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ORIGINAL ARTICLE

Adomian solution of a nonlinear quadratic integral equation

E.A.A. Ziada *

Faculty of Engineering, Delta University for Science and Technology, Gamasa, Egypt

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Abstract We are concerned here with a nonlinear quadratic integral equation (QIE). The existence of a unique solution will be proved. Convergence analysis of Adomian decomposition method (ADM) applied to these type of equations is discussed. Convergence analysis is reliable enough to estimate the maximum absolute truncated error of Adomian's series solution. Two methods are used to solve these type of equations; ADM and repeated trapezoidal method. The obtained results are compared.

MATHEMATICS SUBJECT CLASSIFICATION: 26A33, 65L05, 65L20

1. Introduction

Quadratic integral equations (QIEs) are often applicable in the theory of radiative transfer, kinetic theory of gases, theory of neutron transport and traffic theory [1–7]. In this paper, ADM is used to solve QIEs. This method has many advantages, it is efficiently works with different types of linear and nonlinear equations in deterministic or stochastic fields and gives an analytic solution for all these types of equations without linearization or discretization [8–15]. Repeated trapezoidal method [16] is also used to solve these type of equations and the results obtained from the two methods are compared. The contribution of this work can be summarized in the following four points:

- Introducing the sufficient condition that guarantees the existence of a unique solution to our problem (see Theorem 1).
- Based on the above point and using Adomian polynomials formula suggested in [17] convergence of ADM is discussed (see Theorem 2).
- Using point two, the maximum absolute truncated error of the Adomian's series solution is estimated (see Theorem 3).
- Preparation of algorithms using MATHEMATICA package to solve the given numerical examples.

* Tel.: +20 1009296257.
E-mail address: eng_emanziada@yahoo.com

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2. The Problem with the Solution Algorithm

Let $J = [0, T]$, $T \in R^+$. Denote by $E = C(J)$ the space of continuous functions defined on J with norm $\|x\| = \max_{t\in J}|x(t)|$. Consider the nonlinear QIE,

$$x(t) = p(t) + \left(\int_0^t k_1(t,\tau)f(\tau, x(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau, x(\tau))d\tau\right). \tag{1}$$

Assume the following assumptions:

(i) f, g: $[0, T] \times R \to R$ are continuous.
(ii) k_1, k_2: $[0, T] \times [0, T] \to R$ are continuous.
(iii) f, g satisfy the Lipschitz condition with Lipschitz constants c_1 and c_2

$$|f(t,x) - f(t,y)| \leqslant c_1|x-y|, \qquad |g(t,x) - g(t,y)| \leqslant c_2|x-y|. \tag{2}$$

(iv) $k_i = \max\{|k_i(t,s)| : t, s \in [0, T]\}$, $i = 1, 2$.
(v) $Q = sup\{|f(t,0)| : t \in [0, T]\}$, $R = sup\{|g(t,0)| : t \in [0, T]\}$, $|x(t)| < H$.

The solution algorithm of Eq. (1) using ADM is,

$$x_0(t) = p(t), \tag{3}$$

$$x_i(t) = \left(\int_0^t k_1(t,\tau)A_{i-1}(\tau)d\tau\right)\left(\int_0^t k_2(t,\tau)B_{i-1}(\tau)d\tau\right). \tag{4}$$

where A and B are Adomian polynomials of the nonlinear terms $f(t,x)$ and $g(t,x)$ respectively, which take the form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{\infty}\lambda^i x_i\right)\right]_{\lambda=0}$$

Finally, the solution will be

$$x(t) = \sum_{i=0}^{\infty} x_i(t) \tag{5}$$

This problem was discussed before in [18], the authors proved the existence of at least one positive nondecreasing solution but here we prove that it gives a unique solution.

3. Convergence analysis

3.1. Existence and uniqueness theorem

Theorem 1. *Let $p(t) \in C(J)$, $f(t,x)$ and $g(t,x)$ satisfies the Lipschitz condition (2). If*

$$T < \frac{1}{[k_1k_2[2c_1c_2H + c_1R + c_2Q]]^{1/2}}$$

then the QIE (1) has a unique solution $x \in C(J)$.

Proof. The mapping $F: E \to E$ is defined as,

$$Fx = p(t) + \left(\int_0^t k_1(t,\tau)f(\tau, x(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau, x(\tau))d\tau\right),$$

Let x, $y \in E$, then

$$\begin{aligned}
Fx - Fy &= \left(\int_0^t k_1(t,\tau)f(\tau,x(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau,x(\tau))d\tau\right) - \left(\int_0^t k_1(t,\tau)f(\tau,y(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau,y(\tau))d\tau\right)\\
&= \left(\int_0^t k_1(t,\tau)f(\tau,x(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau,x(\tau))d\tau\right) - \left(\int_0^t k_1(t,\tau)f(\tau,y(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau,x(\tau))d\tau\right)\\
&\quad + \left(\int_0^t k_1(t,\tau)f(\tau,y(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau,x(\tau))d\tau\right) - \left(\int_0^t k_1(t,\tau)f(\tau,y(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau,y(\tau))d\tau\right)\\
&= \left(\int_0^t k_1(t,\tau)[f(\tau,x(\tau)) - f(\tau,y(\tau))]d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau,x(\tau))d\tau\right)\\
&\quad + \left(\int_0^t k_1(t,\tau)f(\tau,y(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)[g(\tau,x(\tau)) - g(\tau,y(\tau))]d\tau\right)
\end{aligned}$$

which implies thatwhich implies that

$$\begin{aligned}
\|Fx - Fy\| &\leqslant \max_{t\in J}\left|\left(\int_0^t k_1(t,\tau)[f(\tau,x(\tau)) - f(\tau,y(\tau))]d\tau\right)\left(\int_0^t k_2(t,\tau)g(\tau,x(\tau))d\tau\right)\right|\\
&\quad + \max_{t\in J}\left|\left(\int_0^t k_1(t,\tau)f(\tau,y(\tau))d\tau\right)\left(\int_0^t k_2(t,\tau)[g(\tau,x(\tau)) - g(\tau,y(\tau))]d\tau\right)\right|\\
&\leqslant \max_{t\in J}\left(\int_0^t |k_1(t,\tau)||f(\tau,x(\tau)) - f(\tau,y(\tau))|d\tau\right)\left(\int_0^t |k_2(t,\tau)||g(\tau,x(\tau))|d\tau\right)\\
&\quad + \max_{t\in J}\left(\int_0^t |k_1(t,\tau)||f(\tau,y(\tau))|d\tau\right)\left(\int_0^t |k_2(t,\tau)||g(\tau,x(\tau)) - g(\tau,y(\tau))|d\tau\right)\\
&\leqslant k_1k_2c_1\max_{t\in J}|x(t) - y(t)|\left(\int_0^t d\tau\right)\left(\int_0^t (|g(\tau,x(\tau)) - g(\tau,0)| + |g(\tau,0)|)d\tau\right)\\
&\quad + k_1k_2c_2\max_{t\in J}|x(t) - y(t)|\left(\int_0^t (|f(\tau,x(\tau)) - f(\tau,0)| + |f(\tau,0)|)d\tau\right)\left(\int_0^t d\tau\right)\\
&\leqslant k_1k_2T\max_{t\in J}|x(t) - y(t)|\left[c_1\int_0^t (c_2|x(\tau)| + R)d\tau + c_2\int_0^t (c_1|x(\tau)| + Q)d\tau\right]\\
&\leqslant k_1k_2T^2[c_1(c_2H + R) + c_2(c_1H + Q)]\|x - y\|\\
&\leqslant k_1k_2T^2[2c_1c_2H + c_1R + c_2Q]\|x - y\| \leqslant L\|x - y\|
\end{aligned}$$

under the condition $0 < L = k_1k_2T^2[2c_1c_2H + c_1R + c_2Q] < 1$, the mapping F is contraction and hence for $T < \frac{1}{[k_1k_2[2c_1c_2H + c_1R + c_2Q]]^{1/2}}$ there exists a unique solution $x \in C(J)$of the problem (1) and this completes the proof. □

Corollary 1. *Consider the nonlinear QIE,*

$$x(t) = p(t) + \left(\int_0^t k(t,\tau)f(\tau, x(\tau))d\tau\right)^2, \tag{6}$$

from problem (1), the QIE (6) has a unique solution $x \in C(J)$, if

$$T < \frac{1}{k[2c(cH + M)]^{1/2}},$$

where $c_1 = c_2 = c$, $k_1 = k_2 = k$, $R = Q = M$.

3.2. Proof of convergence

Theorem 2. *Let the solution of the QIE (1) be exist. If $|x_1(t)| < k$, k is a positive constant then the series solution (5) of the QIE (1) using ADM converges.*

Proof. Define the sequence $\{S_p\}$ such that, $S_p = \sum_{i=0}^{p} x_i(t)$ is the sequence of partial sums from the series solution $\sum_{i=0}^{\infty} x_i(t)$, and we have,

$$f(t,x) = A = \sum_{i=0}^{\infty} A_i,$$

$$g(t,x) = B = \sum_{i=0}^{\infty} B_i.$$

Let S_p and S_q be two arbitrary partial sums with $p > q$. Now, we are going to prove that $\{S_p\}$ is a Cauchy sequence in this Banach space E.

$$\begin{aligned}
S_p - S_q &= \sum_{i=0}^{p} x_i - \sum_{i=0}^{q} x_i = \left(\int_0^t k_1(t,\tau)\sum_{i=0}^{p} A_{i-1}(\tau)d\tau\right)\left(\int_0^t k_2(t,\tau)\sum_{i=0}^{p} B_{i-1}(\tau)d\tau\right)\\
&\quad - \left(\int_0^t k_1(t,\tau)\sum_{i=0}^{q} A_{i-1}(\tau)d\tau\right)\left(\int_0^t k_2(t,\tau)\sum_{i=0}^{q} B_{i-1}(\tau)d\tau\right)\\
&= \left(\int_0^t k_1(t,\tau)\sum_{i=0}^{p} A_{i-1}(\tau)d\tau\right)\left(\int_0^t k_2(t,\tau)\sum_{i=0}^{p} B_{i-1}(\tau)d\tau\right)\\
&\quad - \left(\int_0^t k_1(t,\tau)\sum_{i=0}^{q} A_{i-1}(\tau)d\tau\right)\left(\int_0^t k_2(t,\tau)\sum_{i=0}^{p} B_{i-1}(\tau)d\tau\right)\\
&\quad + \left(\int_0^t k_1(t,\tau)\sum_{i=0}^{q} A_{i-1}(\tau)d\tau\right)\left(\int_0^t k_2(t,\tau)\sum_{i=0}^{p} B_{i-1}(\tau)d\tau\right)\\
&\quad - \left(\int_0^t k_1(t,\tau)\sum_{i=0}^{q} A_{i-1}(\tau)d\tau\right)\left(\int_0^t k_2(t,\tau)\sum_{i=0}^{q} B_{i-1}(\tau)d\tau\right)\\
&= \left(\int_0^t k_1(t,\tau)\left[\sum_{i=0}^{p} A_{i-1}(\tau) - \sum_{i=0}^{q} A_{i-1}(\tau)\right]d\tau\right)\\
&\quad \times \left(\int_0^t k_2(t,\tau)\sum_{i=0}^{p} B_{i-1}(\tau)d\tau\right) + \left(\int_0^t k_1(t,\tau)\sum_{i=0}^{q} A_{i-1}(\tau)d\tau\right)\\
&\quad \times \left(\int_0^t k_2(t,\tau)\left[\sum_{i=0}^{p} B_{i-1}(\tau) - \sum_{i=0}^{q} B_{i-1}(\tau)\right]d\tau\right) \|S_p - S_q\| \leqslant \max_{t\in J} |\\
&\quad \times \left(\int_0^t k_1(t,\tau)\sum_{i=q+1}^{p} A_i(\tau)d\tau\right)\left(\int_0^t k_2(t,\tau)\sum_{i=0}^{p} B_{i-1}(\tau)d\tau\right) + \max_{t\in J} |\\
&\quad \times \left(\int_0^t k_1(t,\tau)\sum_{i=0}^{q} A_{i-1}(\tau)d\tau\right)\left(\int_0^t k_2(t,\tau)\sum_{i=q+1}^{p} B_{i-1}(\tau)d\tau\right)\Bigg|\\
&\leqslant \max_{t\in J}\left|\int_0^t k_1(t,\tau)\sum_{i=q}^{p-1} A_i(\tau)d\tau\right|\left|\int_0^t k_2(t,\tau)\sum_{i=0}^{p} B_{i-1}(\tau)d\tau\right|\\
&\quad + \max_{t\in J}\left|\int_0^t k_1(t,\tau)\sum_{i=0}^{q} A_{i-1}(\tau)d\tau\right|\left|\int_0^t k_2(t,\tau)\sum_{i=q}^{p-1} B_i(\tau)d\tau\right|\\
&\leqslant k_1k_2 \max_{t\in J}\left[\left(\int_0^t |f(\tau,S_{p-1}) - f(\tau,S_{q-1})|d\tau\right)\left(\int_0^t |g(\tau,S_p)|d\tau\right)\right]\\
&\quad + k_1k_2 \max_{t\in J}\left[\left(\int_0^t |f(\tau,S_q)|d\tau\right)\left(\int_0^t |g(\tau,S_{p-1}) - g(\tau,S_{q-1})|d\tau\right)\right]\\
&\leqslant k_1k_2\left[c_1\left(\int_0^t d\tau\right)\left(\int_0^t (|g(\tau,S_p) - g(\tau,0)| + |g(\tau,0)|)d\tau\right)\right.\\
&\quad \left. + c_2\left(\int_0^t (|f(\tau,S_q) - f(\tau,0)| + |f(\tau,0)|)d\tau\right)\left(\int_0^t d\tau\right)\right]\|S_{p-1} - S_{q-1}\|\\
&\leqslant k_1k_2T^2[2c_1c_2H + c_1R + c_2Q]\|S_{p-1} - S_{q-1}\| \leqslant L\|S_{p-1} - S_{q-1}\|
\end{aligned}$$

Let $p = q + 1$ then,

$$\begin{aligned}
\|S_{q+1} - S_q\| &\leqslant L\|S_q - S_{q-1}\| \leqslant L^2\|S_{q-1} - S_{q-2}\| \leqslant \cdots\\
&\leqslant L^q\|S_1 - S_0\|
\end{aligned}$$

From the triangle inequality we have,

$$\begin{aligned}
\|S_p - S_q\| &\leqslant \|S_{q+1} - S_q\| + \|S_{q+2} - S_{q+1}\| + \cdots + \|S_p - S_{p-1}\|\\
&\leqslant \left[L^q + L^{q+1} + \cdots + L^{p-1}\right]\|S_1 - S_0\|\\
&\leqslant L^q\left[1 + L + \cdots + L^{p-q-1}\right]\|S_1 - S_0\|\\
&\leqslant L\left[\frac{1 - L^{p-q}}{1 - L}\right]\|x_1\|
\end{aligned}$$

Now $0 < L < 1$, and $p > q$ implies that $(1 - L^{p-q}) \leqslant 1$. Consequently,

$$\|S_p - S_q\| \leqslant \frac{L^q}{1 - L}\|x_1\| \leqslant \frac{L^q}{1 - L}\max_{t\in J}|x_1(t)|$$

but, $|x_1(t)| < k$ and as $q \to \infty$ then, $\|S_p - S_q\| \to 0$ and hence, $\{S_p\}$ is a Cauchy sequence in this Banach space E and the series $\sum_{i=0}^{\infty} x_i(t)$ converges. □

3.3. Error analysis

Theorem 3. *The maximum absolute truncation error of the series solution (5) to the QIE (1) is estimated to be,*

$$\max_{t\in J}\left|x(t) - \sum_{i=0}^{q} x_i(t)\right| \leqslant \frac{L^q}{1 - L}\max_{t\in J}|x_1(t)|$$

Proof. From Theorem 2 we have,

$$\|S_p - S_q\| \leqslant \frac{L^q}{1 - L}\max_{t\in J}|x_1(t)|$$

but, $S_p = \sum_{i=0}^{p} x_i(t)$ as $p \to \infty$ then, $S_p \to x(t)$ so,

$$\|x - S_q\| \leqslant \frac{L^q}{1 - L}\max_{t\in J}|x_1(t)|$$

so, the maximum absolute truncation error in the interval J is,

$$\max_{t\in J}\left|x(t) - \sum_{i=0}^{q} x_i(t)\right| \leqslant \frac{L^q}{1 - L}\max_{t\in J}|x_1(t)|$$

and this completes the proof. □

4. Numerical examples

Example 1. Consider the following nonlinear QIE,

$$x(t) = \left(t^2 - \frac{t^{15}}{1350}\right) + \left(\int_0^t \tau x^2(\tau)d\tau\right)\left(\int_0^t \frac{\tau^2}{25}x^3(\tau)d\tau\right), \qquad (7)$$

and has the exact solution $x(t) = t^2$. Applying ADM to Eq. (7), we get

$$x_0(t) = \left(t^2 - \frac{t^{15}}{1350}\right),$$

$$x_i(t) = \left(\int_0^t \tau A_{i-1}(\tau)\ d\tau\right)\left(\int_0^t \frac{\tau^2}{25}B_{i-1}(\tau)\ d\tau\right), \quad i \geqslant 1.$$

where A and B are Adomian polynomials of the nonlinear terms x^2 and x^3 respectively and the solution will be,

$$x(t) = \sum_{i=0}^{q} x_i(t)$$

Table 1 Absolute error.

t	Error of ADM ($q = 5$)	Error of RT ($h = 0.01$)
0.1	3.15041×10^{-51}	1.73472×10^{-18}
0.2	2.73766×10^{-26}	5.20417×10^{-16}
0.3	2.33331×10^{-21}	1.00531×10^{-13}
0.4	7.34949×10^{-18}	4.22859×10^{-12}
0.5	3.79959×10^{-15}	7.68964×10^{-11}
0.6	6.26342×10^{-13}	8.22618×10^{-10}
0.7	4.69157×10^{-11}	6.10202×10^{-9}
0.8	1.97275×10^{-9}	3.46237×10^{-8}
0.9	5.33651×10^{-8}	1.6013×10^{-7}
1	1.01893×10^{-6}	6.30669×10^{-7}

Table 2 Max. absolute error.

q	Max. error of ADM
5	0.000036246
10	9.23545×10^{-7}
15	2.35318×10^{-8}
20	5.99588×10^{-10}

Table 3 Max. absolute error.

h	Error of RT
0.1	0.0000638458
0.01	6.30669×10^{-7}
0.001	6.3059×10^{-9}

Table 4 Absolute error.

t	Error of ADM ($q = 1$)	Error of RT ($h = 0.1$)
0.1	1.26517×10^{-10}	1.78902×10^{-6}
0.2	1.87763×10^{-8}	8.01745×10^{-6}
0.3	3.82452×10^{-7}	0.0000209399
0.4	3.49231×10^{-6}	0.0000444139
0.5	0.0000206935	0.0000844492
0.6	0.0000936433	0.000150055
0.7	0.000352391	0.000254643
0.8	0.00115842	0.000418523
0.9	0.00342886	0.000673599
1	0.00931516	0.00107275

this series solution converges if $T < 1.15812$. Table 1 shows a caparison between the absolute error of ADM solution and repeated trapezoidal (RT) solution, while Table 2 shows the maximum absolute truncated error (using Theorem 3) at different values of q when $t = 1$ and Table 3 shows the the maximum absolute error of RT at different values of h (h is the step size).

Example 2. Consider the following nonlinear QIE,

Table 5 Max. absolute error.

q	Max. error of ADM
1	0.000278622
5	1.30744×10^{-8}
10	5.07784×10^{-14}
15	1.97213×10^{-19}

Table 6 Max. absolute error.

h	Error of RT
0.1	0.0000844492
0.01	8.44338×10^{-7}
0.001	8.44337×10^{-9}

$$x(t) = \left(t - (e^t - 1)\left(\frac{t^3}{30} + \frac{t^5}{50}\right)\right) + \left(\int_0^t \left(\frac{\tau^2 + 1}{10}\right) x^2(\tau) d\tau\right)\left(\int_0^t e^{x(\tau)}\, d\tau\right), \tag{8}$$

and has the exact solution $x(t) = t$. Applying ADM to Eq. (8), we get

$$x_0(t) = \left(t - (e^t - 1)\left(\frac{t^3}{30} + \frac{t^5}{50}\right)\right),$$

$$x_i(t) = \left(\int_0^t \left(\frac{\tau^2 + 1}{10}\right) A_{i-1}(\tau)\, d\tau\right)\left(\int_0^t B_{i-1}(\tau)\, d\tau\right), \quad i \geqslant 1.$$

where A and B are Adomian polynomials of the nonlinear terms x^2 and e^x respectively. The series solution converges if $T < 0.723199$. Table 4 shows a caparison between the absolute error of ADM solution and RT solution, while Table 5 shows the maximum absolute truncated error (using Theorem 3) at different values of q (when $t = 0.5$) and Table 6 shows the the maximum absolute error of RT at different values of h.

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