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ORIGINAL ARTICLE

On *b*-connectedness and *b*-disconnectedness and their applications

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Abstract In this paper, by using *b*-open (= γ-open) sets we study the concept of *b*-separated sets. With this concept we study the notion of *b*-connected sets and strongly *b*-connected sets. We give some properties of such concepts with some *b*-separation axioms and compact spaces. Finally, we construct a new topological space on a connected graph.

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1. Introduction

Connectedness [4] is a well-known notion in topology. Numerous authors studied connectedness. In [5], *P*-spaces and extremal disconnectedness are studied. Connectedness in [19–21] are used to expand some topological spaces. In [27], authors proved that neither first countable nor C~ech-complete spaces are maximal Tychonoff connected. Many other topologists defined and studied connectedness in bitopological spaces [8,26]. It is important to study some types of connectedness in digital spaces. A point with integer coordinates is called a digital point. The problem of finding a topology for the digital plane and the digital 3-space is of importance in image processing and more generally in all situations where spatial relations are modeled on a computer. In all these applications it is essential to have a data structure on the computer which shares as many as possible features with the real topological situation.

Connectedness and compactness are powerful tools in topology but they have many dissimilar properties. The concept of Hausdorff spaces is almost an integral part of compactness. Investigations into the properties of cut points of topological spaces which are connected, compact and Hausdorff date back to the 1920s. Connectedness together with compactness with the assumption of Hausdorff has been studied in [28] from the view point of cut points. In [22], authors studied some types of connected topological spaces. The class of *b*-open sets in the sense of Andrijević [3] was discussed by El-Atik [17] under the name of γ-open sets. Since then these concepts have used to define and investigate many topological properties. The aim of this paper is to study γ-connectedness. Also digital spaces are examined in the context of these new

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concepts. However, our main interest shall be digital spaces that are also topological spaces.

2. Preliminaries

In [3], Andrijević introduced a new class of generalized open sets called b-open ($=\gamma$-open [17]) sets into the field of topology. This class is a subset of the class of semi-preopen sets [2], i.e. a subset of a topological space which is contained in the closure of the interior of its closure. Also the class of b-open sets is a superset of the class of semi-open sets [23], i.e. a set which is contained in the closure of its interior, and the class of locally dense sets [7] or preopen sets [24], i.e. a set which is contained in the interior of its closure. Andrijević studied several fundamental and interesting properties of b-open sets. In [6,9–11] authors discussed some applications of b-open sets.

Throughout the present paper, the space (X,τ) and (Y,σ) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. A subset A is said to be semi-open [23] (resp. β-open [1], preopen [24], α-open [25]) if $A \subset Cl(Int(A))$ (resp. $A \subset Cl(Int(Cl(A)))$, $A \subset Int(Cl(A))$, $A \subset Int(Cl(Int(A)))$, where $Cl(A)$ and $Int(A)$ denote the closure and the interior of A in (X,τ), respectively. The complement of a semiopen (resp. β-open, preopen, α-open) set is said to be semi-closed (resp. β-closed, preclosed,α -closed). We denote the collection of all semiopen (resp. β-open, preopen, α-open) sets by SO(X) (resp. βO(X), PO(X), αO(X)). We set $SO(X,x) = \{U: x \in U \in SO(X)\}$, $\beta O(X,x) = \{U: x \in U \in \beta O(X)\}$, $PO(X,x) = \{U: x \in U \in PO(X)\}$ and $\alpha O(X,x) = \{U: x \in U \in \alpha O(X)\}$.

Let $A \subset X$, then A is said to be b-open [3] if $A \subset Cl(Int(A)) \cup Int(Cl(A))$. The complement $X - A$ of a b- open set A is called b-closed and the b-closure of a set A, denoted by $bCl(A)$, is the intersection of all b-closed sets containing A. $bCl(A)$ is the smallest b-closed set containing A. The b-interior of a set A denoted by $bInt(A)$, is the union of all b-open sets contained in A. $bInt(A)$ is the largest b-open set contained in A. The family of all b-open (resp. b-closed) sets in a space X will be denoted by $BO(X)$ (resp. $BC(X)$).

Proposition 2.1 [3]. *(i) The union of any family of b-open sets is a b-open set. (ii) The intersection of an open and a b-open set is a b-open set.*

Lemma 2.2. *The b-closure of a subset A of X, denoted by $bCl(A)$, is the set of all $x \in X$ such that $O \cap A \neq \phi$ for every $O \in BO(X,x)$, where $BO(X,x) = \{U: x \in U \in BO(X,\tau)\}$.*

Definition 2.3 12. The b-boundary of a set A of a space X is defined by $b\text{-}bd(A) = bCl(A) \cap bCl(X - A)$.

Definition 2.4 [13]. A space X is said to be b-connected if X cannot be expressed as the union of two disjoint nonempty b-open sets of X.

Lemma 2.5 17. *Let A be a subset of a topological space X. Then $A \in BO(X)$ if and only if $bCl(A)$ is b-clopen in X (i.e., b-open and b-closed).*

Definition 2.6 17. A subset $N \subseteq X$ is called a b-neighborhood (breifly b-nbd) of a point $x \in X$ if there exists a b-open set $U \subseteq N$ such that $x \in U \subseteq N$.

3. b-Separateness and b-connectedness

Definition 3.1 13. Two subsets A and B in a space X are said to be b-separated if and only if $A \cap bCl(B) = \phi$ and $bCl(A) \cap B = \phi$.

From the fact that $bCl(A) \subset Cl(A)$, for every subset A of X, every separated set is b-separated. But the converse may not be true as shown in the following example.

Example 3.2. Let $X = \{a,b,c,d\}$ with a topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$. The subsets $\{a\}$, $\{c,d\}$ are b-separated but not separated.

Remark 3.3. Each two b-separated sets are always disjoint, since $A \cap B \subseteq A \cap bCl(B) = \phi$. The converse may not be true in general.

Example 3.4. Let $X = \{a,b,c,d\}$ with a topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. $BO(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$ The subsets $\{a,c\}$, $\{b,d\}$ are disjoint but not b-separated.

Theorem 3.5. *Let A and B be nonempty sets in a space X. The following statements hold:*

(i) If A and B are b-separated and $A_1 \subseteq A$ and $B_1 \subseteq B$, then A_1 and B_1 are so.
(ii) If $A \cap B = \phi$ such that each of A and B are both b-closed (b-open), then A and B are b-separated.
(iii) If each of A and B are both b-closed (b-open) and if $H = A \cap (X - B)$ and $G = B \cap (X - A)$, then H and G are b-separated.

Proof.

(i) Since $A_1 \subset A$, then $bCl(A_1) \subset bCl(A)$. Then $B \cap bCl(A) = \phi$ implies $B_1 \cap bCl(A) = \phi$ and $B_1 \cap bCl(A_1) = \phi$. Similarly $A_1 \cap bCl(B_1) = \phi$. Hence A_1 and B_1 are b-separated.
(ii) Since $A = bCl(A)$ and $B = bCl(B)$ and $A \cap B = \phi$, then $bCl(A) \cap B = \phi$ and $bCl(B) \cap A = \phi$. Hence A and B are b-separated. If A and B are b-open, then their complements are b-closed.
(iii) If A and B are b-open, then $X - A$ and $X - B$ are b-closed. Since $H \subset X - B$, $bCl(H) \subset bCl(X - B) = X - B$ and so $bCl(H) \cap B = \phi$. Thus $G \cap bCl(H) = \phi$. Similarly, $H \cap bCl(G) = \phi$. Hence H and G are b-separated. □

Theorem 3.6. *The sets A and B of a space X are b-separated if and only if there exist U and V in $BO(X)$ such that $A \subset U$, $B \subset V$ and $A \cap V = \phi$, $B \cap U = \phi$.*

Proof. Let A and B be b-separated sets. Set $V = X - bCl(A)$ and $U = X - bCl(B)$. Then U, $V \in BO(X)$ such that $A \subset U$, $B \subset V$ and $A \cap V = \phi$, $B \cap U = \phi$. On the other hand, let U, $V \in BO(X)$ such that $A \subset U$, $B \subset V$ and $A \cap V = \phi$, $B \cap U = \phi$. Since $X - V$ and $X - U$ are b-closed, then

$bCl(A) \subset X - V \subset X - B$ and $bCl(B) \subset X - U \subset X - A$. Thus $bCl(A) \cap B = \phi$ and $bCl(B) \cap A = \phi$. □

Definition 3.7 17. A point $x \in X$ is called a b-limit point of a set $A \subset X$ if every b-open set $U \subseteq X$ containing x contains a point of A other than x.

Theorem 3.8. *Let A and B be nonempty disjoint subsets of a space X and $E = A \cup B$. Then A and B are b-separated if and only if each of A and B is b-closed (b-open) in E.*

Proof. Let A and B are b-separated sets. By Definition 3.1, A contains no b-limit points of B. Then B contains all b-limit points of B which are in $A \cup B$ and B is b-closed in $A \cup B$. Therefore B is b-closed in E. Similarly A is b-closed in E. □

Definition 3.9 13. A subset S of a space X is said to be b-connected relative to X if there is not exist two b-separated subsets A and B relative to X and $S = A \cup B$. Otherwise, S is said to be b-disconnected.

By Definition 3.9, one can show that each b-connected set is connected. The converse may not be true in general as shown in [13] and in the below examples. In other words, each disconnected is b-disconnected.

Example 3.10. A space $X = \{a,b,c\}$ with Sierpinski topology is connected but not b-connected.

Example 3.11. Any space with indiscrete topology is connected but not b-connected since b-open sets establish a discrete topology.

Example 3.12. Let $X = \{a,b,c,d\}$ with a topology $\tau = \{X, \phi, \{a\}, \{a,b\}\}$. The subset $\{a,b,c\}$ is connected but not b-connected.

Theorem 3.13. *Let A and B be subsets in a space X such that $A \subset B \subset bCl(A)$. If A is b-connected, then B is b-connected.*

Proof. If B is b-disconnected, then there exist two b-separated subsets U and V relative to X such that $B = U \cup V$. Then either $A \subseteq U$ or $A \subseteq V$. Without loss of generality, let $A \subseteq U$. As $A \subseteq U \subseteq B$, $bCl_B(A) \subseteq bCl_B(U) \subset bCl(U)$. Also $bCl_B(A) = B \cap bCl(A) = B \supseteq bCl(U)$. This implies to $B = bCl(U)$. So U and V are not b-separated and B is b-connected. □

Definition 3.14. A space X is locally b-connected at a point p if every b-nbd of p contains a b-connected b-nbd of p. X is said to be locally b-connected if it is locally b-connected at each of its points.

The following lemma is due to El-Atik [17].

Lemma 3.15. *Let A and X_0 be subsets of a space X. If $A \in BO(X)$ and $X_0 \in \alpha O(X)$, then $A \cap X_0 \in BO(X_0)$.*

Lemma 3.16. *Every α-open subspace X_0 of a locally b-connected space is locally b-connected in X_0.*

Proof. This is an immediate consequence of Definitions 3.14, 2.6 and Lemma 3.15. □

Theorem 3.17. *Any space X is locally b-connected if the components of every α-open subspace U of X are b-open in U.*

Proof. Let X be a locally b-connected space and let U be an α-open subspace of X. By Lemma 3.16, U is a locally b-connected space in U. Also, the components of U say, S_i are α-open sets of U. Since U is α-open, then by Lemma 3.15, $A_i = A_i \cap U \in BO(U)$. □

Theorem 3.18. *If E is b-connected, then bCl(E) is b-connected.*

Proof. By contradiction, suppose that $bCl(E)$ is b-disconnected. Then there are two nonempty b-separated sets G and H in X such that $bcl(E) = G \cup H$. Since $E = (G \cap E) \cup (H \cap E)$ and $bCl(G \cap E) \subset bCl(G)$ and $bCl(H \cap E) \subset bcl(H)$ and $G \cap H = \phi$, then $(bCl(G \cap E)) \cap H = \phi$. Hence $(bCl(G \cap E)) \cap (H \cap E) = \phi$. Similarly $(bcl(H \cap E)) \cap (G \cap E) = \phi$. Therefore E is b-disconnected. □

Lemma 3.19. *Let $A \subseteq B \cup C$ such that A be a nonempty b-connected set in a space X and B, C are b-separated. Then only one of the following conditions holds:*

(i) $A \subseteq B$ and $A \cap C = \phi$.
(ii) $A \subseteq C$ and $A \cap B = \phi$.

Proof. Since $A \cap C = \phi$, then $A \subseteq B$. Also, if $A \cap B = \phi$, then $A \subseteq C$. Since $A \subseteq B \cap C$, then both $A \cap B = \phi$ and $A \cap C = \phi$ cannot hold simultaneously. Similarly, suppose that $A \cap B \neq \phi$ and $A \cap C \neq \phi$, then, by Theorem 3.5(i), $A \cap B$ and $A \cap C$ are b-separated such that $A = (A \cap B) \cup (A \cap C)$ which contradicts with the b-connectedness of A. Hence one of the conditions (i) and (ii) must be hold. □

Definition 3.20 13, 17. A function f: $X \to Y$ is said to be:

(i) b-continuous if the inverse image of each open set in Y is b-open X.
(ii) b-open if the image of each open set in X is b-open Y.
(iii) b-closed if the image of each closed set in X is b-closed Y.

Lemma 3.21 17. *Let f: $X \to Y$ be a b-continuous function. Then $bCl(f^{-1}(B)) \subseteq f^{-1}(Cl(B))$, for each $B \subseteq Y$.*

Theorem 3.22. *For a b-continuous function f: $X \to Y$, if K is b-connected in X, then f(K) is connected in Y.*

Proof. Suppose that $f(K)$ is disconnected in Y. There exist two separated sets P and Q of Y such that $f(K) = P \cup Q$. Set $A = K \cap f^{-1}(P)$ and $B = K \cap f^{-1}(Q)$. Since $f(K) \cap P \neq \phi$, then $K \cap f^{-1}(P) \neq \phi$ and so $A \neq \phi$. Similarly $B \neq \phi$. Since $P \cap Q = \phi$, then $A \cap B = K \cap f^{-1}(P \cap Q) = \phi$ and so $A \cap B = \phi$. Since f is b-continuous, then by Lemma 3.21,

$bCl(f^{-1}(Q)) \subset f^{-1}(Cl(Q))$ and $B \subset f^{-1}(Q)$, then $bCl(B) \subset f^{-1}(Cl(Q))$. Since $P \cap Cl(Q) = \phi$, then $A \cap f^{-1}(Cl(Q)) \subset f^{-1}(P) \cap f^{-1}(Cl(Q)) = \phi$ and then $A \cap bCl(B) = \phi$. Thus A and B are b-separated. □

Corollary 3.23. *For a b-continuous function f: $X \to Y$, if K is disconnected in X, then f(K) is b-disconnected in Y.*

Proof. Obvious. □

Theorem 3.24. *For a bijective b-closed f: $X \to Y$, if K is b-connected Y, then $f^{-1}(K)$ is connected X.*

Proof. The proof is similar to that of Theorem 3.22. Thus we omit it. □

To avoid the confusion for definitions of b-open and b-closed functions in [17,16] and [6], we set M-b-open and M-b-closed functions instead of b-open and b-closed functions.

Definition 3.25. A function f: $X \to Y$ is said to be:

(i) b-Irresolute [6] if for each point $x \in X$ and each b-open set V of Y containing $f(x)$, there exists a b-open set U of X containing x such that $f(U) \subset V$.
(ii) b-Irresolute [17] if $f^{-1}(V) \in BO(X)$ for every $V \in BO(Y)$.
(iii) M-b-open [16] if $f(V) \in BO(Y)$ for every $V \in BO(X)$.
(iv) M-b-closed [14] if $f(V) \subset BC(Y)$ for every $V \in BC(X)$.
(v) Strongly b-irresolute [17] if $f^{-1}(V) \in BO(X)$ for every open set V in Y.
(vi) Strongly M-b-open [17] if $f(V) \in BO(Y)$ for every open set V in X.
(vii) Strongly M-b-closed [17] if $f(V) \in BC(Y)$ for every closed set V in X.

Lemma 3.26. *[17] A function f: $X \to Y$ is a b-irresolute if and only if $bcl(f^{-1}(B)) \subset f^{-1}(bcl(B)) \subset f^{-1}$ $(cl(B))$, for each $B \subset Y$.*

Theorem 3.27. *Let f: $(X,\tau) \to (Y,\sigma)$ be a b-irresolute function. If K is b-connected in X, then f(K) is b-connected in Y.*

Proof. By using Definition 3.25 and Lemma 3.26, it is direct consequence of Theorem 3.22. □

4. Strongly *b*-connectedness in compact spaces

Definition 4.1. A space X is strongly b-connected if and only if it is not a disjoint union of countably many but more than one b-closed set i.e. if E_i are nonempty disjoint closed sets of X, then $X \neq E_1 \cup E_2 \cup E_2 \cup \cdots$. otherwise X is said to be strongly b-disconnected

Note the similarity between Definition 4.1 and that of b-connectedness. If X is b-connected, and E_1 and E_2 are any two nonempty disjoint closed sets of X, then $X \neq E_1 \cup E_2$.

Lemma 4.2. *For any surjective b-irresolute function f: $X \to Y$. The image f(X) is strongly b-connected if X is strongly b-connected.*

Proof. Suppose $f(X)$ is strongly b-disconnected, by Definition 4.1 it is a disjoint union of countably many but more than one b-closed sets. Since f is b-irresolute, then the inverse image of b-closed sets are still b-closed, X is also a disjoint union of b-closed sets. Therefore, f(X) is strongly b-connected. □

Theorem 4.3. *A space X is strongly b-connected if there exists a constant surjective b-irresolute function f: $X \to D$, where D denote to a discrete space of X.*

Proof. Let X be strongly b-connected and f: $X \to D$ be a surjective b-irresolute function, then by Lemma 4.2, $f(X)$ is strongly b-connected. The only strongly b-connected subset of D are the one-point spaces. Hence f is constant. Conversely, suppose X is a disjoint union of countably many but more than one b-closed sets, $X = \cup_i E_i$. Then define f: $X \to D$ by taking $f(x) = i$ whenever $x \in E_i$. This f is a surjective b-irresolute and not constant. So X is strongly b-connected. □

Strongly b-connectedness is a stronger notion of b-connectedness. In other words, given a b-connected space, we can make it strongly b-connected by adding some conditions. But what conditions should be added is the difficulty. Our starting point is b-connected spaces, thus a b-continuum may be useful. The concept of a b-continuum is defined on a b-connected set.

Definition 4.4. A compact b-connected set is called a b-continuum.

Definition 4.5. A space X is called:

(i) bT_1 [15] if for each $x, y \in X$, $x \neq y$, there exist two disjoint b-open sets U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.
(ii) bT_2 ([15,18]) if for each $x, y \in X$, $x \neq y$, there exist two disjoint b-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \phi$.
(iii) b-normal (γ-normal [14]) for any pair of disjoint b-closed sets F_1 and F_2, there exist disjoint b -open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$ such that $U \cap V = \phi$.

Lemma 4.6. *If A is any b-continuum in a bT_2 space X and B is any b-open set such that $A \cap B \neq \phi \neq A \cap (X - B)$, then every component of $(A \cap bCl(B)) \cap b - bd(B) \neq \phi$.*

Proof. It is obvious by Definitions 2.3, 4.4 and 4.5. □

Theorem 4.7. *Let X be a compact bT_2-space. Then X is b-connected if and only if X is strongly b-connected.*

Proof. It is clear that if X is strongly b-connected, then X is b-connected. Now, suppose that X is a compact bT_2 b-connected space and it is strongly b-disconnected, then X is a union of a countably many but more than one disjoint b-closed sets. Then $X = \cup K_i$, where K_i are b-closed disjoint sets. Since a compact bT_2-space is b-normal, then X, by Definition 4.5, is a b-normal space. So there exist a b-open sets U such that $K_2 \subset U$ and $bCl(U) \cap K_1 = \phi$. Let X_1 be a component of $bCl(U)$ which intersects K_2. Then X_1 is compact and b-connected. Now by Lemma 4.6, $X_1 \cap b - bd(U) \neq \phi$ i.e. X_1 contains a point

$p \in b - bd(U)$ such that $p \notin U$ and $p \notin K_1$. Hence $X_1 \cap K_i \neq \phi$ for some $i > 2$. Let K_{n_2} be the first K_i for $i > 2$ which intersects X_1, and let V be a b-open set satisfying $K_{n_2} \subset V$, and $bCl(V) \cap K_2 = \phi$. Then let X_2 be a component of $X_1 \cap bCl(V)$ which contains a point of K_{n_2}. Again we have $X_2 \cap b - bd(V) \neq \phi$, and X_2 contains some point $p \in b - bd(V)$ such that $p \notin V$, $p \notin K_1 \cup K_2$. Hence $X_2 \cap K_i \neq \phi$ for some $i > n_2$, and $X_2 \cap K_i = \phi$ for $i < n_2$. Let K_{n_3} be the first K_i for $i > n_2$, which intersects X_2, then by methods similar to the above we can find a compact b-connected X_3 such that $X_3 \subset X_2 \subset X_1$, and X_3 intersects some K_i with $i > n_3$ but $X_3 \cap K_i = \phi$for $i < n_3$. In this manner, we obtain a sequence of subcontinua of X: X_1 X_2 X_3 $\cdots$, such that for each j, $X_j \cap K_i = \phi$ for $i < n_j$ and $n_j \to \infty$ as $j \to \infty$. we know that $\cap_i X_i \neq \phi$. Also, $(\cap_i X_i) \cap K_j = \phi$ for all j, so that $(\cap_i X_i) \cap (\cup_i K_j) = \phi$ or $(\cap_i X_i) \cap X = \phi$. But $(\cap_i X_i) \subset X$, which contradicts the fact that $\cap_i X_i \neq \phi$. Therefore X strongly b-connected. □

Theorem 4.8. *Let X be a locally compact bT_2-space. If X is locally b-connected, then X is locally strongly b-connected.*

Proof. Let O be a b-open b-nbd of a point $x \in X$. Then there exists a compact b-nbd V of x lying inside O. Let C be a b-connected component of V containing x. Since V is a b-nbd of x and X is locally b-connected, C is a b-nbd of x. Since C is b-closed in V and V is compact, then C is compact. So C is a compact b-connected b-nbd of x lying inside O. By Theorem 4.7, C is strongly b-connected. □

Theorem 4.9. *Let X be a locally compact bT_2-space. If X is locally b-connected and b-connected, then X is strongly b-connected.*

Proof. This follows from Theorems 4.7 and 4.8. □

Lemma 4.10 (*15,18*). *For a space X the following statements are equivalent:*

(i) X is a bT_1-space.
(ii) For any point $x \in X$, the singleton set $\{x\}$ is b-closed.

Corollary 4.11. *A strongly b-connected b T_1-space having more than one point is uncountable.*

Proof. By Lemma 4.10, a one-point set in a b T_1-space is b-closed. Thus by Definition 4.1, a b T_1-space cannot have countably many but more than one point. □

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