and each* $V \in SO(Y, f(x))$, *there is a* $U \in \beta\theta O(X, x)$ *such that* $f(U) \subset Cl(V)$;
(5) $f(\beta Cl_\theta(A)) \subset rker(f(A))$ *for every subset A of X;*
(6) $\beta Cl_\theta(f^{-1}(B)) \subset f^{-1}(rker(B))$ *for every subset B of Y;*
(7) $f^{-1}(Cl(V))$ *is* $\beta\theta$*-open for every* $V \in \beta O(Y)$;
(8) $f^{-1}(Cl(V))$ *is* $\beta\theta$*-open for every* $V \in SO(Y)$;
(9) $f^{-1}(Int(Cl(V)))$ *is* $\beta\theta$*-closed for every* $V \in PO(Y)$;
(10) $f^{-1}(Int(Cl(V)))$ *is* $\beta\theta$*-closed for every* $V \in O(Y)$;
(11) $f^{-1}(Cl(Int(V)))$ *is* $\beta\theta$*-open for every* $V \in C(Y)$.

Proof. (1)⇔(2): see Definition 2.

(2)⇔(4): Let $x \in X$ and V be any semiopen set of Y containing $f(x)$, then $Cl(V)$ is regular closed. By (2) $f^{-1}(Cl(V))$ is $\beta\theta$-open and therefore there exists $U \in \beta\theta O(X, x)$ such that $U \subset f^{-1}(Cl(V))$. Hence $f(U) \subset Cl(V)$.

Conversely, suppose that (4) holds. Let V be any regular closed set of Y and $x \in f^{-1}(V)$. Then V is a semiopen set containing $f(x)$ and there exists $U \in \beta\theta O(X, x)$ such that $U \subset f^{-1}(Cl(V)) = f^{-1}(V)$. Therefore, $x \in U \subset f^{-1}(V)$ and hence $x \in U \subset \beta Int_\theta(f^{-1}(V))$. Consequently, we have $f^{-1}(V) \subset \beta Int_\theta(f^{-1}(V))$. Therefore $f^{-1}(V) = \beta Int_\theta(f^{-1}(V))$, i.e., $f^{-1}(V)$ is $\beta\theta$-open.

(2)⇒(3): Let $x \in X$ and V be a regular closed set of Y containing $f(x)$. Then $x \in f^{-1}(V)$. Since by hypothesis $f^{-1}(V)$ is $\beta\theta$-open, there exists $U \in \beta\theta O(X, x)$ such that $x \in U \subset f^{-1}(V)$. Hence $x \in U$ and $f(U) \subset V$.

(3)⇒(5): Let A be any subset of X. Suppose that $y \notin rker(f(A))$. Then, by Lemma 2.1 there exists $V \in RC(Y, y)$ such that $f(A) \cap V = \emptyset$. For any $x \in f^{-1}(V)$, by (3) there exists $U_x \in \beta\theta O(X, x)$ such that $f(U_x) \subset V$. Hence $f(A \cap U_x) \subset f(A) \cap f(U_x) \subset f(A) \cap V = \emptyset$ and $A \cap U_x = \emptyset$. This shows that $x \notin \beta Cl_\theta(A)$ for any $x \in f^{-1}(V)$. Therefore, $f^{-1}(V) \cap \beta Cl_\theta(A) = \emptyset$ and hence $V \cap f(\beta Cl_\theta(A)) = \emptyset$. Thus, $y \notin f(\beta Cl_\theta(A))$. Consequently, we obtain $f(\beta Cl_\theta(A)) \subset rker(f(A))$.

(5)⇔(6): Let B be any subset of Y. By (5) and Lemma 2.1, we have $f(\beta Cl_\theta(f^{-1}(B))) \subset rker(ff^{-1}(B)) \subset rker(B)$ and $\beta Cl_\theta(f^{-1}(B)) \subset f^{-1}(rker(B))$.

Conversely, suppose that (6) holds. Let $B = f(A)$, where A is a subset of X. Then $\beta Cl_\theta(A) \subset \beta Cl_\theta(f^{-1}(B)) \subset f^{-1}(rker(f(A)))$. Therefore $f(\beta Cl_\theta(A)) \subset rker(f(A))$.

(6)⇒(1): Let V be any regular open set of Y. Then, by (6) and Lemma 2.1(2) we have $\beta Cl_\theta(f^{-1}(V)) \subset f^{-1}(rker(V)) = f^{-1}(V)$ and $\beta Cl_\theta(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\beta\theta$-closed in X. Therefore f is almost contra $\beta\theta$-continuous.

(2)⇒(7): Let V be any β-open set of Y. It follows from ([16], Theorem 2.4) that $Cl(V)$ is regular closed. Then by (2) $f^{-1}(Cl(V))$ is $\beta\theta$-open in X.

(7)⇒(8): This is clear since every semiopen set is β-open. (8)⇒(9): Let V be any preopen set of Y. Then $Int(Cl(V))$ is regular open. Therefore $Y \backslash Int(Cl(V))$ is regular closed and hence it is semiopen. Then by (8) $X \backslash f^{-1}(Int(Cl(V))) = f^{-1}(Y \backslash Int(Cl(V))) = f^{-1}(Cl(Y \backslash Int(Cl(V))))$ is $\beta\theta$-open. Hence $f^{-1}(Int(Cl(V)))$ is $\beta\theta$-closed.

(9)⇒(1): Let V be any regular open set of Y. Then V is preopen and by (9) $f^{-1}(V) = f^{-1}(Int(Cl(V)))$ is $\beta\theta$-closed. It shows that f is almost contra $\beta\theta$-continuous.

(1)⇔(10): Let V be an open subset of Y. Since $Int(Cl(V))$ is regular open, $f^{-1}(Int(Cl(V)))$ is $\beta\theta$-closed. The converse is similar.

(2)⇔(11): Similar to (1)⇔(10). □

Lemma 2.3 [17]. *For a subset A of a topological space* (Y, σ), *the following properties hold:*

(1) $\alpha Cl(A) = Cl(A)$ *for every* $A \in \beta O(Y)$.
(2) $pCl(A) = Cl(A)$ *for every* $A \in SO(Y)$.
(3) $sCl(A) = Int(Cl(A))$ *for every* $A \in PO(Y)$.

Corollary 2.4. *For a function* $f: X \to Y$, *the following properties are equivalent:*

(1) *f is almost contra* $\beta\theta$*-continuous;*
(2) $f^{-1}(\alpha Cl(A))$ *is* $\beta\theta$*-open for every* $A \in \beta O(Y)$;
(3) $f^{-1}(pCl(A))$ *is* $\beta\theta$*-open for every* $A \in SO(Y)$;
(4) $f^{-1}(sCl(A))$ *is* $\beta\theta$*-closed for every* $A \in PO(Y)$.

Proof. It follows from Lemma 2.3. □

Theorem 2.5. *For a function $f: X \to Y$, the following properties are equivalent:*

(1) *f is almost contra $\beta\theta$-continuous;*
(2) *the inverse image of a θ-semi-open set of Y is $\beta\theta$-open;*
(3) *the inverse image of a θ-semi-closed set of Y is $\beta\theta$-closed;*
(4) *$f^{-1}(V) \subset \beta Int_\theta(f^{-1}(Cl(V)))$ for every $V \in SO(Y)$;*
(5) *$f(\beta Cl_\theta(A)) \subset \theta sCl(f(A))$ for every subset A of X;*
(6) *$\beta Cl_\theta(f^{-1}(B)) \subset f^{-1}(\theta sCl(B))$ for every subset B of Y;*
(7) *$\beta Cl_\theta(f^{-1}(V)) \subset f^{-1}(\theta sCl(V))$ for every open subset V of Y;*
(8) *$\beta Cl_\theta(f^{-1}(V)) \subset f^{-1}(sCl(V))$ for every open subset V of Y;*
(9) *$\beta Cl_\theta(f^{-1}(V)) \subset f^{-1}(Int(Cl(V)))$ for every open subset V of Y.*

Proof. (1)⇒(2): Since any θ-semiopen set is a union of regular closed sets, by using (1) and Theorem 2.2, we obtain that (2) holds.

(2)⇒(1): Let $x \in X$ and $V \in SO(Y)$ containing $f(x)$. Since $Cl(V)$ is θ-semiopen in Y, there exists a $\beta\theta$-open set U in X containing x such that $x \in U \subset f^{-1}(Cl(V))$. Hence $f(U) \subset Cl(V)$.

(1)⇒(4): Let $V \in SO(Y)$ and $x \in f^{-1}(V)$. Then $f(x) \in V$. By (1) and Theorem 2.2, there exists a $U \in \beta\theta O(X, x)$ such that $f(U) \subset Cl(V)$. It follows that $x \in U \subset f^{-1}(Cl(V))$. Hence $x \in \beta Int_\theta(f^{-1}(Cl(V)))$. Thus $f^{-1}(V) \subset \beta Int_\theta(f^{-1}(Cl(V)))$.

(4)⇒(1): Let F be any regular closed set of Y. Since $F \in SO(Y)$, then by (4), $f^{-1}(F) \subset \beta Int_\theta(f^{-1}(F))$. This shows that $f^{-1}(F)$ is $\beta\theta$-open, by Theorem 2.2, (1) holds.

(2)⇔(3): Obvious.

(1)⇒(5): Let A be any subset of X. Suppose that $x \in \beta Cl_\theta(A)$ and G is any semiopen set of Y containing $f(x)$. By (1) and Theorem 2.2, there exists $U \in \beta\theta O(X, x)$ such that $f(U) \subset Cl(G)$. Since $x \in \beta Cl_\theta(A)$, $U \cap A \neq \emptyset$ and hence $\emptyset \neq f(U) \cap f(A) \subset Cl(G) \cap f(A)$. Therefore, we obtain $f(x) \in \theta sCl(f(A))$ an hence $f(\beta Cl_\theta(A)) \subset \theta sCl(f(A))$.

(5)⇒(6): Let B be any subset of Y. Then $f(\beta Cl_\theta(f^{-1}(B))) \subset \theta sCl(f(f^{-1}(B)) \subset \theta sCl(B)$ and $\beta Cl_\theta(f^{-1}(B)) \subset f^{-1}(\theta sCl(f(B))$.

(6)⇒(1): Let V be any semiopen set of Y containing $f(x)$. Since $Cl(V) \cap (Y \backslash Cl(V)) = \emptyset$. we have $f(x) \notin \theta sCl(Y \backslash ClV))$ and $x \notin f^{-1}(\theta sCl(Y \backslash Cl(V)))$. By (6), $x \notin \beta Cl_\theta(f^{-1}(Y \backslash Cl(V)))$. Hence, there exists $U \in \beta\theta O(X, x)$ such that $U \cap f^{-1}(Y \backslash Cl(V)) = \emptyset$ and $f(U) \cap (Y \backslash Cl(V)) = \emptyset$. It follows that $f(U) \subset Cl(V)$. Thus, by Theorem 2.2, we have that (1) holds.

(6)⇒(7): Obvious.

(7)⇒(8): Obvious from the fact that $\theta sCl(V) = sCl(V)$ for an open set V.

(8)⇒(9): Obvious from Lemma 2.3.

(9)⇒(1): Let $V \in RO(Y)$. Then by (9) $\beta Cl_\theta(f^{-1}(V)) \subset f^{-1}(Int(Cl(V))) = f^{-1}(V)$. Hence, $f^{-1}(V)$ is $\beta\theta$-closed which proves that f is almost contra $\beta\theta$-continuous. □

Corollary 2.6. *For a function $f: X \to Y$, the following properties are equivalent:*

(1) *f is almost contra $\beta\theta$-continuous;*
(2) *$\beta Cl_\theta(f^{-1}(B)) \subset f^{-1}(\theta sCl(B))$ for every $B \in SO(Y)$.*
(3) *$\beta Cl_\theta(f^{-1}(B)) \subset f^{-1}(\theta sCl(B))$ for every $B \in PO(Y)$.*
(4) *$\beta Cl_\theta(f^{-1}(B)) \subset f^{-1}(\theta sCl(B))$ for every $B \in \beta O(Y)$.*

Proof. In Theorem 2.5, we have proved that the following are equivalent:

(1) f is almost contra $\beta\theta$-continuous;
(2) $\beta Cl_\theta(f^{-1}(B)) \subset f^{-1}(\theta sCl(B))$ for every subset B of Y.

Hence the corollary is proved. □

Recall that a topological space (X, τ) is said to be extremally disconnected if the closure of every open set of X is open in X.

Theorem 2.7. *If (Y, σ) is extremally disconnected, then the following properties are equivalent for a function $f: X \to Y$:*

(1) *f is almost contra $\beta\theta$-continuous;*
(2) *f is almost $\beta\theta$-continuous.*

Proof. (1)⇒(2): Let $x \in X$ and U be any regular open set of Y containing $f(x)$. Since Y is extremally disconnected, by Lemma 5.6 of [18] U is clopen and hence U is regular closed. Then $f^{-1}(U)$ is $\beta\theta$-open in X. Thus f is almost $\beta\theta$-continuous.

(2)⇒(1): Let B be any regular closed set of Y. Since Y is extremally disconnected, B is regular open and $f^{-1}(B)$ is $\beta\theta$-open in X. Thus f is almost contra $\beta\theta$-continuous.

The following implications are hold for a function $f: X \to Y$:

$$\begin{array}{ccccccc} & & A & \leftarrow & B & \leftarrow & C \\ & \nearrow & \uparrow & \nwarrow & & & \downarrow \\ H & \leftarrow & D & \rightarrow & E & & G \\ & \nwarrow & \uparrow & \nearrow & & & \\ & & F & & & & \end{array}$$

Notation: $A =$ almost contra β-continuity, $B =$ almost contra $\beta\theta$-continuity, $C =$ contra $\beta\theta$-continuity, $D =$ almost contra-continuity, $E =$ almost contra pre-continuity, $F =$ contra R-map, $G =$ contra β-continuity, $H =$ almost contra semi-continuity. □

Example 2.8. Let (X, τ) be a topological space such that $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$. Clearly $\beta\theta O(X, \tau) = \{\emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Let $f: X \to X$ be defined by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is almost contra $\beta\theta$-continuous but f is not contra $\beta\theta$-continuous, not $\beta\theta$-continuous and also is not contra continuous.

Other implications not reversible are shown in [2,3,5,6,13,15].

Theorem 2.9. *If $f: X \to Y$ is an almost contra $\beta\theta$-continuous function which satisfies the property $\beta Int_\theta((f^{-1}(Cl(V)))) \subset f^{-1}(V)$ for each open set V of Y, then f is $\beta\theta$-continuous.*

Proof. Let V be any open set of Y. Since f is almost contra $\beta\theta$-continuous by Theorem 2.2 $f^{-1}(V) \subset f^{-1}(Cl(V)) = \beta Int_\theta(\beta Int_\theta(f^{-1}(Cl(V)))) \subset \beta Int_\theta(f^{-1}(V)) \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is $\beta\theta$-open and therefore f is $\beta\theta$-continuous.

Recall that a topological space is said to be P_Σ [19] if for any open set V of X and each $x \in V$, there exists a regular closed set F of X containing x such that $x \in F \subset V$. □

Theorem 2.10. *If $f: X \to Y$ is an almost contra $\beta\theta$-continuous function and Y is P_Σ, then f is $\beta\theta$-continuous.*

Proof. Suppose that V is any open set of Y. By the fact that Y is P_Σ, so there exists a subfamily Ω of regular closed sets of Y such that $V = \bigcup\{F \mid F \in \Omega\}$. Since f is almost contra $\beta\theta$-continuous, then $f^{-1}(F)$ is $\beta\theta$-open in X for each $F \in \Omega$. Therefore $f^{-1}(V)$ is $\beta\theta$-open in X. Hence f is $\beta\theta$-continuous.

Recall that a function $f: X \to Y$ is said to be:

a) R-map [20] (resp. pre $\beta\theta$-closed [21]) if $f^{-1}(V)$ is regular closed in X for every regular closed V of Y (resp. $f(V)$ is $\beta\theta$-closed in Y for every $\beta\theta$-closed V of X).
b) weakly β-irresolute [12] if $f^{-1}(V)$ is $\beta\theta$-open in X for every $\beta\theta$-open set V in Y. □

Theorem 2.11. *Let $f: X \to Y$ and $g: Y \to Z$ be functions. Then the following properties hold:*

(1) *If f is almost contra-$\beta\theta$-continuous and g is an R-map, then $g \circ f: X \to Z$ is almost contra $\beta\theta$-continuous.*
(2) *If f is almost $\beta\theta$-continuous and g is a contra R-map, then $g \circ f: X \to Z$ is almost contra $\beta\theta$-continuous.*
(3) *If f is weakly β-irresolute and g is almost contra $\beta\theta$-continuous, then $g \circ f$ is almost contra $\beta\theta$-continuous.*

Theorem 2.12. *If $f: X \to Y$ is a pre $\beta\theta$-closed surjection and $g: Y \to Z$ is a function such that $g \circ f: X \to Z$ is almost contra $\beta\theta$-continuous, then g is almost contra $\beta\theta$-continuous.*

Proof. Let V be any regular open set in Z. Since $g \circ f$ is almost contra $\beta\theta$-continuous, $f^{-1}(g^{-1}((V))) = (g \circ f)^{-1}(V)$ is $\beta\theta$-closed. Since f is a pre $\beta\theta$-closed surjection, $f(f^{-1}(g^{-1}((V)))) = g^{-1}(V)$ is $\beta\theta$-closed. Therefore g is almost contra $\beta\theta$-continuous. □

Theorem 2.13. *Let $\{X_i : i \in \Omega\}$ be any family of topological spaces. If $f: X \to \prod X_i$ is an almost contra $\beta\theta$-continuous function, then $Pr_i \circ f: X \to X_i$ is almost contra $\beta\theta$-continuous for each $i \in \Omega$, where Pr_i is the projection of $\prod X_i$ onto X_i.*

Proof. Let U_i be an arbitrary regular open set in X_i. Since Pr_i is continuous and open, it is an R-map and hence $Pr_i^{-1}(U_i)$ is regular open in $\prod X_i$. Since f is almost contra $\beta\theta$-continuous, we have by definition $f^{-1}(Pr_i^{-1}(U_i)) = (Pr_i \circ f)^{-1}(U_i)$ is $\beta\theta$-closed in X. Therefore $Pr_i \circ f$ is almost contra $\beta\theta$-continuous for each $i \in \Omega$. □

Definition 4. A function $f: X \to Y$ is called weakly $\beta\theta$-continuous if for each $x \in X$ and every open set V of Y containing $f(x)$, there exists a $\beta\theta$-open set U in X containing x such that $f(U) \subset Cl(V)$.

Theorem 2.14. *For a function $f: X \to Y$, the following properties hold:*

(1) *If f is almost contra $\beta\theta$-continuous, then it is weakly $\beta\theta$-continuous,*
(2) *If f is weakly $\beta\theta$-continuous and Y is extremally disconnected, then f is almost contra $\beta\theta$-continuous.*

Proof.

(1) Let $x \in X$ and V be any open set of Y containing $f(x)$. Since $Cl(V)$ is a regular closed set containing $f(x)$, by Theorem 2.2 there exists a $\beta\theta$-open set U containing x such that $f(U) \subset Cl(V)$. Therefore, f is weakly $\beta\theta$-continuous.
(2) Let V be a regular closed subset of Y. Since Y is extremally disconnected, we have that V is a regular open set of Y and the weak $\beta\theta$-continuity of f implies that $f^{-1}(V) \subset \beta Int_\theta(f^{-1}(Cl(V))) = \beta Int_\theta f^{-1}(V)$. Therefore $f^{-1}(V)$ is $\beta\theta$-open in X. This shows that f is almost contra $\beta\theta$-continuous. □

Definition 5. A function $f: X \to Y$ is said to be:

a) neatly $(\beta\theta, s)$-continuous if for each $x \in X$ and each $V \in SO(Y, f(x))$, there is a $\beta\theta$-open set U in X containing x such that $Int(f(U)) \subset Cl(V)$.
b) $(\beta\theta, s)$-open if $f(U) \in SO(Y)$ for every $\beta\theta$-open set U of X.

Theorem 2.15. *If a function $f: X \to Y$ is neatly $(\beta\theta, s)$-continuous and $(\beta\theta, s)$-open, then f is almost contra $\beta\theta$-continuous.*

Proof. Suppose that $x \in X$ and $V \in SO(Y, f(x))$. Since f is neatly $(\beta\theta, s)$-continuous, there exists a $\beta\theta$-open set U of X containing x such that $Int(f(U)) \subset Cl(V)$. By hypothesis, f is $(\beta\theta, s)$-open. This implies that $f(U) \in SO(Y)$. It follows that $f(U) \subset Cl(Int(f(U))) \subset Cl(V)$. This shows that f is almost contra $\beta\theta$-continuous. □

3. Some fundamental properties

Definition 6 [6,22]**.** A topological space (X, τ) is said to be:

(1) $\beta\theta$-T_0 (resp. $\beta\theta$-T_1) if for any distinct pair of points x and y in X, there is a $\beta\theta$-open set U in X containing x but not y or (resp. and) a $\beta\theta$-open set V in X containing y but not x.
(2) $\beta\theta$-T_2 (resp. β-T_2 [7]) if for every pair of distinct points x and y, there exist two $\beta\theta$-open (resp. β-open) sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Theorem 3.1. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is $\beta\theta$-T_0;*
(2) *(X, τ) is $\beta\theta$-T_1;*
(3) *(X, τ) is $\beta\theta$-T_2;*
(4) *(X, τ) is β-T_2;*
(5) *For every pair of distinct points $x, y \in X$, there exist $U, V \in \beta O(X)$ such that $x \in U$, $y \in V$ and $\beta Cl(U) \cap \beta Cl(V) = \emptyset$;*
(6) *For every pair of distinct points $x, y \in X$, there exist $U, V \in \beta R(X)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*
(7) *For every pair of distinct points $x, y \in X$, there exist $U \in \beta\theta O(X, x)$ and $V \in \beta\theta O(X, y)$ such that $\beta Cl_\theta(U) \cap \beta Cl_\theta(V) = \emptyset$.*

Proof. It follows from ([6], Remark 3.2 and Theorem 3.4).

Recall that a topological space (X, τ) is said to be:

(i) Weakly Hausdorff [23] (briefly weakly-T_2) if every point of X is an intersection of regular closed sets of X.
(ii) s-Urysohn [24] if for each pair of distinct points x and y in X, there exist $U \in SO(X, x)$ and $V \in SO(X, x)$ such that $Cl(U) \cap Cl(V) \neq \emptyset$. □

Theorem 3.2. *If X is a topological space and for each pair of distinct points x_1 and x_2 in X, there exists a map f of X into a Urysohn topological space Y such that $f(x_1) \neq f(x_2)$ and f is almost contra $\beta\theta$-continuous at x_1 and x_2, then X is $\beta\theta$-T_2.*

Proof. Let x_1 and x_2 be any distinct points in X. Then by hypothesis, there is a Urysohn space Y and a function $f: X \to Y$, which satisfies the conditions of the theorem. Let $y_i = f(x_i)$ for $i = 1, 2$. Then $y_1 \neq y_2$. Since Y is Urysohn, there exist open sets U_{y_1} and U_{y_2} of y_1 and y_2, respectively, in Y such that $Cl(U_{y_1}) \cap Cl(U_{y_2}) = \emptyset$. Since f is almost contra $\beta\theta$ -continuous at x_i, there exists a $\beta\theta$-open set W_{x_i} containing x_i in X such that $f(W_{x_i}) \subset Cl(U_{y_i})$ for $i = 1, 2$. Hence we get $W_{x_1} \cap W_{x_2} = \emptyset$ since $Cl(U_{y_1}) \cap Cl(U_{y_2}) = \emptyset$. Hence X is $\beta\theta$-T_2. □

Corollary 3.3. *If f is an almost contra $\beta\theta$-continuous injection of a topological space X into a Urysohn space Y, then X is $\beta\theta$-T_2.*

Proof. For each pair of distinct points x_1 and x_2 in X , f is an almost contra $\beta\theta$-continuous function of X into a Urysohn space Y such that $f(x_1) \neq f(x_2)$ since f is injective. Hence by Theorem 3.2, X is $\beta\theta$-T_2. □

Theorem 3.4.

(1) *If f is an almost contra $\beta\theta$-continuous injection of a topological space X into a s-Urysohn space Y, then X is $\beta\theta$-T_2.*
(2) *If f is an almost contra $\beta\theta$-continuous injection of a topological space X into a weakly Hausdorff space Y, then X is $\beta\theta$-T_1.*

Proof.

(1) Let Y be s-Urysohn. Since f is injective, we have $f(x) \neq f(y)$ for any distinct points x and y in X. Since Y is s-Urysohn, there exist $V_1 \in SO(Y, f(x))$ and $V_2 \in SO(Y, f(y))$ such that $Cl(V_1) \cap Cl(V_2) = \emptyset$. Since f is almost contra $\beta\theta$-continuous, there exist $\beta\theta$-open sets U_1 and U_2 in X containing x and y, respectively, such that $f(U_1) \subset Cl(V_1)$ and $f(U_2) \subset Cl(V_2)$. Therefore $U_1 \cap U_2 = \emptyset$. This implies that X is $\beta\theta$-T_2.
(2) Since Y is weakly Hausdorff and f is injective, for any distinct points x_1 and x_2 of X, there exist $V_1, V_2 \in RC(Y)$ such that $f(x_1) \in V_1$, $f(x_2) \notin V_1$, $f(x_2) \in V_2$ and $f(x_1) \notin V_2$. Since f is almost contra $\beta\theta$-continuous, by Theorem 2.2 $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $\beta\theta$-open sets and $x_1 \in f^{-1}(V_1)$, $x_2 \notin f^{-1}(V_1)$, $x_2 \in f^{-1}(V_2)$, $x_1 \notin f^{-1}(V_2)$. Then, there exists U_1, $U_2 \in \beta\theta O(X)$ such that $x_1 \in U_1 \subset f^{-1}(V_1)$, $x_2 \notin U_1$, $x_2 \in U_2 \subset f^{-1}(V_2)$ and $x_1 \notin U_2$. Thus X is $\beta\theta$-T_1. □

The union of two $\beta\theta$-closed sets is not necessarily $\beta\theta$-closed as shown in the following example.

Example 3.5. Let $X=\{a,b,c\}$, $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$. The subsets $\{a\}$ and $\{b\}$ are $\beta\theta$-closed in (X,τ) but $\{a,b\}$ is not $\beta\theta$-closed.

Recall that a topological space is called a $\beta\theta$c-space [25] if the union of any two $\beta\theta$-closed sets is a $\beta\theta$-closed set.

Theorem 3.6. *If f, $g\colon X\to Y$ are almost contra $\beta\theta$-continuous functions, X is a $\beta\theta$c-space and Y is s-Urysohn, then $E=\{x\in X\mid f(x)=g(x)\}$ is $\beta\theta$-closed in X.*

Proof. If $x\in X\backslash E$, then $f(x)\neq g(x)$. Since Y is s-Urysohn, there exist $V_1\in SO(Y,f(x))$ and $V_2\in SO(Y,g(x))$ such that $Cl(V_1)\cap Cl(V_2)=\emptyset$. By the fact that f and g are almost contra $\beta\theta$-continuous, there exist $\beta\theta$-open sets U_1 and U_2 in X containing x such that $f(U_1)\subset Cl(V_1)$ and $g(U_2)\subset Cl(V_2)$. We put $U=U_1\cap U_2$. Then U is $\beta\theta$-open in X. Thus $f(U)\cap g(U)=\emptyset$. It follows that $x\notin \beta Cl_\theta(E)$. This shows that E is $\beta\theta$-closed in X.

We say that the product space $X=X_1\times\ldots\times X_n$ has Property $P_{\beta\theta}$ if A_i is a $\beta\theta$-open set in a topological space X_i, for $i=1,2,\ldots n$, then $A_1\times\ldots\times A_n$ is also $\beta\theta$-open in the product space $X=X_1\times\ldots\times X_n$. □

Theorem 3.7. *Let $f_1\colon X_1\to Y$ and $f_2\colon X_2\to Y$ be two functions, where*

(1) *$X=X_1\times X_2$ has the Property $P_{\beta\theta}$.*
(2) *Y is a Urysohn space.*
(3) *f_1 and f_2 are almost contra $\beta\theta$-continuous. Then $\{(x_1,x_2): f_1(x_1)=f_2(x_2)\}$ is $\beta\theta$-closed in the product space $X=X_1\times X_2$.*

Proof. Let A denote the set $\{(x_1,x_2): f_1(x_1)=f_2(x_2)\}$. In order to show that A is $\beta\theta$-closed, we show that $(X_1\times X_2)\backslash A$ is $\beta\theta$-open. Let $(x_1,x_2)\notin A$. Then $f_1(x_1)\neq f_2(x_2)$. Since Y is Urysohn , there exist open sets V_1 and V_2 containing $f_1(x_1)$ and $f_2(x_2)$, respectively, such that $Cl(V_1)\cap Cl(V_2)=\emptyset$. Since f_i $(i=1,2)$ is almost contra $\beta\theta$-continuous and $Cl(V_i)$ is regular closed, then $f_i^{-1}(Cl(V_i))$ is a $\beta\theta$-open set containing x_i in X_i $(i=1,2)$. Hence by (1), $f_1^{-1}(Cl(V_1))\times f_2^{-1}(Cl(V_2))$ is $\beta\theta$-open. Furthermore $(x_1,x_2)\in f_1^{-1}(Cl(V_1))\times f_2^{-1}(Cl(V_2))\subset (X_1\times X_2)\backslash A$. It follows that $(X_1\times X_2)\backslash A$ is $\beta\theta$-open. Thus A is $\beta\theta$ -closed in the product space $X=X_1\times X_2$. □

Corollary 3.8. *Assume that the product space $X\times X$ has the Property $P_{\beta\theta}$. If $f\colon X\to Y$ is almost contra $\beta\theta$-continuous and Y is a Urysohn space. Then $A=\{(x_1,x_2): f(x_1)=f(x_2)\}$ is $\beta\theta$ -closed in the product space $X\times X$.*

Theorem 3.9. *Let $f\colon X\to Y$ be a function and $g\colon X\to X\times Y$ the graph function, given by $g(x)=(x,f(x))$ for every $x\in X$. Then f is almost contra $\beta\theta$-continuous if g is almost contra $\beta\theta$-continuous.*

Proof. Let $x\in X$ and V be a regular open subset of Y containing $f(x)$. Then we have that $X\times V$ is regular open. Since g is almost contra $\beta\theta$-continuous, $g^{-1}(X\times V)=f^{-1}(V)$ is $\beta\theta$-closed. Hence f is almost contra $\beta\theta$-continuous. □

Recall that for a function $f\colon X\to Y$, the subset $\{(x,f(x)): x\in X\}\subset X\times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 7. A function $f\colon X\to Y$ has a $\beta\theta$-closed graph if for each $(x,y)\in(X\times Y)\backslash G(f)$, there exists $U\in\beta\theta O(X,x)$ and an open set V of Y containing y such that $(U\times V)\cap G(f)=\emptyset$.

Lemma 3.10. *The graph, $G(f)$ of a function $f\colon X\to Y$ is $\beta\theta$-closed if and only if for each $(x,y)\in(X\times Y)\backslash G(f)$ there exists $U\in\beta\theta O(X,x)$ and an open set V of Y containing y such that $f(U)\cap V=\emptyset$.*

Theorem 3.11. *If $f\colon X\to Y$ is a function with a $\beta\theta$-closed graph, then for each $x\in X$, $f(x)=\cap\{Cl(f(U)): U\in\beta\theta O(X,x)\}$.*

Proof. Suppose the theorem is false. Then there exists a $y\neq f(x)$ such that $y\in\cap\{Cl(f(U)): U\in\beta\theta O(X,x)\}$. This implies that $y\in Cl(f(U))$, for every $U\in\beta\theta O(X,x)$. So $V\cap f(U)\neq\emptyset$, for every $V\in O(Y,y)$. which contradicts the hypothesis that f is a function with a $\beta\theta$-closed graph. Hence the theorem. □

Theorem 3.12. *If $f\colon X\to Y$ is almost contra $\beta\theta$-continuous and Y is Haudsorff, then $G(f)$ is $\beta\theta$-closed.*

Proof. Let $(x,y)\in(X\times Y)\backslash G(f)$. Then $y\neq f(x)$. Since Y is Hausdorff, there exist disjoint open sets V and W of Y such that $y\in V$ and $f(x)\in W$. Then $f(x)\notin Y\backslash Cl(W)$. Since $Y\backslash Cl(W)$ is a regular open set containing V, it follows that $f(x)\notin \mathrm{rker}(V)$ and hence $x\notin f^{-1}(\mathrm{rker}(V))$. Then by Theorem 2.2(6) $x\notin\beta Cl_\theta(f^{-1}(V))$. Therefore we have $(x,y)\in(X\backslash\beta Cl_\theta((f^{-1}(V)))\times V\subset(X\times Y)\backslash G(f)$, which proves that $G(f)$ is $\beta\theta$-closed. □

Theorem 3.13. *Let $f\colon X\to Y$ have a $\beta\theta$-closed graph.*

(1) *If f is injective, then X is $\beta\theta$-T_1.*
(2) *If f is surjective, then Y is T_1.*

Proof.

(1) Let x_1 and x_2 be any distinct points in X. Then $(x_1,f(x_2))\in(X\times Y)\backslash G(f)$. Since f has a $\beta\theta$-closed graph, there exist $U\in\beta\theta O(X,x_1)$ and an open set V of Y containing $f(x_2)$ such that $f(U)\cap V=\emptyset$. Then $U\cap f^{-1}(V)=\emptyset$. Since $x_2\in f^{-1}(V)$, $x_2\notin U$. Therefore U is a $\beta\theta$-open set containing x_1 but not x_2, which proves that X is $\beta\theta$-T_1.
(2) Let y_1 and y_2 be any distinct points in Y. Since Y is surjective, there exists $x\in X$ such that $f(x)=y_1$. Then $(x,y_2)\in(X\times Y)\backslash G(f)$. Since f has a $\beta\theta$-closed graph, there exist $U\in\beta\theta O(X,x)$ and an open set V of Y containing y_2 such that $f(U)\cap V=\emptyset$. Since $y_1=f(x)$ and $x\in U$, $y_1\in f(U)$. Therefore $y_1\notin V$, which proves that Y is T_1. □

Theorem 3.14. *If $f\colon X\to Y$ has a $\beta\theta$-closed graph and X is a $\beta\theta$c-space, then $f^{-1}(K)$ is $\beta\theta$-closed for every compact subset K of Y.*

Proof. Let K be a compact subset of Y and let $x\in X\backslash f^{-1}(K)$. Then for each $y\in K$, $(x,y)\in(X\times Y)\backslash G(f)$. So there exist $U_y\in\beta\theta O(X,x)$ and an open set V_y of Y containing y such that $f(U_y)\cap V_y=\emptyset$. The family $\{V_y: y\in K\}$ is an open cover of K and hence there is a finite subcover $\{V_{y_i}: i=1,\ldots,n\}$. Let $U=\cap_{i=1}^{n}U_{y_i}$. Then $U\in\beta\theta O(X,x)$ and $f(U)\cap K=\emptyset$. Hence $U\cap f^{-1}(K)=\emptyset$, which proves that $f^{-1}(K)$ is $\beta\theta$-closed in X. □

Definition 8. A topological space X is said to be:

(1) strongly $\beta\theta$C-compact [6] if every $\beta\theta$-closed cover of X has a finite subcover. (resp. $A\subset X$ is strongly $\beta\theta$C-compact if the subspace A is strongly $\beta\theta$C-compact).
(2) nearly-compact [26] if every regular open cover of X has a finite subcover.

Theorem 3.15. *If $f\colon X\to Y$ is an almost contra $\beta\theta$-continuous surjection and X is strongly $\beta\theta$C-compact, then Y is nearly compact.*

Proof. Let $\{V_\alpha:\alpha\in I\}$ be a regular open cover of Y. Since f is almost contra $\beta\theta$-continuous, we have that $\{f^{-1}(V_\alpha):\alpha\in I\}$ is a cover of X by $\beta\theta$-closed sets. Since X is strongly $\beta\theta$C-compact, there exists a finite subset I_0 of I such that $X=\bigcup\{f^{-1}(V_\alpha):\alpha\in I_0\}$. Since f is surjective $Y=\bigcup\{V_\alpha:\alpha\in I_0\}$ and therefore Y is nearly compact.

A topological space X is said to be almost-regular [27] if for each regular closed set F of X and each point $x\in X\backslash F$, there exist disjoint open sets U and V such that $F\subset V$ and $x\in U$. □

Theorem 3.16. *If a function $f: X \to Y$ is almost contra $\beta\theta$-continuous and Y is almost-regular, then f is almost $\beta\theta$-continuous.*

Proof. Let x be an arbitrary point of X and V an open set of Y containing $f(x)$. Since Y is almost-regular, by Theorem 2.2 of [27] there exists a regular open set W in Y containing $f(x)$ such that $Cl(W) \subset Int(Cl(V))$. Since f is almost contra $\beta\theta$ -continuous, and $Cl(W)$ is regular closed in Y, by Theorem 3.1 there exists $U \in \beta\theta O(X, x)$ such that $f(U) \subset Cl(W)$. Then $f(U) \subset Cl(W) \subset Int(Cl(V))$. Hence, f is almost $\beta\theta$-continuous.

The $\beta\theta$-frontier of a subset A, denoted by $Fr_{\beta\theta}(A)$, is defined as $Fr_{\beta\theta}(A) = \beta Cl_\theta(A) \setminus \beta Int_\theta(A)$, equivalently $Fr_{\beta\theta}(A) = \beta Cl_\theta(A) \cap \beta Cl_\theta(X \setminus A)$. □

Theorem 3.17. *The set of points $x \in X$ which $f: (X, \tau) \to (Y, \sigma)$ is not almost contra $\beta\theta$-continuous is identical with the union of the $\beta\theta$-frontiers of the inverse images of regular closed sets of Y containing f(x).*

Proof. Necessity. Suppose that f is not almost contra $\beta\theta$-continuous at a point x of X. Then there exists a regular closed set $F \subset Y$ containing $f(x)$ such that $f(U)$ is not a subset of F for every $U \in \beta\theta O(X, x)$. Hence we have $U \cap (X \setminus f^{-1}(F)) \neq \emptyset$ for every $U \in \beta\theta O(X, x)$. It follows that $x \in \beta Cl_\theta(X \setminus f^{-1}(F))$. We also have $x \in f^{-1}(F) \subset \beta Cl_\theta(f^{-1}(F))$. This means that $x \in Fr_{\beta\theta}(f^{-1}(F))$.

Sufficiency. Suppose that $x \in Fr_{\beta\theta}(f^{-1}(F))$ for some $F \in RC(Y, f(x))$ Now, we assume that f is almost contra $\beta\theta$-continuous at $x \in X$. Then there exists $U \in \beta\theta O(X, x)$ such that $f(U) \subset F$. Therefore, we have $x \in U \subset f^{-1}(F)$ and hence $x \in \beta Int_\theta(f^{-1}(F)) \subset X \setminus Fr_{\beta\theta}(f^{-1}(F))$. This is a contradiction. This means that f is not almost contra $\beta\theta$-continuous. □

References

[1] C.W. Baker, On contra almost β-continuous functions in topological spaces, Kochi J. Math. 1 (2006) 1–8.
[2] E. Ekici, Almost contra-precontinuous functions, Bull. Malaysian Math. Sci. Soc. 27 (2006) 53–65.
[3] T. Noiri, V. Popa, Some properties of almost contra-precontinuous functions, Bull. Malaysian Math. Sci. Soc. 28 (2005) 107–116.
[4] M. Caldas, S. Jafari, Some properties of contra-β-continuous functions, Mem. Fac. Sci. Kochi Univ. Ser. A (Math.) 22 (2001) 19–28.
[5] S. Jafari, T. Noiri, On contra-precontinuous functions, Bull. Malaysian Math. Sci. Soc. 25 (2) (2002) 115–128.
[6] M. Caldas, On contra $\beta\theta$-continuous functions, Proyecciones J. Math. 39 (4) (2013) 333–342.
[7] A.S. Mashhour, M.E.A. El-Monsef, S.N. El-Deeb, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt 53 (1982) 47–53.
[8] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 68 (1961) 44–46.
[9] M.E.A. El-Monsef, S.N. Ei-Deeb, R.A. Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Assiut Univ. 12 (1983) 77–90.
[10] O. Njastad, On some classes of nearly open sets, Pacific J. Math. 15 (1965) 961–970.
[11] J.E. Joseph, M.H. Kwack, On s-closed spaces, Proc. Amer. Math. Soc. 80 (1980) 341–348.
[12] T. Noiri, Weak and strong forms of β-irresolute functions, Acta Math. Hungar. 99 (2003) 315–328.
[13] E. Ekici, Another form of contra-continuity, Kochi J. Math. 1 (2006) 21–29.
[14] J. Dontchev, Contra-continuous functions and strongly s-closed spaces, Internat. J. Math. Math. Sci. 19 (1996) 303–310.
[15] J. Dontchev, M. Ganster, I. Reilly, More on almost s-continuity, Indian J. Math. 41 (1999) 139–146.
[16] D. Andrijevic, Semi-preopen sets, Mat. Vesnik 38 (1986) 24–32.
[17] T. Noiri, On almost continuous functions, Indian J. Pure Appl. Math. 20 (1989) 571–576.
[18] M.C. Pal, P. Bhattacharyya, Faint precontinuous functions, Soochow J. Math. 21 (1995) 273–289.
[19] G.J. Wang, On s-closed spaces, Acta Math. Sinica 24 (1981) 55–63.
[20] D. Carnahan, Some properties related to compactness in topological spaces, University of Arkansas, 1973 Ph. d. thesis.
[21] M. Caldas, On θ-β-generalized closed sets and θ-β-generalized continuity in topolological spaces, J. Adv. Math. Studies 4 (2011) 13–24.
[22] M. Caldas, Weakly sp-θ-closed functions and semipre-hausdorff spaces, Creative Math. Inform. 20 (2) (2011) 112–123.
[23] T. Soundarajan, Weakly hausdorff space and the cardinality of topological spaces, general topology and its relation to modern analysis and algebra III, in: Proceedings Conference Kampur, 168, Academy Prague, 1971, pp. 301–306.
[24] S.P. Arya, M.P. Bhamini, Some generalizations of pairwise urysohn spaces, Indian J. Pure Appl. Math. 18 (1987) 1088–1093.
[25] M. Caldas, Functions with strongly β-θ-closed graphs, J. Adv. Stud. Top. 3 (2012) 1–6.
[26] M.K. Singal, A. Mathur, On nearly compact spaces, Boll. Un. Mat. Ital. 4 (2) (1969) 702–710.
[27] M.K. Singal, S.P. Arya, On almost-regular spaces, Glasnik Mat. III 4 (24) (1969) 89–99.