Journal of the Egyptian Mathematical Society 25 (2017) 59–64

Contents lists available at ScienceDirect

Journal of the Egyptian Mathematical Society

journal homepage: www.elsevier.com/locate/joems

Original Article

Fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous functions

S.E. Abbas[a], E. El-sanowsy[b], A. Atef[c,*]

[a] Department of Mathematics, Faculty of Science, Jazan University, Saudi-Arabia
[b] Department of Mathematics, Faculty of Science, Sohag 82524, Egypt
[c] Preparatory Year Deanship, King Saud University, Saudi-Arabia

ARTICLE INFO

Article history:
Received 3 May 2016
Revised 19 June 2016
Accepted 23 June 2016
Available online 12 July 2016

MSC:
54A40
54A10
54C10

Keywords:
Fuzzy soft continuity
Fuzzy soft operator
Fuzzy soft ideal

ABSTRACT

In this paper, we introduce the concept of fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous functions. In order to unify several characterizations and properties of some kinds of modifications of fuzzy soft continuous functions and fuzzy soft open functions, we introduce and explore a generalized form of fuzzy soft continuous and fuzzy soft open functions, namely fuzzy soft $\eta\eta'$-continuous functions and fuzzy soft $\eta\eta'$-open functions.

1. Introduction and preliminaries

In 1999, Molodtsov [1] introduced the soft set theory, which is completely new approach for modeling uncertainty. He applied his concept of soft sets in several directions of applications, such that smoothness of functions, game theory, Riemann integrations and theory of probability. In 2001, Maji et al. [2,3], introduced the fuzzy soft set which is a combination of fuzzy set [4] and soft set [1] and they studied their properties. Later some researchers studied the concept of fuzzy soft sets [5–7]. Moreover, Shabir and Naz [8] presented soft topological spaces and defined some concepts based on soft sets. Tanay and Kandemir [9] initially introduced the concept of fuzzy soft topological space using fuzzy soft sets, and studied the basic notions by following Chang's fuzzy topology [10]. Pazar and Aygün [11] defined the fuzzy soft topology in sense of Lowen. Aygünoglu et al., [12] defined fuzzy soft topology in Šostak's sense [13].

In this paper, we introduce the concept of fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous functions and prove that if α, β are operators on the fuzzy soft topological space (X, τ_E) and θ, θ^* are operators on the fuzzy soft topological space (Y, τ_K^*) and $\mathcal{I}$ a fuzzy soft ideal on X, then a function $\varphi_\psi : (X, \tau_E) \to (Y, \tau_K^*)$ is fuzzy soft $(\alpha, \beta, \theta \sqcap \theta^*, \delta, \mathcal{I})$-continuous if and only if φ_ψ is both of fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous and fuzzy soft $(\alpha, \beta, \theta^*, \delta, \mathcal{I})$-continuous. Additional results on fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I}^0)$-continuous functions are given. In Section 3, we introduce new generalized notions that cover many of the generalized forms of fuzzy soft continuity and fuzzy soft open functions.

Throughout this paper, X refers to an initial universe, E is the set of all parameters for X. A fuzzy soft set f_E on X is called λ-absolute fuzzy soft set and denoted by $\tilde{E}^\lambda$, if $f_e = \underline{\lambda}$, for each $e \in E$, for $\lambda \in I$, $\underline{\lambda}(x) = \lambda$, for all $x \in X$, (where $(\tilde{E}^\lambda)^c = \tilde{E}^{1-\lambda}$, $I = [0, 1]$ and $I_0 = (0, 1]$) and $\widetilde{(X, E)}$ is the set of all fuzzy soft sets on X. Also, The concept of an operation associated with a fuzzy soft topology τ on a set X as a map $\alpha : E \times \widetilde{(X, E)} \times I_0 \to \widetilde{(X, E)}$ such that $f_A \sqsubseteq \alpha(e, f_A, r)$ for each $f_A \in \widetilde{(X, E)}, r \in I_0$ and $e \in E$ with $\tau_e(f_A) \geq r$. This type of maps is called an expansion on X. The above operators, by allowing the operator α to be defined on $\widetilde{(X, E)}$ are called fuzzy soft operators on (X, τ_E). All definitions and properties of fuzzy soft sets and fuzzy soft topology are found in [5–7,12,14]. In fact, let (X, τ_E) and (Y, τ_K^*) be two fuzzy soft topological spaces, α and β are fuzzy soft operators on (X, τ_E), θ and δ are fuzzy soft operators on (Y, τ_K^*), respectively. Recall that a fuzzy soft ideal $\mathcal{I}$ on X is a mapping $\mathcal{I} : E \to I^{\widetilde{(X,E)}}$ that satisfies the following conditions for each $e \in E$;

(1) $\mathcal{I}_e(\Phi) = 1, \mathcal{I}_e(\tilde{\mathrm{E}}) = 0$,
(2) $\mathcal{I}_e(f_A \sqcup g_B) \geq \mathcal{I}_e(f_A) \wedge \mathcal{I}_e(g_B)$, for each $f_A, g_B \in \widetilde{(X, E)}$,
(3) if $f_A \sqsubseteq g_B$, then $\mathcal{I}_e(f_A) \geq \mathcal{I}_e(g_B)$.

* Corresponding author.
E-mail address: aatef3000@gmail.com (A. Atef).

http://dx.doi.org/10.1016/j.joems.2016.06.009

Define the fuzzy soft ideal $\mathcal{I}^0$ by,

$$\mathcal{I}^0_e(f_A) = \begin{cases} 1, & if \;\; f_A = \Phi, \\ 0, & \text{otherwise}. \end{cases}$$

The difference of two fuzzy soft sets f_A and g_B, denoted by $(f_A \bar{\wedge} g_B)$ is defined as;

$$f_A \bar{\wedge} g_B = \begin{cases} \Phi, & if \;\; f_A \sqsubseteq g_B, \\ f_A \sqcap g_B^c, & \text{otherwise}. \end{cases}$$

2. Fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous functions

Definition 2.1. Let φ: $X \to Y$ and ψ: $E \to K$ be mappings. Then, the mapping $\varphi_\psi : (X, \tau_E) \to (Y, \tau^*_K)$ is called fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous if for every $g_B \in \widetilde{(Y, K)}, r \in I_0$ and $e \in E$,

$$\mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \bar{\wedge} \beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)), r)] \geq \tau^*_{\psi(e)}(g_B).$$

We can see that, the above definition is generalized of the concept of fuzzy soft continuity [12], when we choose, $\alpha =$ identity operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta =$ identity operator and $\mathcal{I} = \mathcal{I}^0$.

From the above definition we can present different cases of the fuzzy soft continuity as follow:

(1) In (2016), Abbas et al., [15] defined the concept of fuzzy soft semi-continuous mappings: For every $g_B \in \widetilde{(Y, K)}, r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then;

$$\varphi_\psi^{-1}(g_B) \sqsubseteq cl_\tau(e, int_\tau(e, \varphi_\psi^{-1}(g_B), r), r)$$

Here, $\alpha =$ identity operator, $\beta =$ closure interior operator, $\delta =$ identity operator, $\theta =$ identity operator and $\mathcal{I} = \mathcal{I}^0$.

(2) φ_ψ is fuzzy soft precontinuous mapping, iff for every $g_B \in \widetilde{(Y, K)}, r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then;

$$\varphi_\psi^{-1}(g_B) \sqsubseteq int_\tau(e, cl_\tau(e, \varphi_\psi^{-1}(g_B), r), r)$$

Here, $\alpha =$ identity operator, $\beta =$ interior closure operator, $\delta =$ identity operator, $\theta =$ identity operator and $\mathcal{I} = \mathcal{I}^0$.

(3) φ_ψ is fuzzy soft strongly semi-continuous mapping, iff for every $g_B \in \widetilde{(Y, K)}$, $r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then;

$$\varphi_\psi^{-1}(g_B) \sqsubseteq int_\tau(e, cl_\tau(e, int_\tau(e, \varphi_\psi^{-1}(g_B), r), r), r)$$

Here, $\alpha =$ identity operator, $\beta =$ interior closure interior operator, $\delta =$ identity operator, $\theta =$ identity operator and $\mathcal{I} = \mathcal{I}^0$.

(4) φ_ψ is fuzzy soft semi-precontinuous mapping, iff for every $g_B \in \widetilde{(Y, K)}, r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then;

$$\varphi_\psi^{-1}(g_B) \sqsubseteq cl_\tau(e, int_\tau(e, cl_\tau(e, \varphi_\psi^{-1}(g_B), r), r), r)$$

Here, $\alpha =$ identity operator, $\beta =$ closure interior closure operator, $\delta =$ identity operator, $\theta =$ identity operator and $\mathcal{I} = \mathcal{I}^0$.

(5) φ_ψ is fuzzy soft weakly continuous mapping, iff for every $g_B \in \widetilde{(Y, K)}, r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then;

$$\varphi_\psi^{-1}(g_B) \sqsubseteq int_\tau(e, \varphi_\psi^{-1}(cl_{\tau^*}(\psi(e), g_B, r)), r)$$

Here, $\alpha =$ identity operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta =$ closure operator and $\mathcal{I} = \mathcal{I}^0$.

(6) φ_ψ is fuzzy soft almost continuous mapping, iff for every $g_B \in \widetilde{(Y, K)}, r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then;

$$\varphi_\psi^{-1}(g_B) \sqsubseteq int_\tau(e, \varphi_\psi^{-1}(int_{\tau^*}(\psi(e), cl_{\tau^*}(\psi(e), g_B, r), r)), r)$$

Here, $\alpha =$ identity operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta =$ interior closure operator and $\mathcal{I} = \mathcal{I}^0$.

(7) φ_ψ is fuzzy soft almost weakly continuous mapping, iff for every $g_B \in \widetilde{(Y, K)}$, $r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then;

$$\varphi_\psi^{-1}(g_B) \sqsubseteq int_\tau(e, cl_\tau(e, \varphi_\psi^{-1}(cl_{\tau^*}(\psi(e), g_B, r)), r), r)$$

Here, $\alpha =$ identity operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta =$ interior closure operator and $\mathcal{I} = \mathcal{I}^0$.

(8) φ_ψ is fuzzy soft perfectly continuous mapping, iff for every $g_B \in \widetilde{(Y, K)}$, $r \in I_0$, and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then $\varphi_\psi^{-1}(g_B)$ is r-fuzzy soft clopen set. Here, $\alpha =$ closure operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta =$ identity operator and $\mathcal{I} = \mathcal{I}^0$.

(9) φ_ψ is fuzzy soft weak almost continuous mapping, iff for every $g_B \in \widetilde{(Y, K)}$, $r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then;

$$\varphi_\psi^{-1}(g_B) \sqsubseteq int_\tau(e, \varphi_\psi^{-1}(int_{\tau^*}(\psi(e), Ker_{\tau^*}(\psi(e), cl_{\tau^*}(\psi(e), g_B, r), r), r)), r)$$

Here, $\alpha =$ identity operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta =$ interior kernel closure operator and $\mathcal{I} = \mathcal{I}^0$.

(10) φ_ψ is fuzzy soft very weakly continuous mapping, iff for every $g_B \in \widetilde{(Y, K)}$, $r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$, then;

$$\varphi_\psi^{-1}(g_B) \sqsubseteq int_\tau(e, \varphi_\psi^{-1}(Ker_{\tau^*}(\psi(e), (cl_{\tau^*}(\psi(e), g_B, r), r)), r)$$

Here, $\alpha =$ identity operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta =$ kernel closure operator and $\mathcal{I} = \mathcal{I}^0$.

(11) φ_ψ is called fuzzy soft P-continuous iff $\tau_e(\varphi_\psi^{-1}(g_B)) \geq \tau^*_{\psi(e)}(g_B)$ for each $g_B \in \widetilde{(Y, K)}$, $r \in I_0$, $e \in E$ such that g_B satisfying the property P. Let $\theta_P : K \times \widetilde{(Y, K)} \times I_0 \longrightarrow \widetilde{(Y, K)}$ be an operator in (Y, τ^*_K) defined as follows: For each $g_B \in \widetilde{(Y, K)}$, $r \in I_0$, $k \in K$;

$$\theta_p(k, g_B, r) = \begin{cases} g_B \text{ if } \tau^*_k(g_B) \geq r \text{ and } g_B \text{ satisfies the property } P, \\ \tilde{K} \text{ otherwise}. \end{cases}$$

Here, $\alpha =$ identity operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta = \theta_p$ and $\mathcal{I} = \mathcal{I}^0$.

Example 2.2. Let $X = \{a, b, c\}$, $Y = \{x, y\}$, $E = \{e_1, e_2\}$, and $K = \{k_1, k_2\}$. Define $A \subseteq E, B \subseteq K, f_A = \{(e_1, \{0.5, 0.5, 0.3\}), (e_2, \{0.4, 0.4, 0.2\})\} \in \widetilde{(X, E)}$ and $g_B = \{(k_1, \{0.5, 0.3\}), (k_2, \{0.4, 0.2\})\} \in \widetilde{(Y, K)}$. Define fuzzy soft topologies $\tau_E : E \to I^{\widetilde{(X,E)}}$ and $\tau^*_K : K \to I^{\widetilde{(Y,K)}}$ as follow:

$$\tau_e(h_G) = \begin{cases} 1, & \text{if } \;\; h_G = \Phi, \; \tilde{E}, \\ \frac{1}{2}, & \text{if } \;\; h_G = f_A, \tilde{E}^{0.4} \\ \frac{1}{2}, & \text{if } \;\; h_G = f_A \sqcap \tilde{E}^{0.4}, \\ \frac{2}{3}, & \text{if } \;\; h_G = f_A \sqcup \tilde{E}^{0.4}, \\ 0, & \text{otherwise}, \end{cases}$$

$$\tau^*_k(w_D) = \begin{cases} 1, & \text{if } \;\; w_D = \Phi, \tilde{K}, \\ \frac{1}{2}, & \text{if } \;\; w_D = g_B, \\ 0, & \text{otherwise}, \end{cases}$$

Consider the maps φ: $X \to Y$ and ψ: $E \to K$ defined by $\varphi(a) = \varphi(b) = x$, $\varphi(c) = y$, $\psi(e_1) = k_1$ and $\psi(e_2) = k_2$. Therefore, for each $e \in E, k \in K$ and $r \in I_\circ$, define the fuzzy soft operators $\alpha, \beta : E \times \widetilde{(X, E)} \times I_0 \to \widetilde{(X, E)}$ and $\theta, \delta : K \times \widetilde{(Y, K)} \times I_0 \to \widetilde{(Y, K)}$, as follow:

$$\alpha(e, w_D, r) = w_D, \;\; \theta(k, w_D, r) = \delta(k, w_D, r) = w_D$$

and

$$\beta(e,w_D,r) = \begin{cases} \tilde{E}, & \text{if } w_D=\tilde{E}, \quad \forall r\in I_\circ,\\ f_A, & \text{if } (f_A)_1\sqsubseteq w_D\sqsubset (f_A\sqcup \tilde{E}^{0.4}), \quad 0<r\le \frac{1}{2},\\ \tilde{E}^{0.4}, & \text{if } \tilde{E}^{0.4}\sqsubseteq w_D\sqsubset (f_A\sqcup \tilde{E}^{0.4}),\\ & w_D\not\sqsupseteq f_A \quad 0<r\le \frac{1}{2},\\ f_A\sqcup \tilde{E}^{0.4}, & \text{if } f_A\sqcup \tilde{E}^{0.4}\sqsubseteq w_D\ne \tilde{E}, \quad 0<r\le \frac{2}{3},\\ f_A\sqcap \tilde{E}^{0.4}, & \text{if } f_A\sqcap \tilde{E}^{0.4}\sqsubseteq w_D\sqsubset f_A\sqcup \tilde{E}^{0.4},\\ & f_A\not\sqsubseteq w_D, \tilde{E}^{0.4}\not\sqsubseteq w_D \quad 0<r\le \frac{1}{2},\\ \Phi, & \text{otherwise,}\end{cases}$$

Then the map $\varphi_\psi : (X,\tau_E)\to (Y,\tau^*_K)$ is fuzzy soft continuous.

Remark 2.3. If $\mathcal{I}^*$ is a fuzzy soft ideal on Y, then the mapping $\varphi_\psi : (X,\tau_E)\to (Y,\tau^*_K)$ is said to be fuzzy soft $(\alpha,\beta,\theta,\delta,\mathcal{I}^*)$-open if for every $f_A\in \widetilde{(X,E)}$, $r\in I_0$ and $e\in E$,

$$\mathcal{I}^*_{\psi(e)}[\alpha(\psi(e),\varphi_\psi(\delta(e,f_A,r)),r)\barwedge \beta(\psi(e),\varphi_\psi(\theta(e,f_A,r)),r)] \ge \tau_e(f_A).$$

We can see that the fuzzy soft $(\alpha,\beta,\theta,\delta,\mathcal{I}^*)$-open mapping is generalized of the concept of fuzzy soft open [12], when we choose, $\alpha=$ identity operator, $\beta=$ interior operator, $\delta=$ identity operator, $\theta=$ identity operator and $\mathcal{I}^*=\mathcal{I}^{*0}$. Also, from Remark 2.3 we can present some types of fuzzy soft open functions as follow:

(1) In (2016), Abbas et al., [15] defined the concept of fuzzy soft semi-open mappings: for every $f_A\in \widetilde{(X,E)}$, $r\in I_0$ and $e\in E$ with $\tau_e(f_A)\ge r$, then;

$$\varphi_\psi(f_A)\sqsubseteq cl_{\tau^*}(\psi(e), int_{\tau^*}(\psi(e),\varphi_\psi(f_A),r),r)$$

Here, $\alpha=$ identity operator, $\beta=$ closure interior operator, $\delta=$ identity operator, $\theta=$ identity operator and $\mathcal{I}^*=\mathcal{I}^{*0}$.

(2) φ_ψ is fuzzy soft pre-open mapping, iff for every $f_A\in \widetilde{(X,E)}$, $r\in I_0$ and $e\in E$ with $\tau_e(f_A)\ge r$, then;

$$\varphi_\psi(f_A)\sqsubseteq int_{\tau^*}(\psi(e), cl_{\tau^*}(\psi(e),\varphi_\psi(f_A),r),r)$$

Here, $\alpha=$ identity operator, $\beta=$ interior closure operator, $\delta=$ identity operator, $\theta=$ identity operator and $\mathcal{I}^*=\mathcal{I}^{*0}$.

(3) φ_ψ is fuzzy soft strongly semi-open mapping, iff for every $f_A\in \widetilde{(X,E)}$, $r\in I_0$ and $e\in E$ with $\tau_e(f_A)\ge r$, then;

$$\varphi_\psi(f_A)\sqsubseteq int_{\tau^*}(\psi(e), cl_{\tau^*}(\psi(e), int_{\tau^*}(\psi(e),\varphi_\psi(f_A),r),r),r)$$

Here, $\alpha=$ identity operator, $\beta=$ interior closure interior operator, $\delta=$ identity operator, $\theta=$ identity operator and $\mathcal{I}^*=\mathcal{I}^{*0}$.

(4) φ_ψ is fuzzy soft semi-preopen mapping, iff for every $f_A\in \widetilde{(X,E)}$, $r\in I_0$ and $e\in E$ with $\tau_e(f_A)\ge r$, then;

$$\varphi_\psi(f_A)\sqsubseteq cl_{\tau^*}(\psi(e), int_{\tau^*}(\psi(e), cl_{\tau^*}(\psi(e),\varphi_\psi(f_A),r),r),r)$$

Here, $\alpha=$ identity operator, $\beta=$ closure interior closure operator, $\delta=$ identity operator, $\theta=$ identity operator and $\mathcal{I}^*=\mathcal{I}^{*0}$.

(5) φ_ψ is fuzzy soft weakly open mapping, iff for every $f_A\in \widetilde{(X,E)}$, $r\in I_0$ and $e\in E$ with $\tau_e(f_A)\ge r$, then;

$$\varphi_\psi(f_A)\sqsubseteq int_{\tau^*}(\psi(e),\varphi_\psi(cl_\tau(e,f_A,r)),r)$$

Here, $\alpha=$ identity operator, $\beta=$ interior operator, $\delta=$ identity operator, $\theta=$ closure operator and $\mathcal{I}^*=\mathcal{I}^{*0}$.

(6) φ_ψ is fuzzy soft almost open mapping, iff for every $f_A\in \widetilde{(X,E)}$, $r\in I_0$ and $e\in E$ with $\tau_e(f_A)\ge r$, then;

$$\varphi_\psi(f_A)\sqsubseteq int_{\tau^*}(\psi(e),\varphi_\psi(int_\tau(e, cl_\tau(e,f_A,r),r)),r)$$

Here, $\alpha=$ identity operator, $\beta=$ interior operator, $\delta=$ identity operator, $\theta=$ interior closure operator and $\mathcal{I}^*=\mathcal{I}^{*0}$.

Example 2.4. Let $X=\{a,b,c\}$, $Y=\{x,y,z\}$, $E=\{e_1,e_2\}$, and $K=\{k_1,k_2\}$. Define fuzzy soft topologies $\tau_E:E\to I^{\widetilde{(X,E)}}$ and $\tau^*_K:K\to I^{\widetilde{(Y,K)}}$ as follows:

$$\tau_e(h_G)=\begin{cases}1, & \text{if } h_G=\Phi,\ \tilde{E},\\ \frac{1}{2}, & \text{if } h_G=\tilde{E}^{0.3}\\ 0, & \text{otherwise,}\end{cases} \qquad \tau^*_k(w_D)=\begin{cases}1, & \text{if } w_D=\Phi,\tilde{K},\\ \frac{1}{2}, & \text{if } w_D=\tilde{K}^{0.3},\\ \frac{2}{3}, & \text{if } w_D=\tilde{K}^{0.6},\\ 0, & \text{otherwise,}\end{cases}$$

Consider the maps $\varphi: X\to Y$ and $\psi: E\to K$ defined by $\varphi(a)=x$, $\varphi(b)=y$, $\varphi(c)=z$, $\psi(e_i)=k_i$, $i\in\{1,2,3\}$. Therefore, for each $e\in E$, $k\in K$ and $r\in I_\circ$, define the fuzzy soft operators α, β,θ and δ, as follow:

$\alpha(k,w_D,r)=w_D$, $\theta(e,w_D,r)=\delta(e,w_D,r)=w_D$ and

$$\beta(k,w_D,r)=\begin{cases}\tilde{K}, & \text{if } w_D=\tilde{K}, \quad \forall r\in I_\circ,\\ \tilde{K}^{0.3}, & \text{if } \tilde{K}^{0.3}\sqsubseteq w_D\sqsubset \tilde{K}^{0.6}, \quad 0<r\le\frac{1}{2},\\ \tilde{K}^{0.6}, & \text{if } \tilde{K}^{0.6}\sqsubseteq w_D\sqsubset \tilde{K}, \quad 0<r\le \frac{2}{3},\\ \Phi, & \text{otherwise.}\end{cases}$$

Then the map $\varphi_\psi : (X,\tau_E)\to (Y,\tau^*_K)$, is fuzzy soft open.

Definition 2.5. If β and β^* are fuzzy soft operators on X, then the operator $\beta\sqcap\beta^*$ is defined by, $(\beta\sqcap\beta^*)(e,f_A,r)=\beta(e,f_A,r)\sqcap\beta^*(e,f_A,r)$ for each $f_A\in\widetilde{(X,E)}$, $e\in E$ and $r\in I_0$. The fuzzy soft operators β and β^* are said to be mutually dual if $\beta\sqcap\beta^*$ is the identity operator.

Theorem 2.6. *Let $(X,\ \tau_E)$ and (Y,τ^*_K) be two fuzzy soft topological spaces and $\mathcal{I}$ be a fuzzy soft ideal on X. Let α, β and β^* be fuzzy soft operators on X and δ, θ and θ^* be fuzzy soft operators on Y. Then $\varphi_\psi : \widetilde{(X,E)}\longrightarrow \widetilde{(Y,K)}$ is:*

(1) fuzzy soft $(\alpha,\beta,(\theta\sqcap\theta^*),\delta,\mathcal{I})$-continuous if and only if it is both fuzzy soft $(\alpha,\ \beta,\ \theta,\ \delta,\ \mathcal{I})$-continuous and fuzzy soft $(\alpha,\beta,\theta^*,\delta,\mathcal{I})$-continuous, provided that $\beta(e,f_A\sqcap g_B,r)=\beta(e,f_A,r)\sqcap\beta(e,g_B,r)$, for each $f_A,g_B\in\widetilde{(X,E)}$, $e\in E$ and $r\in I_0$.

(2) fuzzy soft $(\alpha,(\beta\sqcap\beta^*),\theta,\delta,\mathcal{I})$-continuous, if and only if it is both fuzzy soft $(\alpha,\ \beta,\ \theta,\ \delta,\ \mathcal{I})$-continuous and fuzzy soft $(\alpha,\beta^*,\theta,\delta,\mathcal{I})$-continuous.

Proof. (1) If φ_ψ is both fuzzy soft $(\alpha,\beta,\theta,\delta,\mathcal{I})$-continuous and fuzzy soft $(\alpha,\beta,\theta^*,\delta,\mathcal{I})$-continuous then for each $g_B\in\widetilde{(Y,K)}$, $e\in E$ and $r\in I_0$ we have that, $\mathcal{I}_e[\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\barwedge\beta(e,\varphi^{-1}_\psi(\theta(\psi(e),g_B,r)),r)]\ge\tau^*_{\psi(e)}(g_B)$ and $\mathcal{I}_e[\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\barwedge\beta(e,\varphi^{-1}_\psi(\theta^*(\psi(e),g_B,r)),r)]\ge\tau^*_{\psi(e)}(g_B)$, then $\mathcal{I}_e[(\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\barwedge\beta(e,\varphi^{-1}_\psi(\theta(\psi(e),g_B,r)),r))\sqcup(\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\barwedge\beta(e,\varphi^{-1}_\psi(\theta^*(\psi(e),g_B,r)),r))]\ge\tau^*_{\psi(e)}(g_B)$. But, $(\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\barwedge\beta(e,\varphi^{-1}_\psi(\theta(\psi(e),g_B,r)),r))\sqcup(\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\barwedge\beta(e,\varphi^{-1}_\psi(\theta^*(\psi(e),g_B,r)),r))$

$$\begin{cases} =\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\barwedge(\beta(e,\varphi^{-1}_\psi(\theta(\psi(e),g_B,r)),r)\\ \quad\sqcap\beta(e,\varphi^{-1}_\psi(\theta^*(\psi(e),g_B,r)),r))\\ =\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\barwedge\beta(e,\varphi^{-1}_\psi(\theta(\psi(e),g_B,r)\\ \quad\sqcap\theta^*(\psi(e),g_B,r)))\\ =\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\\ \quad\barwedge\beta(e,\varphi^{-1}_\psi((\theta\sqcap\theta^*)(\psi(e),g_B,r))).\end{cases}$$

That is $\mathcal{I}_e[\alpha(e,\varphi^{-1}_\psi(\delta(\psi(e),g_B,r)),r)\barwedge\beta(e,\varphi^{-1}_\psi((\theta\sqcap\theta^*)(\psi(e),g_B,r)))]\ge\tau^*_{\psi(e)}(g_B)$. Hence, φ_ψ is fuzzy soft $(\alpha,\beta,(\theta\sqcap\theta^*),\delta,\mathcal{I})$-continuous.

Conversely, if φ_ψ is fuzzy soft $(\alpha,\beta,(\theta\sqcap\theta^*),\delta,\mathcal{I})$-continuous, then for each $g_B\in\widetilde{(Y,K)}$, $r\in I_0$ and $e\in$

E, we have $\mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}((\theta \sqcap \theta^*)(\psi(e), g_B, r)))] \geq \tau^*_{\psi(e)}(g_B)$. Now, by the above equalities, we get that, $\mathcal{I}_e[(\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)), r)) \sqcup (\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta^*(\psi(e), g_B, r)), r))] \geq \tau^*_{\psi(e)}(g_B)$ which implies that $\mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)), r)] \geq \tau^*_{\psi(e)}(g_B)$ and so, $\mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta^*(\psi(e), g_B, r)), r)] \geq \tau^*_{\psi(e)}(g_B)$, which means that φ_ψ is both fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous and fuzzy soft $(\alpha, \beta, \theta^*, \delta, \mathcal{I})$-continuous.

(2) Similar to the proof in (1). □

Definition 2.7. Let (X, τ_E) and (Y, τ^*_K) be two fuzzy soft topological spaces and $\mathcal{A}$ be an expansion on Y. Then a mapping $\varphi_\psi : \widetilde{(X, E)} \longrightarrow \widetilde{(Y, K)}$ is said to be fuzzy soft $\mathcal{A}$-continuous expansion if $\varphi_\psi^{-1}(g_B) \sqsubseteq int_\tau(e, \varphi_\psi^{-1}(\mathcal{A}(g_B)), r)$ for each $g_B \in \widetilde{(Y, K)}$, $r \in I_0$ and $e \in E$ with $\tau^*_{\psi(e)}(g_B) \geq r$.

Corollary 2.8. *Let (X, τ_E) and (Y, τ^*_K) be two fuzzy soft topological spaces and $\mathcal{A}$ and $\mathcal{B}$ are two mutually dual expansions on Y. Then a mapping $\varphi_\psi : \widetilde{(X, E)} \longrightarrow \widetilde{(Y, K)}$ is a fuzzy soft continuous if and only if φ_ψ is fuzzy soft $\mathcal{A}$-continuous expansion and fuzzy soft $\mathcal{B}$-continuous expansion.*

Proof. Take $\alpha = \delta =$ identity operator, $\beta =$ interior operator, $\theta = \mathcal{A}$, $\theta^* = \mathcal{B}$ and $\mathcal{I} = \mathcal{I}^0$. Then the result is fulfilled directly from Theorem 2.6(1). □

Corollary 2.9. *Let (X, τ_E) and (Y, τ^*_K) be two fuzzy soft topological spaces. A mapping $\varphi_\psi : \widetilde{(X, E)} \longrightarrow \widetilde{(Y, K)}$ is fuzzy soft continuous if and only if φ_ψ is fuzzy soft almost continuous and fuzzy soft $(id_X, int_\tau, \gamma, id_Y, \mathcal{I}^0)$-continuous, where the operators γ and $int_{\tau^*}(cl_{\tau^*})$ are mutually dual operators on Y such that; $(\gamma g_B) = g_B \sqcup (int_{\tau^*}(\psi(e), cl_{\tau^*}(\psi(e), g_B, r), r))^c$, $\forall\, g_B \in \widetilde{(Y, K)}, e \in E$ and $r \in I_0$.*

Proof.. Fuzzy soft almost continuous equivalents a fuzzy soft $(id_X, int_\tau, int_{\tau^*}(cl_{\tau^*}), id_Y, \mathcal{I}^0)$-continuous. But the operators γ and $int_{\tau^*}(cl_{\tau^*})$ are mutually dual operators on Y. Hence, from Theorem 2.6, we get the required proof. □

Let Ψ be the set of all fuzzy soft operators on X and $\alpha, \beta \in \Psi$. Then a partial order relation could be given as; $\alpha \leq \beta$ if and only if $\alpha(e, f_A, r) \sqsubseteq \beta(e, f_A, r)$. Also, an operator α on X is called monotone if $f_A \sqsubseteq g_B$ for each $f_A, g_B \in \widetilde{(X, E)}$, $e \in E$ and $r \in I_0$ then, $\alpha(e, f_A, r) \sqsubseteq \alpha(e, g_B, r)$,

Theorem 2.10. *Let (X, τ_E) and (Y, τ^*_K) be two fuzzy soft topological spaces, $\mathcal{I}$ be a fuzzy soft ideal on X and let α, α^*, β and β^* be fuzzy soft operators on X, θ, θ^* and δ be fuzzy soft operators on Y and $\varphi_\psi : \widetilde{(X, E)} \longrightarrow \widetilde{(Y, K)}$ be a function. Then:*

(1) *If β is a monotone, $\theta \leq \theta^*$ and φ_ψ is a fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous function, then φ_ψ is fuzzy soft $(\alpha, \beta, \theta^*, \delta, \mathcal{I})$-continuous.*
(2) *If $\alpha^* \leq \alpha$ and φ_ψ is a fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous function, then φ_ψ is fuzzy soft $(\alpha^*, \beta, \theta, \delta, \mathcal{I})$-continuous.*
(3) *If $\beta \leq \beta^*$ and φ_ψ is a fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous function, then φ_ψ is fuzzy soft $(\alpha, \beta^*, \theta, \delta, \mathcal{I})$-continuous.*

Proof. (1) Let φ_ψ be fuzzy soft$(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous, then for each $g_B \in \widetilde{(Y, K)}$, $e \in E$ and $r \in I_0$, we have that $\mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)), r)] \geq \tau^*_{\psi(e)}(g_B)$. Now we know that $\theta \leq \theta^*$, then $\theta(\psi(e), g_B, r) \sqsubseteq \theta^*(\psi(e), g_B, r)$, thus $\varphi_\psi^{-1}(\theta(\psi(e), g_B, r)) \sqsubseteq \varphi_\psi^{-1}(\theta^*(\psi(e), g_B, r))$ and $\beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)), r) \sqsubseteq \beta(e, \varphi_\psi^{-1}(\theta^*(\psi(e), g_B, r)), r)$, then $[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta^*(\psi(e), g_B, r)), r)] \sqsubseteq [\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)).r)]$ Therefore, $\mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta^*(\psi(e), g_B, r)), r)] \geq \mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)), r)]$ which means that, $\mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta^*(\psi(e), g_B, r)), r)] \geq \tau^*_{\psi(e)}(g_B)$. Hence, φ_ψ is fuzzy soft $(\alpha, \beta, \theta^*, \delta, \mathcal{I})$-continuous.

(2) and (3) Similarly. □

Definition 2.11. An operator β on the fuzzy soft topological space (X, τ_E) induces another fuzzy soft operator $(int_\tau \beta)$ defined as follows;

$(int_\tau \beta)(e, f_A, r) = int_\tau(e, \beta(e, f_A, r), r)$. Note that, $int_\tau\, \beta \leq \beta$.

Definition 2.12. A function $\varphi_\psi : (X, \tau_E) \to (Y, \tau^*_K)$ satisfies the fuzzy soft openness condition with respect to the fuzzy soft operator β on X if for every $g_B \in \widetilde{(Y, K)}$, $e \in E$ $r \in I_0$, we get that:

$$\beta(e, \varphi_\psi^{-1}(g_B), r) \sqsubseteq \beta(e, \varphi_\psi^{-1}(int_{\tau^*}(\psi(e), g_B, r)), r).$$

Whenever $\beta = int_\tau$, then the definition will be equivalent to that usual one of fuzzy soft open mapping.

Theorem 2.13. *Let (X, τ_E) and (Y, τ^*_K) be two fuzzy soft topological spaces. If $\varphi_\psi : \widetilde{(X, E)} \longrightarrow \widetilde{(Y, K)}$ is fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous and satisfies the openness condition with respect to the fuzzy soft operator β, then φ_ψ is fuzzy soft$(\alpha, \beta, (int_{\tau^*}\theta), \delta, \mathcal{I})$-continuous.*

Proof. Let $g_B \in \widetilde{(Y, K)}$, $e \in E$ and $r \in I_0$. Since φ_ψ is fuzzy soft $(\alpha, \beta, \theta, \delta, \mathcal{I})$-continuous, we have, $\mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)), r)] \geq \tau^*_{\psi(e)}(g_B)$. But φ_ψ satisfies the openness condition with respect to the fuzzy soft operator β, then $\beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)), r) \sqsubseteq \beta(e, \varphi_\psi^{-1}((int_{\tau^*}\theta)(\psi(e), g_B, r)), r)$. Hence, $\mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B, r)), r) \barwedge \beta(e, \varphi_\psi^{-1}((int_{\tau^*}\theta)(\psi(e), g_B, r)), r)] \geq \mathcal{I}_e[\alpha(e, \varphi_\psi^{-1}(\delta(\psi(e), g_B)), r) \barwedge \beta(e, \varphi_\psi^{-1}(\theta(\psi(e), g_B, r)), r)] \geq \tau^*_{\psi(e)}(g_B)$. Thus, φ_ψ is fuzzy soft$(\alpha, \beta, (int_{\tau^*}\theta), \delta, \mathcal{I})$ -continuous. □

Corollary 2.14. *Let (X, τ_E) and (Y, τ^*_K) be two fuzzy soft topological spaces. If $\varphi_\psi : \widetilde{(X, E)} \longrightarrow \widetilde{(Y, K)}$ is fuzzy soft weakly continuous and fuzzy soft open mapping, then φ_ψ is a fuzzy soft almost continuous.*

Proof. Let $\alpha =$ identity operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta =$ closure operator and $\mathcal{I} = \mathcal{I}^0$. Since φ_ψ satisfies the openness condition, by Theorem 2.11, we have φ_ψ is fuzzy soft $(\alpha, \beta, (int_{\tau^*}\theta), \delta, \mathcal{I})$-continuous, then φ_ψ is fuzzy soft $(id_X, int_\tau, (int_{\tau^*} cl_{\tau^*}), id_Y, \mathcal{I}^0)$-continuous. Hence, φ_ψ is fuzzy soft almost continuous. □

Corollary 2.15. *Let (X, τ_E) and (Y, τ^*_K) be two fuzzy soft topological spaces. If $\varphi_\psi : \widetilde{(X, E)} \longrightarrow \widetilde{(Y, K)}$ is fuzzy soft very weakly continuous and fuzzy soft open mapping, then φ_ψ is a fuzzy soft weak almost continuous.*

Proof. Let $\alpha =$ identity operator, $\beta =$ interior operator, $\delta =$ identity operator, $\theta =$ Kernel closure operator and $\mathcal{I} = \mathcal{I}^0$. Then the proof of this result comes easily from Theorem 2.13, and as similar to Corollary 2.14. □

Definition 2.16. Let (X, τ_E) be a fuzzy soft topological space. Then X is called fuzzy soft θ-compact space if for each family $\{(f_A)_i \in \widetilde{(X, E)} \mid \tau_e((f_A)_i) \geq r,\ r \in I_0,\ i \in \Gamma,\ e \in$

$E\}$ with $\bigsqcup_{i\in\Gamma}(f_A)_i = \tilde{E}$, there exists a finite subset $\Gamma_\circ$ of Γ such that $\bigsqcup_{i\in\Gamma_\circ}\theta(e,(f_A)_i,r) = \tilde{E}$.

(1) If $\theta =$ closure operator we get the fuzzy soft almost compact space.
(2) If $\theta =$ interior closure operator we get the fuzzy soft nearly compact space.

Theorem 2.17. *Let (X,τ_E) and (Y,τ^*_K) be two fuzzy soft topological spaces, α be an operator on (X,τ_E), θ and δ be operators on (Y,τ^*_K) and $f_A \sqsubseteq \alpha(e,f_A,r)$, for each $f_A \in \widetilde{(X,E)}$ If $\varphi_\psi : \widetilde{(X,E)} \to \widetilde{(Y,K)}$ is fuzzy soft $(\alpha, int_\tau, \theta, \delta, \mathcal{I}^0)$-continuous and X is fuzzy soft compact, then Y is fuzzy soft θ-compact.*

Proof. For each family $\{(g_B)_i \in \widetilde{(Y,K)} : \tau^*_{\psi(e)}((g_B)_i) \geq r,\ r\in I_0,\ i\in\Gamma,\ e\in E\}$ with $\bigsqcup_{i\in\Gamma}(g_B)_i = \tilde{K}$. By fuzzy soft $(\alpha, int_\tau, \theta, \delta, \mathcal{I}^0)$-continuous, for each $i\in\Gamma$ there exists $(h_C)_i \in \widetilde{(X,E)}$ with $\tau_e((h_C)_i) \geq r$ such that $\alpha(e, \varphi^{-1}_\psi(\delta(\psi(e),(g_B)_i,r)),r) \sqsubseteq (h_C)_i \sqsubseteq \varphi^{-1}_\psi(\theta(\psi(e),(g_B)_i,r))$. Also, since $\varphi^{-1}_\psi(\delta(\psi(e),(g_B)_i,r)) \sqsubseteq \alpha(e,\varphi^{-1}_\psi(\theta(\psi(e),(g_B)_i,r)),r)$ for every $i\in\Gamma$, we have that $\bigsqcup_{i\in\Gamma}(h_C)_i \sqsubseteq \bigsqcup_{i\in\Gamma}\varphi^{-1}_\psi(\theta(\psi(e),(g_B)_i,r)) \sqsubseteq \varphi^{-1}_\psi(\bigsqcup_{i\in\Gamma}\theta(\psi(e),(g_B)_i,r))$ and $\bigsqcup_{i\in\Gamma}(h_C)_i = \tilde{E}$ By fuzzy soft compactness of X there exists a finite subset $\Gamma_\circ$ of Γ with $\bigsqcup_{i\in\Gamma_\circ}(h_C)_i = \tilde{E}$. Then $\bigsqcup_{i\in\Gamma_\circ}\varphi_\psi((h_C)_i) = \tilde{K}$. Thus, $\bigsqcup_{i\in\Gamma_\circ}\theta(\psi(e),(g_B)_i,r) = \tilde{K}$ which means that Y is fuzzy soft θ-compact. □

Corollary 2.18. *Let (X,τ_E) and (Y,τ^*_K) be two fuzzy soft topological spaces, If $\varphi_\psi : \widetilde{(X,E)} \to \widetilde{(Y,K)}$ is a fuzzy soft weakly continuous map and X is fuzzy soft compact, then Y is fuzzy soft almost compact space.*

Proof. Let $\alpha =$ identity operator on X, $\beta =$ interior operator, $\theta =$ closure operator on Y, $\delta =$ identity operator and $\mathcal{I} = \mathcal{I}^0$. □

Corollary 2.19. *Let (X,τ_E) and (Y,τ^*_K) be two fuzzy soft topological spaces, If $\varphi_\psi : \widetilde{(X,E)} \to \widetilde{(Y,K)}$ is a fuzzy soft almost continuous map and X is fuzzy soft compact, then Y is fuzzy soft nearly compact space.*

Proof. Let $\alpha =$ identity operator on X, $\beta =$ interior operator, $\theta =$ interior closure operator on Y, $\delta =$ identity operator and $\mathcal{I} = \mathcal{I}^0$. □

3. Fuzzy soft $\eta\eta'$-continuous functions

Let X and Y be nonempty sets, E and K be parameters sets for X and Y respectively, $\eta : E \longrightarrow I^{\widetilde{(X,E)}}$ and $\eta' : K \longrightarrow I^{\widetilde{(Y,K)}}$.

Definition 3.1. A function $\varphi_\psi : \widetilde{(X,E)} \longrightarrow \widetilde{(Y,K)}$ is said to be:

(1) fuzzy soft $\eta\eta'$-continuous if $\eta_e(\varphi^{-1}_\psi(g_B)) \geq \eta'_{\psi(e)}(g_B)$, $\forall g_B \in \widetilde{(Y,K)}$.

(2) fuzzy soft $\eta\eta'$-open if $\eta'_{\psi(e)}(\varphi_\psi(f_A)) \geq \eta_e(f_A)$, $\forall\ f_A \in \widetilde{(X,E)}$.

Definition 3.2. A mapping $\tau : E \to I^{\widetilde{(X,E)}}$ is called a supra fuzzy soft topology on X if it satisfies the following conditions for each $e\in E$;

(S1) $\tau_e(\Phi) = \tau_e(\tilde{E}) = 1$,

(S2) $\tau_e(\bigsqcup_{i\in\Gamma}(f_A)_i) \geq \bigwedge_{i\in\Gamma}\tau_e((f_A)_i)$, for all $(f_A)_i \in \widetilde{(X,E)}$, $i\in\Gamma$.

Definition 3.3. A mapping $m_X : E \to I^{\widetilde{(X,E)}}$ is said to have a fuzzy soft minimal structure on X if $(m_X)_e(\Phi) = (m_X)_e(\tilde{E}) = 1$. And m_X is said to have the property U if for $(m_X)_e((f_A)_j) \geq r, r\in I_0, j\in J$;

$$(m_X)_e(\bigsqcup_{j\in J}(f_A)_j) \geq \bigwedge_{j\in J}(m_X)_e((f_A)_j).$$

Observe that if in Definition 3.1, η and η' are exactly the supra fuzzy soft topologies on X and Y, respectively, then we obtain the notions of supra fuzzy soft $\eta\eta'$-continuous function and supra fuzzy soft $\eta\eta'$-open function.

By the notion of fuzzy soft minimal structures, if in Definition 3.1, $\eta = m_X$ and $\eta' = m_Y$ are fuzzy soft minimal structures on X and Y, respectively, then we obtain the notion of fuzzy soft (m_X, m_Y)-continuous function and fuzzy soft (m_X, m_Y)-open function. For $\eta : E \longrightarrow I^{\widetilde{(X,E)}}$, determine in a natural form an operator $\theta_\eta : E \times \widetilde{(X,E)} \times I_0 \longrightarrow \widetilde{(X,E)}$, $e\in E$, $r\in I_0$ and $f_A \in \widetilde{(X,E)}$:

$$\theta_\eta(e,f_A,r) = \begin{cases} f_A & \text{if } \eta(f_A) \geq r, \\ \tilde{E} & \text{in other case.} \end{cases}$$

In the case that η is a supra fuzzy soft topology on X we obtain other operations that are important for their applications:

$$C_\eta(e,f_A,r) = \sqcap\{\, g_B \mid f_A \sqsubseteq g_B, \quad \eta_e(g^c_B) \geq r\}$$

$$I_\eta(e,f_A,r) = \sqcup\{\, g_B \mid g_B \sqsubseteq f_A, \quad \eta_e(g_B) \geq r\}$$

Note that, usually, $I_\eta \sqsubseteq id_X \sqsubseteq \theta_\eta$. Similarly, in the case of a fuzzy soft minimal structure m_X on X, we have;

$$cl_{m_X}(e,f_A,r) = \sqcap\{\, g_B \mid f_A \sqsubseteq g_B, \quad (m_X)_e(g^c_B) \geq r\}$$

$$int_{m_X}(e,f_A,r) = \sqcup\{\, g_B \mid g_B \sqsubseteq f_A, \quad (m_X)_e(g_B) \geq r\}$$

Note that, $int_{m_X} \sqsubseteq id_X \sqsubseteq \theta_\eta$. Also, $int_{m_X}(e,f_A,r) = f_A$ if $(m_X)_e(f_A) \geq r$, while $(m_X)_e(int_{m_X}(e,f_A,r)) \geq r$, whenever m_X is a fuzzy soft minimal structure with the property U.

The following results give the relationship between fuzzy soft $\eta\eta'$-continuity and fuzzy soft $(\alpha,\beta,\theta,\delta,\mathcal{I})$-continuity.

Theorem 3.4. *Let $\varphi: X \to Y$, $\psi: E \to K$, $\eta : E \to I^{\widetilde{(X,E)}}$ with $\eta_e(\tilde{E}) = 1$ and $\eta' : K \to I^{\widetilde{(Y,K)}}$ be functions. Then $\varphi_\psi : \widetilde{(X,E)} \longrightarrow \widetilde{(Y,K)}$ is a fuzzy soft $\eta\eta'$-continuous if and only if φ_ψ is fuzzy soft $(\theta_\eta, id_X, \theta_{\eta'}, id_Y, \mathcal{I}^0)$-continuous.*

Proof. Sufficiency. Suppose that $\varphi_\psi : \widetilde{(X,E)} \to \widetilde{(Y,K)}$ is a fuzzy soft $\eta\eta'$-continuous. Let $g_B \in \widetilde{(Y,K)}$, $r\in I_0$ we have two cases: **(Case 1).** If $\eta'_{\psi(e)}(g_B) \geq r$ then $\theta_{\eta'}(\psi(e),g_B,r) = g_B$ and $\theta_\eta(e,\varphi^{-1}_\psi(g_B),r) = \varphi^{-1}_\psi(g_B)$. Hence, $\theta_\eta(e,\varphi^{-1}_\psi(id_Y(\psi(e),g_B,r)),r) = \varphi^{-1}_\psi(g_B) = id_X(e,\varphi^{-1}_\psi\ (\theta_{\eta'}(\psi(e),g_B,\ r)),\ r)$. Consequently, $\theta_\eta(e,\varphi^{-1}_\psi(id_Y(\psi(e),g_B,r)),r) \sqsubseteq id_X(e,\varphi^{-1}_\psi(\theta_{\eta'}(\psi(e),g_B,r)),r)$.

(Case 2). If $\eta'_{\psi(e)}(g_B) \not\geq r$ then we have that, $\theta_{\eta'}(\psi(e),g_B,r) = \tilde{K}$ and $\theta_\eta(e,\varphi^{-1}_\psi(id_Y(\psi(e),g_B,r)\quad),r) \sqsubseteq \tilde{E} = \varphi^{-1}_\psi(\tilde{K}) = id_X(e,\varphi^{-1}_\psi(\theta_{\eta'}(\psi(e),g_B,r)),r)$. So, $\theta_\eta(e,\varphi^{-1}_\psi(id_Y(\psi(e),g_B,r)),r) \barwedge id_X(e,\varphi^{-1}_\psi(\theta_{\eta'}(\psi(e),g_B,r)),r) = \Phi$ for all $g_B \in \widetilde{(Y,K)}$. Thus φ_ψ is fuzzy soft $(\theta_\eta, id_X, \theta_{\eta'}, id_Y, \mathcal{I}^0)$-continuous.

Necessity. Suppose that $\eta_e(\varphi^{-1}_\psi(g_B)) \not\geq \eta'_{\psi(e)}(g_B)$; $\forall\, g_B \in \widetilde{(Y,K)}$, $e\in E$ then there exists $r\in I_0$ such that, $\eta_e(\varphi^{-1}_\psi(g_B)) < r \leq \eta'_{\psi(e)}(g_B)$. Since φ_ψ is a fuzzy soft $(\theta_\eta, id_X, \theta_{\eta'}, id_Y, \mathcal{I}^0)$-continuous, that is, $\mathcal{I}^0_e[\theta_\eta(e,\varphi^{-1}_\psi(id_Y(\psi(e),g_B,r)),r) \barwedge id_X(e,\varphi^{-1}_\psi(\theta_{\eta'}(\psi(e),g_B,r)),r)] \geq \eta'_{\psi(e)}(g_B)$. Then we have that;

$\theta_\eta(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) \barwedge id_X(e, \varphi_\psi^{-1}(\theta_{\eta'}(\psi(e), g_B, r)), r) = \Phi$, which means that, $\theta_\eta(e, \varphi_\psi^{-1}(g_B), r) \sqsubseteq \varphi_\psi^{-1}(\theta_{\eta'}(\psi(e), g_B, r))$. This follows that, if for some h_C such that $\eta'_{\psi(e)}(h_C) \geq r$ and $\eta_e(\varphi_\psi^{-1}(h_C)) < r$, then we obtain that $\tilde{E} \sqsubseteq \varphi_\psi^{-1}(h_C)$ and so $\varphi_\psi^{-1}(h_C) = \tilde{E}$. Now, our hypothesis implies that $\eta_e(\varphi_\psi^{-1}(h_C)) \geq r$, it is a contradiction. Hence, $\eta_e(\varphi_\psi^{-1}(g_B)) \geq \eta'_{\psi(e)}(g_B)$, and moreover φ_ψ is fuzzy soft $\eta\eta'$-continuous. □

Theorem 3.5. *Let $\varphi: X \to Y$, $\psi: E \to K$, and $\eta' : K \to I^{\widetilde{(Y,K)}}$ be functions and let η be a fuzzy soft supra topology on X. Then $\varphi_\psi : \widetilde{(X,E)} \longrightarrow \widetilde{(Y,K)}$ is a fuzzy soft $\eta\eta'$-continuous if and only if φ_ψ is fuzzy soft $(id_X, I_\eta, \theta_{\eta'}, id_Y, \mathcal{I}^0)$-continuous.*

Proof. Sufficiency. Suppose that φ_ψ is a fuzzy soft $\eta\eta'$-continuous. Let $g_B \in \widetilde{(Y,K)}$, $r \in I_0$. Then we consider two cases: **(Case 1).** If $\eta'_{\Psi(e)}(g_B) \geq r$ then $\theta_{\eta'}(\psi(e), g_B, r) = g_B$ and $id_X(e, \varphi_\psi^{-1}(g_B), r) = I_\eta(e, \varphi_\psi^{-1}(g_B), r) = \varphi_\psi^{-1}(g_B)$. Hence, $id_X(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) = \varphi_\psi^{-1}(g_B) = I_\eta(e, \varphi_\psi^{-1}$ $(\theta_{\eta'}(\psi(e), g_B, r))$, $r)$. Consequently, $id_X(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) \sqsubseteq I_\eta(e, \varphi_\psi^{-1}(\theta_{\eta'}(\psi(e), g_B, r)), r)$.

(Case 2). If $\eta'_{\Psi(e)}(g_B) \not\geq r$ then we have that, $\theta_{\eta'}(\psi(e), g_B, r) = \tilde{K}$ and so, $id_X(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) \sqsubseteq \tilde{E} = \varphi_\psi^{-1}(\tilde{K}) = I_\eta(e, \varphi_\psi^{-1}$ $(\theta_{\eta'}(\psi(e), g_B, r))$, $r)$. Hence, $id_X(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) \barwedge I_\eta(e, \varphi_\psi^{-1}$ $(\theta_{\eta'}(\psi(e), g_B, r)), r) = \Phi$ for all $g_B \in \widetilde{(Y,K)}$. Thus φ_ψ is fuzzy soft $(id_X, I_\eta, \theta_{\eta'}, id_Y, \mathcal{I}^0)$-continuous.

Necessity. Suppose that $\eta_e(\varphi_\psi^{-1}(g_B)) \not\geq \eta'_{\psi(e)}(g_B)$, for each $g_B \in \widetilde{(Y,K)}$, $e \in E$ then there exists $r \in I_0$ such that, $\eta_e(\varphi_\psi^{-1}(g_B)) < r \leq \eta'_{\psi(e)}(g_B)$. Since φ_ψ is a fuzzy soft $(id_X, I_\eta, \theta_{\eta'}, id_Y, \mathcal{I}^0)$-continuous, that is, for each $g_B \in \widetilde{(Y,K)}$, $e \in E$, $\mathcal{I}_e^0[id_X(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) \barwedge I_\eta(e, \varphi_\psi^{-1}(\theta_{\eta'}(\psi(e), g_B, r)), r)] \geq \eta'_{\psi(e)}(g_B)$. Then, $id_X(e, \varphi_\psi^{-1}$ $(id_Y(\psi(e), g_B, r)), r) \barwedge I_\eta(e, \varphi_\psi^{-1}(\theta_{\eta'}(\psi(e), g_B, r)), r) = \Phi$, which means that $\varphi_\psi^{-1}(g_B) \sqsubseteq I_\eta(e, \varphi_\psi^{-1}(\theta_{\eta'}(\psi(e), g_B, r)), r)$. This follows that, if for some h_C such that $\eta'_{\psi(e)}(h_C) \geq r$ and $\eta_e(\varphi_\psi^{-1}(h_C) < r$, we obtain $\varphi_\psi^{-1}(h_C) \sqsubseteq I_\eta(e, \varphi_\psi^{-1}(h_C, r)$, and so $\varphi_\psi^{-1}(h_C) = I_\eta(e, \varphi_\psi^{-1}(h_C, r)$, implies that $\eta_e(\varphi_\psi^{-1}(h_C) \geq r$, and there is a contradiction. Consequently, $\eta_e(\varphi_\psi^{-1}(g_B)) \geq \eta'_{\psi(e)}(g_B)$. Hence, φ_ψ is fuzzy soft $\eta\eta'$-continuous. □

Corollary 3.6. *Let $\varphi: X \to Y$, $\psi: E \to K$ and $m_Y : K \to I^{\widetilde{(Y,K)}}$ be functions and let $\varphi_\psi : \widetilde{(X,E)} \to \widetilde{(Y,K)}$ be a fuzzy soft $(id_X, int_{m_X}, \theta_{m_Y}, id_Y, \mathcal{I}^0)$-continuous with m_X has the property U, then φ_ψ is fuzzy soft (m_X, m_Y)-continuous.*

Theorem 3.7. *A function $\varphi_\psi : \widetilde{(X,E)} \longrightarrow \widetilde{(Y,K)}$ is fuzzy soft $\eta\eta'$-open function if and only if φ_ψ is fuzzy soft $(I_\eta, I_\eta, I_{\eta'}, id_Y, \mathcal{I}^0)$-continuous.*

Proof. Sufficiency. Let $f_A \in \widetilde{(X,E)}$, $r \in I_0$, $g_B = \varphi_\psi(f_A)$ and φ_ψ be fuzzy soft $\eta\eta'$-open, then $\eta'_{\psi(e)}(\varphi_\psi(f_A)) = \eta'_{\Psi(e)}(g_B) \geq r$, and $I_{\eta'}(\psi(e), g_B, r) = g_B$. Since, $f_A \sqsubseteq \varphi_\psi^{-1}(\varphi_\psi(f_A))$, we have, $I_\eta(e, f_A, r) \sqsubseteq I_\eta(e, \varphi_\psi^{-1}(\varphi_\psi(f_A)), r) = I_\eta(e, \varphi_\psi^{-1}(g_B), r) \sqsubseteq \varphi_\psi^{-1}(g_B) = \varphi_\psi^{-1}(I_{\eta'}(\psi(e), g_B, r))$, then we have the relation $I_\eta(e, \varphi_\psi^{-1}$ $(id_Y(\psi(e), g_B, r), r) \sqsubseteq I_\eta(e, \varphi_\psi^{-1}(I_{\eta'}(\psi(e), g_B, r)), r)$, which means that, $I_\eta(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) \barwedge I_\eta(e, \varphi_\psi^{-1}(I_{\eta'}(\psi(e),$ $g_B, r)), r) = \Phi$. Hence, $\mathcal{I}_e^0[I_\eta(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) \barwedge I_\eta(e, \varphi_\psi^{-1}(I_{\eta'}(\psi(e), g_B, r)), r)] \geq \eta'_{\psi(e)}(g_B)$. Thus, φ_ψ is fuzzy soft $(I_\eta, I_\eta, I_{\eta'}, id_Y, \mathcal{I}^0)$-continuous.

Necessity. Suppose that $\eta'_{\psi(e)}(\varphi_\psi(f_A)) \not\geq \eta_e(f_A)$; $f_A \in \widetilde{(X,E)}$ and $e \in E$, then there exists $r \in I_0$ such that, $\eta'_{\psi(e)}(\varphi_\psi(f_A)) < r \leq \eta_e(f_A)$. Since φ_ψ is a fuzzy soft $(I_\eta, I_\eta, I_{\eta'}, id_Y, \mathcal{I}^0)$-continuous, that is, for each $g_B \in \widetilde{(Y,K)}$, $e \in E, \mathcal{I}_e^0[I_\eta(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) \barwedge I_\eta(e, \varphi_\psi^{-1}(I_{\eta'}(\psi(e), g_B, r)), r)] \geq \eta'_{\psi(e)}(g_B)$. Then, $I_\eta(e, \varphi_\psi^{-1}(id_Y(\psi(e), g_B, r)), r) \barwedge I_\eta(e, \varphi_\psi^{-1}(I_{\eta'}(\psi(e), g_B, r)), r) = \Phi$, which means that, $I_\eta(e, \varphi_\psi^{-1}(g_B), r) \sqsubseteq I_\eta(e, \varphi_\psi^{-1}(I_{\eta'}(\psi(e), g_B, r)), r)$. Assume that $\eta_e(f_A) \geq r$ and $g_B = \varphi_\psi(f_A)$, then we obtain that, $f_A = I_\eta(e, f_A, r) \sqsubseteq I_\eta(e, \varphi_\psi^{-1}(\varphi_\psi(f_A)), r) \sqsubseteq I_\eta(e, \varphi_\psi^{-1}(I_{\eta'}(\psi(e), \varphi_\psi(f_A), r)), r) \sqsubseteq \varphi_\psi^{-1}(I_{\eta'}(\psi(e), \varphi_\psi(f_A), r))$. This follows that, $\varphi_\psi(f_A) \sqsubseteq I_{\eta'}(\psi(e), \varphi_\psi(f_A), r)$, then $\eta'_{\psi(e)}(\varphi_\psi(f_A)) \geq r$, it is a contradiction. Consequently, $\eta'_{\psi(e)}(\varphi_\psi(f_A)) \geq \eta_e(f_A)$. Hence, φ_ψ is fuzzy soft $\eta\eta'$-open function. □

Corollary 3.8. *Let $\varphi: X \to Y$, $\psi: E \to K$, be functions. If $\varphi_\psi : \widetilde{(X,E)} \longrightarrow \widetilde{(Y,K)}$ is a fuzzy soft $(int_{m_X}, int_{m_X}, int_{m_Y}, id_Y, \mathcal{I}^0)$-continuous with m_Y has the property U, then φ_ψ is fuzzy soft (m_X, m_Y)-open function.*

Acknowledgements

The authors would like thank the referees for the helpful suggestions.

References

[1] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37 (1999) 19–31.
[2] K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (8-9) (2002) 1077–1083.
[3] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (4-5) (2003) 555–562.
[4] L.A. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 338–353.
[5] B. Ahmad, A. Kharal, On fuzzy soft sets, Adv. Fuzzy (2009) 6. Article ID 586507
[6] A. Kharal, B. Ahmad, Mappings on fuzzy soft classes, Adv. Fuzzy Syst. (2009). Article ID 407890, 6 pages.
[7] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (2001) 589–602.
[8] M. Shabir, M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (7) (2011) 1786–1799.
[9] B. Tanay, M.B. Kandemir, Topological structure of fuzzy soft sets, Comput. Math. Appl. 61 (2011) 2952–2957.
[10] C.L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1) (1968) 182–190.
[11] B.P. Varol, H. Aygün, Fuzzy soft topology, Hacettepe J. Math. Stat. 41 (3) (2012) 407–419.
[12] A. Aygünoglu, V. Çetkin, H. Aygün, An introduction to fuzzy soft topological spaces, Hacettepe J. Math. Stat. 43 (2) (2014) 193–204.
[13] A.P. Šostak, On a fuzzy topological structure, Rendiconti del Circolo Matematico di Palermo 1 (11) (2004) 53–64.
[14] C. Gunduz, S. Bayramov, Some results on fuzzy soft topological spaces, Math. Prob. Eng. (2013). Article ID 835308, 10 pages.
[15] S.E. Abbas, E. El-sanowsy, A. Atef, on fuzzy soft irresolute functions, J. Fuzzy Math. 24 (2) (2016) 465–482.