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ORIGINAL ARTICLE

A comparison between the reduced differential transform method and the Adomian decomposition method for the Newell–Whitehead–Segel equation

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Abstract In this paper, we will carry out a comparative study between the reduced differential transform method and the Adomian decomposition method. This is been achieved by handling the Newell–Whitehead–Segel equation. Two numerical examples have also been carried out to validate and demonstrate efficiency of the two methods. Furthermost, it is shown that the reduced differential transform method has an advantage over the Adomian decomposition method that it takes less time to solve the nonlinear problems without using the Adomian polynomials.

MATHEMATICS SUBJECT CLASSIFICATION: 44A99; 35Q99

1. Introduction and preliminaries

Nonequilibrium systems are commonly exhibited as equilibrium extended states: uniform, oscillatory, chaotic, and pattern states. Stripe-(or roll-) patterns appear in a variety of spatially extended systems in nature, like ripples in sand, stripes of seashells or on the fur of mammals, such as our domestic cats or markings of the skins of the animals and also in variety of physics laboratory systems, like Rayleigh–Benard convection, Taylor–Couette flow, Faraday instability, directional solidification, nonlinear optics, chemical reactions, and biological systems. This type of systems can be well described by a set of equations called amplitude equations. One of the most well-known amplitude equations in two dimensional systems is the Newell–Whitehead–Segel equation. This model describes the appearance of the stripe pattern in two dimensional systems.

Now, we consider the well-known Newell–Whitehead–Segel equation of the following type:

$$u_t(x,t) = ku_{xx} + au(x,t) - bu^q(x,t), \tag{1}$$

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subject to the initial condition,

$$u(x,0) = f(x), \tag{2}$$

where a and b are real numbers and k and q are positive integers.

The Newell–Whitehead–Segel equation has been given considerable attention in recent years by introducing various methods and techniques, for example, Li et al. [1] used lattice Boltzmann scheme, Malik et al. [2] used $\left(\frac{G'}{G}\right)$ expansion method to get generalized traveling wave solutions. Manaa [3] applied the Adomian decomposition method to get approximate solution and Aasaraai [4] discussed by the differential transform method.

The reduced differential transform method was first proposed by Keskin [5] and successfully employed to solve many types of nonlinear partial differential equations. Also, Keskin and Oturanc used this method to obtain the analytical solution of linear and nonlinear wave equations [6]. The Adomian decomposition method was introduced and developed by George Adomian in [7,8] and is well addressed in the literature [9,10]. Recently, applying Adomian decomposition method, many researchers [9–13] and [14,15] investigated a wide class of linear and nonlinear ordinary differential equations, partial differential equations, and integral equations.

In this paper, we solved Newell–Whitehead–Segel Eq. (1) by the reduced differential transform method, the Adomian decomposition method and discuss the comparison between the reduced differential transform method and the Adomian decomposition method.

2. The reduced differential transform method

As in Ref. [5], the basic definition of reduced differential transform is introduced as follows:

The reduced differential transform of $u(x,t)$ at $t = 0$ is defined as,

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0}, \tag{3}$$

where $u(x,t)$ is the original function and $U_k(x)$ is the transformed function.

The reduced differential inverse transform of $U_k(x)$ is defined as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k, \tag{4}$$

and from (3) and (4), we have,

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0} t^k. \tag{5}$$

The following theorems that can be deduced from (3) and (4) are given below [5,16]:

Theorem 2.1. *If $w(x,t) = u(x,t) \pm v(x,t)$ then $W_k(x) = U_k(x) \pm V_k(x)$.*

Theorem 2.2. *If $w(x,t) = \alpha u(x,t)$ then $W_k(x) = \alpha U_k(x)$.*

Theorem 2.3. *If $w(x,t) = [x^m t^n]$ then $W_k(x) = x^m \delta(k-n)$*

where (the Kronecker delta) $\delta(k-n) = \begin{cases} 1, & \text{when } k = n, \\ 0, & \text{when } k \neq n. \end{cases}$

Theorem 2.4. *If $w(x,t) = [x^m t^n u(x,t)]$ then $W_k(x) = x^m U_{k-n}(x)$.*

Theorem 2.5. *If $w(x,t) = \left[\frac{\partial^r u(x,t)}{\partial t^r}\right]$ then*

$$W_k(x) = (k+1)(k+2)\ldots(k+r)U_{k+r}(x) = \frac{(k+r)!}{k!}U_{k+r}(x).$$

Theorem 2.6. *If $w(x,t) = \left[\frac{\partial u(x,t)}{\partial x}\right]$ then $W_k(x) = \frac{\partial}{\partial x}(U_k(x))$.*

Theorem 2.7. *If $w(x,t) = u(x,t)v(x,t)$ then $W_k(x) = \sum_{r=0}^{k} U_r(x)V_{k-r}(x)$.*

Theorem 2.8. *If $w(x,t) = [u(x,t)]^m$ then $W_k(x) = \begin{cases} U_0(x), & \mathrm{k} = 0, \\ \sum_{n=1}^{k} \frac{(m+1)n-k}{kU_0(x)} U_n(x) W_{k-n}(x), & k \geqslant 1 \end{cases}$.*

3. Main result

Section 3.1

In this section, we use the reduced differential transform method to obtain the solution of (1) and (2).

Consider the Newell–Whitehead–Segel equation of the following type:

$$u_t(x,t) = ku_{xx} + au(x,t) - bu^q(x,t), \tag{6}$$

subject to the initial condition,

$$u(x,0) = f(x), \tag{7}$$

where k, a, b are real numbers k and q are positive integers.

By taking the reduced differential transform on both sides of (6) and (7), we have:

$$RDT[u_t(x,t)] = kRDT[u_{xx}(x,t)] + aRDT[u(x,t)] - bRDT[u^q(x,t)], \tag{8}$$

$$RDT[u(x,0) = f(x)]. \tag{9}$$

After applying the fundamental theorems in (8) and (9), we obtain the following recurrence relation:

$$(k+1)U_{k+1}(x) = k\frac{\partial^2}{\partial x^2}U_k(x) + aU_k(x) - bF_k(x) \tag{10}$$

where $F_k(x) = \begin{cases} U_0(x), & \mathrm{k} = 0, \\ \sum_{n=1}^{k} \frac{(q+1)n-k}{kU_0(x)} U_n(x) F_{k-n}(x), & k \geqslant 1 \end{cases}$.

$$U_0(x) = f(x)\delta(k-0), \tag{11}$$

where $U_k(x)$ and $F_k(x)$ are the transformed values of u(x,t) and $u^q(x,t)$ respectively and $f(x)\delta(k-0)$ is the transformed value of $f(x)$.

By iterative calculations on (10) and (11), we obtain the following values of $U(k,h)$ as

$$U_1(x) = \eta_1(x), U_2(x) = \eta_2(x), U_3(x) = \eta_3(x), \ldots U_n(x) = \eta_n(x), \ldots. \tag{12}$$

From (4), we have,

$$u(x,t) = U_0(x)t^0 + U_1(x)t + U_2(x)t^2 + U_3(x)t^3 + \cdots U_n(x)t^n + \cdots. \tag{13}$$

One can get the exact solution of (6) by substituting (11) and (12) in (13).

Section 3.2

In this section, we use the Adomian decomposition method to obtain the solution of (1) and (2).

In an operator form, (1) becomes,

$$L_t u = kL_{xx}u + au - bu^q, \tag{14}$$

where the differential operator L is given by,

$$L_t = \frac{\partial}{\partial t}$$

and

$$L_{xx} = \frac{\partial^2}{\partial x^2},$$

where each operator is assumed easily invertible, and therefore, the inverse operator L_t^{-1} is defined by,

$$L_t^{-1}(\cdot) = \int_0^t (\cdot)\,dt$$

and

$$L_{xx}^{-1}(\cdot) = \int_0^x \int_0^x (\cdot)\,dx\,dx.$$

Applying L_t^{-1} on both sides of (14) and using the initial conditions, we obtain,

$$L_t^{-1}L_t u = L_t^{-1}[kL_{xx}u + au - bu^q],$$

or equivalently,

$$\begin{aligned} u(x,t) - u(x,0) &= L_t^{-1}[kL_{xx}u + au - bu^q], \\ u(x,t) &= f(x) + L_t^{-1}[kL_{xx}u + au - bu^q]. \end{aligned} \tag{15}$$

Adomian method defines the solution $u(x,t)$ by an infinite series of components, and it is given by:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \tag{16}$$

where the components $u_0, u_1, u_2, \ldots$ are usually recurrently determined and the nonlinear term $F(u) = u^q$ can be expressed by the Adomian polynomials A_n as,

$$u^q = \sum_{n=0}^{\infty} A_n. \tag{17}$$

The Adomian polynomials A_n for the nonlinear term $F(u) = u^q$ can be evaluated by using the following expression,

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]_{\lambda=0}. \tag{18}$$

The general formula (17) can be simplified as follows:

$$\begin{aligned} A_0 &= F(u_0), \\ A_1 &= u_1 F'(u_0), \\ A_2 &= u_2 F'(u_0) + \frac{1}{2!}u_1^2 F''(u_0), \\ A_3 &= u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!}u_1^3 F'''(u_0), \end{aligned} \tag{19}$$

other polynomials can be generated in a similar manner. Based on these assumptions, (14) becomes,

$$\sum_{n=0}^{\infty} u_n(x,t) = f(x) + L_t^{-1}\left[kL_{xx}\sum_{n=0}^{\infty} u_n(x,t) + a\sum_{n=0}^{\infty} u_n(x,t) - b\sum_{n=0}^{\infty} A_n\right]. \tag{20}$$

The components $u_n(x,t), n \geqslant 0$ of the solution $u(x,t)$ can be recursively determined by using the relation:

$$u_0(x,t) = f(x), \tag{21}$$

$$u_{k+1}(x,t) = L_t^{-1}[kL_{xx}[u_k] + au_k] - bL_t^{-1}[A_k]. \tag{22}$$

One can get exact solution by substituting the values of $u_n(x,t)$ in (16).

4. Illustrative examples

Two different examples are considered in this section to illustrate the effectiveness of the reduced differential transform method.

Example 4.1. Consider the linear Newell–Whitehead–Segel equation,

$$u_t = u_{xx} - 2u, \tag{23}$$

with the initial condition,

$$u(x,0) = e^x, \tag{24}$$

whose exact solution was found to be [4]:

$$u(x,t) = e^{x-t}. \tag{25}$$

Case 1. (*By RDTM*)

By taking the reduced differential transform on both sides of (23) and (24), we have:

$$RDT[u_t] = RDT[u_{xx}] - 2RDT[u] \tag{26}$$

and

$$RDT[u(x,0) = e^x]. \tag{27}$$

After applying fundamental theorems in (26) and (27), the following recurrence relation is obtained,

$$(k+1)U_{k+1}(x) = \frac{\partial^2}{\partial x^2}U_k(x) - 2U_k(x), \tag{28}$$

$$U_0(x) = e^x. \tag{29}$$

By iterative calculations on (28) and (29), we have:

$$U_1(x) = -e^x, U_2(x) = \frac{e^x}{2!}, U_3(x) = \frac{-e^x}{3!}, \ldots. \tag{30}$$

From (4),

$$u(x,t) = U_0(x)t^0 + U_1(x)t + U_2(x)t^2 + U_3(x)t^3 + \cdots. \tag{31}$$

Substituting (30) in (31), we have,

$$u(x,t) = e^{x-t}, \tag{32}$$

which is an exact solution of the problem.

Case 2. (*By ADM*)

In an operator form, (23) becomes,

$$L_t u = L_{xx}u - 2u, \tag{33}$$

where the differential operator L is given by,

$$L_t = \frac{\partial}{\partial t}$$

and

$$L_{xx} = \frac{\partial^2}{\partial x^2},$$

where each operator is assumed as easily invertible, and therefore, the inverse operator L_t^{-1} is defined by,

$$L_t^{-1}(\cdot) = \int_0^t (\cdot)dt$$

and

$$L_{xx}^{-1}(\cdot) = \int_0^x \int_0^x (\cdot)\, dx\, dx.$$

Applying L_t^{-1} on both sides of (33) and using the initial condition, we obtain,

$$L_t^{-1}[L_t u] = L_t^{-1}[L_{xx}u - 2u],$$

or equivalently,

$$u(x,t) - u(x,0) = L_t^{-1}[L_{xx}u - 2u],$$
$$u(x,t) = e^x + L_t^{-1}[L_{xx}u - 2u]. \tag{34}$$

Adomian method defines the solution $u(x,t)$ by an infinite series of components, and it is given by,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{35}$$

Substituting the above equation in (34), we have,

$$\sum_{n=0}^{\infty} u_n(x,t) = e^x + L_t^{-1}\left[L_{xx}\sum_{n=0}^{\infty} u_n(x,t) - 2\sum_{n=0}^{\infty} u_n(x,t)\right], \tag{36}$$

where the components $u_n(x,t)$, $n \geqslant 0$ of the solution $u(x,t)$ can be recursively determined by using the relation,

$$u_0(x,t) = e^x, \tag{37}$$
$$u_{k+1}(x,t) = L_t^{-1}[L_{xx}[u_k] - 2u_k]. \tag{38}$$

From the above equation, we have,

$$u_1(x,t) = -e^x t, u_2(x,t) = e^x \frac{t^2}{2!}, u_3(x,t) = -e^x \frac{t^3}{3!} \ldots \tag{39}$$

Substituting the values of $u_n(x,t)$ in (35), we have,

$$u(x,t) = -e^x t + e^x \frac{t^2}{2!} + -e^x \frac{t^3}{3!} \cdots,$$
$$u(x,t) = e^{x-t}. \tag{40}$$

which is an exact solution of the problem. In order to verify the efficiency and accuracy of the proposed reduced differential transform method for solving linear Newell–Whitehead–Segel equations. (23) and (24), graphs are drawn for the numerical solution as well as the exact solution.

From Fig. 1a–c, we can see that the solution obtained by the reduced differential transform method and the Adomian decomposition method coincide with the exact solution. The results of this example show that the reduced differential transform method is a very simple method than the Adomian decomposition method because the reduced differential transform method solves the problem without using the Adomian polynomial which was used in the Adomian decomposition method, due to this fact, it is concluded that the reduced

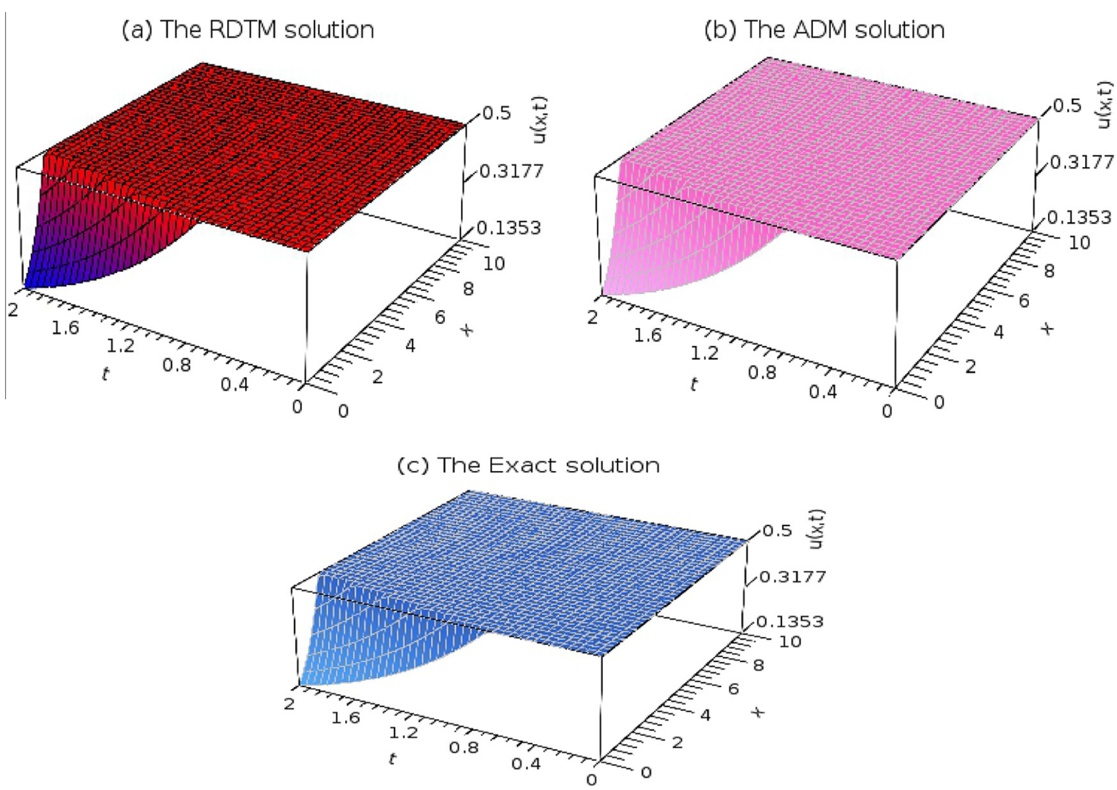

Figure 1 The comparison of RDTM and ADM with the exact solution $u(x,t)$ for different values of 't'.

differential transform method is less consumption of time when compared with the Adomian decomposition method.

Example 4.2. Consider the nonlinear Newell–Whitehead–Segel equation,

$$u_t = u_{xx} + 2u - 3u^2, \tag{41}$$

with the initial condition,

$$u(x, 0) = \lambda, \tag{42}$$

whose exact solution was found to be [4]:

$$u(x, t) = \frac{\frac{-2}{3}\lambda e^{2t}}{\frac{-2}{3} + \lambda - \lambda e^{2t}}. \tag{43}$$

Case 1. (*By RDTM*)

By taking the reduced differential transform on both sides of (41) and (42), we have:

$$RDT[u_t] = RDT[u_{xx}] + 2RDT[u] - 3RDT[u^2] \tag{44}$$

and

$$RDT[u(x, 0) = \lambda]. \tag{45}$$

After applying fundamental theorems in (44) and (45), the following recurrence relation is obtained,

$$(k+1)U_{k+1}(x) = \frac{\partial^2}{\partial x^2}U_k(x) - 2U_k(x) - 3\sum_{r=0}^{k}U_r(x)U_{k-r}(x), \tag{46}$$

$$U_0(x) = \lambda. \tag{47}$$

By iterative calculations on (46), we have,

$$\begin{aligned} U_1(x) &= 2\lambda - 3\lambda^2, \\ U_2(x) &= \frac{2\lambda(2-3\lambda)(1-3\lambda)}{2!}, \\ U_3(x) &= \frac{2\lambda(2-3\lambda)(27\lambda^2 - 18\lambda + 2)}{3!}, \ldots \end{aligned} \tag{48}$$

From (4),

$$u(x, t) = U_0(x)t^0 + U_1(x)t + U_2(x)t^2 + U_3(x)t^3 + \cdots. \tag{49}$$

Substituting (47) and (48) in (49), we have,

$$\begin{aligned} u(x, t) = \lambda + (2\lambda - 3\lambda^2)t + \frac{2\lambda(2-3\lambda)(1-3\lambda)}{2!}t^2 \\ + \frac{2\lambda(2-3\lambda)(27\lambda^2 - 18\lambda + 2)}{3!}t^3 \cdots, \end{aligned} \tag{50}$$

or equivalently,

$$u(x, t) = \frac{\frac{-2}{3}\lambda e^{2t}}{\frac{-2}{3} + \lambda - \lambda e^{2t}}, \tag{51}$$

which is an exact solution of the problem.

Case 2. (*By ADM*)

In an operator form, (41) becomes

$$L_t u = L_{xx}u + 2u - 3u^2, \tag{52}$$

where the differential operator L is given by,

$$L_t = \frac{\partial}{\partial t}$$

and

$$L_{xx} = \frac{\partial^2}{\partial x^2},$$

where each operator is assumed as easily invertible, and therefore, the inverse operator L_t^{-1} is defined by,

$$L_t^{-1}(\cdot) = \int_0^t (\cdot)\, dt$$

and

$$L_{xx}^{-1}(\cdot) = \int_0^x \int_0^x (\cdot)\, dx\, dx.$$

Applying L_t^{-1} on both sides of (52) and using the initial condition, we obtain,

$$L_t^{-1}[L_t u] = L_t^{-1}[L_{xx}u + 2u - 3u^2],$$

or equivalently,

$$\begin{aligned} u(x, t) - u(x, 0) &= L_t^{-1}[L_{xx}u + 2u] - 3L_t^{-1}[u^2], \\ u(x, t) &= \lambda + L_t^{-1}[L_{xx}u + 2u] - 3L_t^{-1}[u^2]. \end{aligned} \tag{53}$$

Adomian method defines the solution $u(x, t)$ by an infinite series of componentsm, and it is given by,

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t), \tag{54}$$

where the components $u_0, u_1, u_2, \ldots$ are usually recurrently determined, and the nonlinear term $F(u) = u^2$ can be expressed by the Adomian polynomials A_n as,

$$u^2 = \sum_{n=0}^{\infty} A_n. \tag{55}$$

Substituting (54) and (55) in (53), we have,

$$\begin{aligned} \sum_{n=0}^{\infty} u_n(x, t) = \lambda + L_t^{-1}\left[L_{xx}\sum_{n=0}^{\infty} u_n(x, t) + 2\sum_{n=0}^{\infty} u_n(x, t)\right] \\ - 3L_t^{-1}\left[\sum_{n=0}^{\infty} A_n\right], \end{aligned} \tag{56}$$

where the components $u_n(x, t)$, $n \geqslant 0$ of the solution $u(x, t)$ can be recursively determined by using the relation,

$$u_0(x, t) = \lambda, \tag{57}$$

$$u_{k+1}(x, t) = L_t^{-1}[L_{xx}[u_k] + 2u_k] - 3L_t^{-1}[A_k]. \tag{58}$$

The Adomian polynomials A_n for the nonlinear term $F(u) = u^2$ is given by,

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]_{\lambda=0}. \tag{59}$$

The general formula (59) can be simplified as follows,

$$\begin{aligned} A_0 &= F(u_0) = u_0^2 = \lambda^2, \\ A_1 &= u_1F'(u_0) = 2u_0u_1 = \lambda^2(2-3\lambda)t^2, \\ A_2 &= u_2F'(u_0) + \frac{1}{2!}u_1^2F''(u_0) = \frac{\lambda^2(2-3\lambda)(4-9\lambda)t^3}{3}\cdots. \end{aligned} \tag{60}$$

By using (57), (58) and (60), we have,

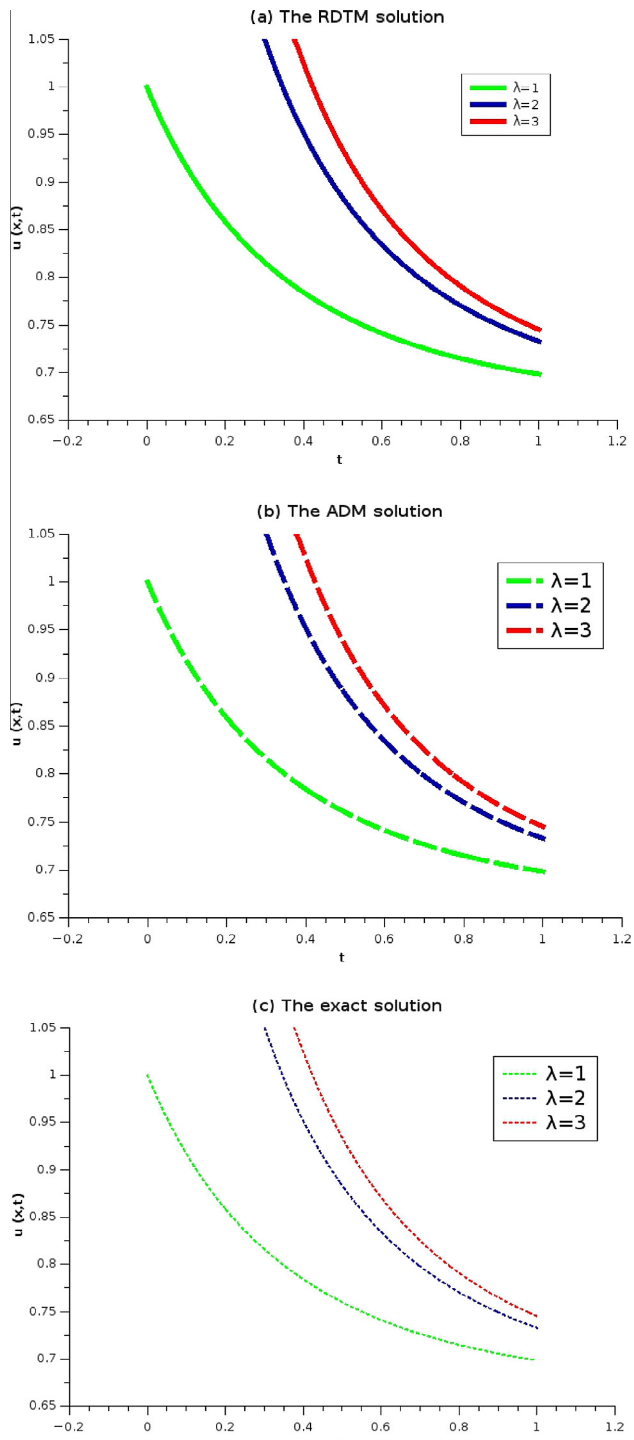

Figure 2 The comparison of RDTM and ADM with the exact solution $u(x,t)$ for different values of 'λ'.

$$u_1(x,t) = (2\lambda - 3\lambda^2)t,$$
$$u_2(x,t) = \frac{2\lambda(2-3\lambda)(1-3\lambda)t^2}{2!},\qquad (61)$$
$$u_3(x,t) = \frac{2\lambda(2-3\lambda)(27\lambda^2-18\lambda+2)t^3}{3!},\cdots.$$

Substituting the values of $u_n(x,t)$ in (54), we have,

$$u(x,t) = \lambda + (2\lambda - 3\lambda^2)t + \frac{2\lambda(2-3\lambda)(1-3\lambda)}{2!}t^2 + \frac{2\lambda(2-3\lambda)(27\lambda^2-18\lambda+2)}{3!}t^3\cdots,\qquad (62)$$

or, equivalently,

$$u(x,t) = \frac{\frac{-2}{3}\lambda e^{2t}}{\frac{-2}{3} + \lambda - \lambda e^{2t}},\qquad (63)$$

which is an exact solution of the problem. In order to verify the efficiency and accuracy of the proposed reduced differential transform method for solving nonlinear Newell–Whitehead–Segel Eqs. (41) and (42), graphs are drawn for the numerical solution as well as the exact solution.

From Fig. 2a–c, we can see that the solution obtained by the reduced differential transform method and the Adomian decomposition method coincide with the exact solution. The results of this example show that the reduced differential transform method is a simple method than the Adomian decomposition method because the reduced differential transform method solves the problem without using the Adomian polynomial which was used in the Adomian decomposition method, due to this fact, it is concluded that the reduced differential transform method is less consumption of time when compared with the Adomian decomposition method.

5. Conclusion

We have carried out the comparative study between the reduced differential transform method and the Adomian decomposition method by handling the Newell–Whitehead–Segel equation. Two numerical examples have shown that the reduced differential transform method is a very simple technique to handle linear and nonlinear Newell–Whitehead–Segel equation than the Adomian decomposition method, and also, it is demonstrated that the reduced differential transform method solves linear and nonlinear Newell–Whitehead–Segel equation without using any complicated polynomials like as the Adomian polynomials. In addition, the obtained series solution by the reduced differential transform method converges faster than those obtained by the Adomian decomposition method. It is concluded that this simple reduced differential transform method is a powerful technique to handle linear and nonlinear initial value problems.

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References

[1] Q. Li, Z. Zheng, S. Wang, J. Liu, Lattice boltzmann model for nonlinear heat equations, Adv. Neural Netw. (2012) 140–148.
[2] A. Malik, F. Chand, H. Kumar, S.C. Mishra, Exact solutions of some physical models using the $(\frac{G'}{G})$ expansion method, Pramana J. Phys. 78 (4) (2012) 513–529.
[3] S.A. Manaa, An approximate solution to the Newell–Whitehead equation by the Adomian decomposition method, Raf. J. Comp. Math. 8 (1) (2011) 171–180.

[4] A. Aasaraai, Analytic solution for Newell–Whitehead–Segel equation by differential transform method, Middle-East J. Sci. Res. 10 (2) (2011) 270–273.
[5] Y. Keskin, G. Oturanc, The reduced differential transform method for partial differential equations, Int. J. Nonlin. Sci. Numer. Simul. 10 (6) (2009) 741–749.
[6] Y. Keskin, G. Oturanc, The reduced differential transform method for solving linear and nonlinear wave equations, Iran. J. Sci. Technol. 34 (2) (2010) 113–122.
[7] G. Adomian, Nonlinear Stochastic Operator Equations, Academic Press, Orlando, FL, 1986.
[8] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Acad. Publ., Dordrecht, 1994.
[9] G. Adomian, A review of the decomposition method and some recent results for nonlinear equations, Comput. Math. Appl. 21 (1991) 101–127.
[10] R. Rach, A bibliography of the theory and applications of the Adomian decomposition method, 1961–2011, Kybernetes 41 (2012) 1087–1148.
[11] Y. Cherruault, Convergence of Adomian's method, Kybernetes. 18 (2) (1989) 31–38.
[12] Y. Cherruault, G. Adomian, Decomposition methods: a new proof of convergence, Math. Comput. Modell. 18 (12) (1993) 103–106.
[13] A.M. Wazwaz, A comparison between the Adomian decomposition method and Taylor series method in the series solutions, Appl. Math. Comput. 97 (1) (1998) 37–44.
[14] A.M. Wazwaz, A new algorithm for solving differential equations of Lane-Emden type, Appl. Math. Comput. 118 (2) (2001) 287–310.
[15] A.M. Wazwaz, Partial Differential Equations: Methods and Applications, Balkema Publishers, Leiden, 2002.
[16] T.M. Elzaki, Solution of nonlinear differential equations using mixture of Elzaki transform and differential transform method, Int. Math. Forum. 7 (13) (2012) 631–638.