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Original Article

A comment on generalized $\alpha\beta$-closed sets

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ABSTRACT

In this note, we will show that the notions of generalized alpha b-closed ($g\alpha b$-closed) sets and b-closed set are equivalent.

1. Introduction

The new class b-closed set was introduced by Andrijevic [1]. The application of b-open sets had been introduced by Caldas and Jafari [2]. Many results had been obtained by using the concept of b-closed sets. Also , Vinayagamoorthi and Nagaveni [3] discussed and established the concept of generalized αb-closed sets as a generalization of b-closed sets. We will show that the concept of b-closed set and a generalized αb-closed set are same. This means that all results in [3–7] are considered as the same well-known results.

Definition 1.1 [1]. Let (X, τ) be a topological spaces. A subset $A \sqsubseteq X$ is said to be b-closed set if $int(cl(A)) \sqcap cl(int(A)) \sqsubseteq A$.

Definition 1.2 [3]. Let (X, τ) be a topological spaces. A subset $A \sqsubseteq X$ is said to be a generalized αb-closed set (briefly $g\alpha b$-closed set) if $bcl(A) \sqsubseteq U$ whenever $A \sqsubseteq U$ and U is an α-open set.

2. Main result

Theorem 2.1. *The concepts of a $g\alpha b$-closed set and b-closed set are equivalent.*

Proof. Clearly b-closeness $\Rightarrow$ $g\alpha b$-closeness. □

Conversely: let (X, τ) be a topological spaces and $A \sqsubseteq X$ be a $g\alpha b$-closed set. We will prove that A is b-closed set, for let $x \in bcl(A)$. Since every singleton is either preopen or nowhere dense, then we have the following two cases.

Case (1): If x is preopen, then it is also b-open, $\{x\} \sqcap A \neq \emptyset$, and hence $x \in A$. Therefore $bcl(A) \sqsubseteq A$, and hence A is b-closed.

Case (2): If $\{x\}$ is nowhere dense, then $int(cl\{x\}) = \emptyset$, this implies $X = cl(int(X\setminus\{x\}))$. Then $X\setminus\{x\} \sqsubseteq X = intX = int(cl(int(X\setminus\{x\})))$. Therefore $X\setminus\{x\}$ is α-open. Suppose that $x \notin A$, then $A \sqsubseteq X\setminus\{x\}$ and, since A is $g\alpha b$-closed, we have $bcl(A) \sqsubseteq X\setminus\{x\}$. Hence $x \notin bcl(A)$, which is a contradiction and hence $x \in A$. Therefore $bcl(A) \sqsubseteq A$, and hence A is b-closed.

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