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SHORT COMMUNICATION

# *CI*-algebra is equivalent to dual *Q*-algebra

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*CI*-algebra;
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**Abstract** In this paper, we introduce the notion of dual *Q*-algebras and we show that the *CI*-algebras are equivalent to the dual *Q*-algebras.

**MATHEMATICS SUBJECT CLASSIFICATION:** 06F35, 03G25, 03B05, 03B52



## 1. Introduction

Several algebras with one binary and one nulary operations were introduced to set up an algebraic counterpart of implication reduct of classical or non-classical propositional logics. In 1966, Imai and Iseki [2] introduced two classes of abstract algebras: *BCK*-algebras and *BCI*-algebras. It is known that the class of *BCK*-algebras is a proper subclass of the class of *BCI*-algebras. In [1], Hu and Li introduced a wide class of abstract algebras: *BCH*-algebra. They shown that the class of *BCI*-algebras is a proper subclass of the class of *BCH*-algebras. In [5], Neggers and Kim introduced the notion of *d*-algebras, which is generalization of *BCK*-algebras and investigated relation between *d*-algebras and *BCK*-algebras. Neggers et al. introduced the notion of *Q*-algebras [6], which is a generalization of $BCH/BCI/BCK$-algebras.

Recently, Kim and Kim defined a *BE*-algebra [3]. Meng, defined notion of *CI*-algebra as a generalization of a *BE*-algebra [4].

In this note, we state and prove the relationship between *Q*-algebras and *CI*-algebras.

## 2. Preliminaries

**Definition 2.1** [6]. A *Q*-algebra is a non-empty set $X$ with a consonant 0 and a binary operation $*$ satisfying the following axioms:

(I) $x * x = 0$,
(II) $x * 0 = x$,
(III) $(x * y) * z = (x * z) * y$,

for all $x, y, z \in X$.

**Example 2.2** [6]. Let $X = \{0, 1, 2, 3\}$ be a set with the following table:

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| $*$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 2 | 1 |
| 1 | 1 | 0 | 2 |
| 2 | 2 | 1 | 0 |

Then $(A, *, 0)$ is a $Q$-algebra.

**Theorem 2.3** [6]. *If $(X,*,0)$ is a Q-algebra, then $(x*(x*y))*y = 0$, for all $x, y \in X$.*

**Definition 2.4** [6]. A non-empty subset $S$ of a $Q$-algebra $X$ is called a subalgebra of $X$ if $x*y \in S$ for any $x, y \in S$.

**Definition 2.5** [4]. A $CI$-algebra is an algebra $(X;*,1)$ of type $(2,0)$ satisfying the following axioms:

(CI1) $x*x = 1$;
(CI2) $1*x = x$;
(CI3) $x*(y*z) = y*(x*z)$ for all $x, y, z \in X$.

In any $CI$-algebra $X$ one can define a binary relation "$\leqslant$" by $x \leqslant y$ if and only if $x*y = 1$. A $CI$-algebra $X$ has the following properties:

(2.1) $y*((y*x)*x) = 1$,
(2.2) $(x*1)*(y*1) = (x*y)*1$,
(2.3) $1 \leqslant x \Rightarrow x = 1$ for all $x, y \in X$.

A non-empty subset $S$ of a $CI$-algebra $X$ is called a subalgebra of $X$ if $x*y \in S$ whenever $x, y \in S$.

## 3. *CI*-algebra is equivalent to the dual *Q*-algebra

**Definition 3.1.** Let $(X,*,0)$ be a $Q$-algebra and binary operation "$*$" on $X$ is defined as follows:

$x*y = y \circ x$

Then $(X, \circ, 1)$ is called dual $Q$-algebra. In fact, the axioms of that are as follows:

$(DQ1)$ $x \circ x = 1$,
$(DQ2)$ $1 \circ x = x$,
$(DQ3)$ $x \circ (y \circ z) = y \circ (x \circ z)$,

for all $x, y, z \in X$.

**Theorem 3.2.** *Any CI-algebra is equivalent to the dual Q-algebra.*

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