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Solitons and other solutions to a new coupled nonlinear Schrodinger type equation

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ABSTRACT

In this paper, the first integral method combined with Liu's theorem is applied to integrate a new coupled nonlinear Schrodinger type equation. Using this combination, more new exact traveling wave solutions are obtained for the considered equation using ideas from the theory of commutative algebra. In addition, more solutions are also obtained via the application of semi-inverse variational principle due to Ji-Huan He. The used approaches with the help of symbolic computations via Mathematica 9, may provide a straightforward effective and powerful mathematical tools for solving nonlinear partial differential equations in mathematical physics.

1. Introduction

In recent years, the investigation of exact solutions to nonlinear partial differential equations (NPDEs.) has played an important role in nonlinear phenomena. Nonlinear phenomena appear in a wide variety of scientific applications such as plasma physics, solid state physics and fluid dynamics .In order to better understanding these nonlinear phenomena, many mathematicians as well as physicists have been made big efforts to seek more exact solutions to NPDEs. Therefore, several powerful methods have been proposed to obtain exact solutions of nonlinear equations, such as inverse scattering method [1], Backlund transformation method [2], Hirota direct method [3,4], tanh-sech method [5–7], extended tanth method [8–10], $(G'/G, 1/G)$ - expansion method [11], modified simplest equation method [12,13], homogeneous balance method [14,15], Jacobi elliptic function expansion method [16], F- expansion method [17], the transformed rational function method [18] and others.

The first integral (FI) method was first proposed by Feng in [19] in solving Burgers-KdV equation which is based on the ring theory of commutative algebra. Recently, this useful method has been widely used by many authors such as [20–25] and by the references therein.

The variational approaches such as Ji-Huan He semi-inverse variational (SIV) method [26] is a powerful mathematical tool for searching the variational principles of nonlinear physical systems from the field equations without using Lagrange multipliers.

Yong et al. [27], have studied the following new coupled nonlinear Schrodinger type (CNLST) equation

$$\begin{cases} u_{xt} = u_{xx} + \dfrac{2}{1-\beta^2}|u|^2\ u + u\,(v-w), \\ v_t = -\dfrac{(|u|^2)_t}{1+\beta} + (1+\beta)\,v_x, \\ w_t = \dfrac{(|u|^2)_t}{1-\beta} + (1-\beta)\,w_x, \end{cases} \tag{1}$$

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by using the truncated singular expansions and direct quadrature method to obtain exact solutions of this equation.

The new coupled nonlinear Schrodinger type (CNLST) Eq. (1) was proposed in (2009) by Ma and Geng via spectral problem and its auxiliary one [28]. In this present paper, we aim to extend the previous works made in [27], to extract more exact solutions of the new coupled nonlinear Schrodinger type (CNLST) Eq. (1) via two distinct algorithms, namely the first integral(FI) method combined with Liu's theorem and Ji-Huan He's semi-inverse variational (SIV) method.

The layout of this paper is as follows: in Section 2 we present basic algorithm of the first integral (FI)method .In Section 3, application to the new coupled nonlinear Schrodinger type (CNLST) Eq. (1) is considered. Also, the algorithm of semi-inverse variational (SIV) method combined with its application to the considered equation are presented in Sections 4 and 5. The graphics of the obtained solutions accompanied with their explanations have been added in Section 6. Section 7 is devoted to some conclusions.

2. Algorithm of the FI method

Consider a general nonlinear partial differential equation (nonlinear PDE) in the form

$$F(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, u_{xxx}, \ldots) = 0, \tag{2}$$

where $u = u(x,t)$ is the solution of this nonlinear PDE (2).

We use the traveling wave transformation

$$u(x,t) = u(\xi), \tag{3}$$

where $\xi = x - \lambda t + \xi_0$, and ξ_0 is an arbitrary constant. This enables us to use the following changes:

$$\frac{\partial}{\partial t}(\bullet) = -\lambda\frac{\partial}{\partial \xi}(\bullet), \quad \frac{\partial}{\partial x}(\bullet) = \frac{\partial}{\partial \xi}(\bullet), \quad \frac{\partial^2}{\partial x^2}(\bullet) = \frac{\partial^2}{\partial \xi^2}(\bullet), \cdots. \tag{4}$$

Using Eq. (4), the nonlinear PDE (2) is transformed to the nonlinear ordinary differential equation (nonlinear ODE)

$$G(u(\xi), \partial u(\xi)/\partial\xi, \partial^2 u(\xi)/\partial\xi^2, \ldots) = 0. \tag{5}$$

Next, we introduce new independent variables

$$X(\xi) = u(\xi), \quad Y(\xi) = \partial u(\xi)/\partial\xi, \tag{6}$$

which lead to a system of nonlinear ODEs.:

$$\partial X(\xi)/\partial\xi = Y(\xi) \tag{7a}$$

$$\partial Y(\xi)/\partial\xi = F(X(\xi), Y(\xi)) \tag{7b}$$

According to the qualitative theory of ordinary differential equations [29], if we can find two first integrals to system (7) under the same conditions, then analytic solutions to Eqs. (7a) and (7b) can be solved directly. However, in general, it is difficult to realize this even for one first integral, because for a given plane autonomous system, there is no systematic theory that can tell us how to find its first integrals, nor is there a logical way for telling us what these first integrals are.

We will apply the Division theorem to obtain one first integral to Eqs. (7a) and (7b) which reduces Eq. (5) to a first order integrable ODE. An exact solution to Eq. (2) is then obtained by solving this equation.

For convenience, first let us recall the Division theorem.

Theorem 1 (Divison theorem)**.** *Suppose that $P(w,z)$ and $Q(w,z)$ are polynomials in $C(w,z)$ and $P(w,z)$ is irreducible in $C(w,z)$. If $Q(w,z)$ vanishes at all zero points of $P(w,z)$, then there exists a polynomial $G(w,z)$ in $C(w,z)$ such that*

$$Q(w,z) = P(w,z)\,G(w,z). \tag{8}$$

The Division theorem follows immediately from the Hilbert–Nullstellensatz Theorem [30], namely,

Theorem 2 (Hilbert – Nullstellensatz theorem)**.** *Let k be a field and L an algebraic closure of k.*

(1) Every ideal γ of $k[X_1, \ldots, X_n]$ not containing 1 admits at least one zero in L^n
(2) Let $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n)$ be two elements of L^n; for the set of polynomials of $k[X_1, \ldots, X_n]$ zero at x to be identical with the set of polynomials of $k[X_1, \ldots, X_n]$ zero at y, it is necessary and sufficient that there exists a $k-$automorphism s of L such that $y_i = s(x_i)$ for $1 \le i \le n$.
(3) For an ideal α of $k[X_1, \ldots, X_n]$ to be maximal, it is necessary and sufficient that there exists an x in L^n such that α is the set of polynomials of $k[X1_1, \ldots, X_n]$ zero at x.
(4) For a polynomial Q of $k[X_1, \ldots, X_n]$ to be zero on the set of zeros in L^n of an ideal γ of $k[X_1, \ldots, X_n]$, it is necessary and sufficient that there exists an integer $m \succ 0$ such that $Q^m \in \gamma$.

Theorem 3 (Liu's theorem [31])**.** *If Eq. (2) has a kink-type solution*

$$u(\xi) = Q_\ell\,(\tanh[A(\xi + \xi_0)]), \tag{9}$$

then, it has certain kink-bell - type solution

$$u(\xi) = Q_\ell\,(\tanh[2A(\xi + \xi_0)] \pm i\,\mathrm{sec}\,h[2A(\xi + \xi_0)]), \tag{10}$$

where Q_ℓ is a polynomial of degree k, i is the imaginary number, namely, $i = \sqrt{-1}$.

3. Application

The authors in [27] have taken the traveling wave transformation [32]

$$u = \phi(\xi)\ e^{i(kx - \omega t)}, \quad v = V(\xi), \quad w = W(\xi), \quad \xi = x - \lambda\,t + \xi_0, \tag{11}$$

where ξ_0 is an arbitrary constant. Conducting the analysis made in [27] on the new coupled nonlinear Schrodinger type (CNLST) Eq. (1), thus, the following results have been obtained as

$$V(\xi) = -\frac{\lambda\,\phi^2}{(\beta+1)(\beta+\lambda+1)}, \quad W(\xi) = \frac{\lambda\,\phi^2}{(\beta-1)(\beta-\lambda-1)} \tag{12}$$

and the reduced nonlinear ODE

$$\phi'' = -k^2\phi \ - \ \frac{2}{(\lambda+1+\beta)\ (\lambda+1-\beta)}\ \phi^3 \tag{13}$$

where $' := d/d\xi$.

Therefore, we are concerned to solve the Lienard Eq. (13).

By introducing new independent variables $X = \phi(\xi)$ and $Y = \phi'(\xi)$ and using (6), we get a system of nonlinear ODEs

$$X'(\xi) \ = \ Y(\xi) \tag{14a}$$

$$Y'(\xi) = (-k^2)X(\xi) - \left(\frac{2}{(\lambda+1+\beta)\ (\lambda+1-\beta)}\right)X^3(\xi). \tag{14b}$$

According to the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ are the nontrivial solutions of (14a) and (14b), and

$$Q(X,Y) = \sum_{i=0}^{m} a_i(X)\,Y^i, \tag{15}$$

is an irreducible polynomial in the complex domain $C[X,Y]$ such that

$$Q(X(\xi),Y(\xi)) = \sum_{i=0}^{m} a_i(X(\xi))\, Y^i(\xi) = 0, \tag{16}$$

where $a_i(X)$, $(i=0,1,2,...,m)$ are polynomials of X and $a_m(X)\neq 0$.

Eq. (16) is called the first integral to (14a) and (14b). Due to the Division Theorem, there exists a polynomial $h(X)+g(X)Y$, in the complex domain $C[X,Y]$ such that

$$\frac{dQ}{d\xi} = \frac{dQ}{dX}\frac{dX}{d\xi} + \frac{dQ}{dY}\frac{dY}{d\xi} = (h(X)+g(X)Y)\left(\sum_{i=0}^{m} a_i(X)Y^i\right) \tag{17}$$

Here, we take two distinct cases, assuming that $m=1$ and $m=2$ in Eq. (16).

Case I: Suppose that $m=1$, by equating the coefficients of $Y^i(i=2,1,0)$ on both sides of (17), we have

$$a_1'(X) = g(X)a_1(X) \tag{18a}$$

$$a_0'(X) = h(X)a_1(X) + g(X)a_0(X) \tag{18b}$$

$$a_1(X)\left[(-k^2)X(\xi) - \left(\frac{2}{(\lambda+1+\beta)\ (\lambda+1-\beta)}\right)X^3(\xi)\right] = h(X)a_0(X) \tag{18c}$$

Since, $a_i(X)(i=0,1)$ are polynomials, then from (18a) we conclude that $a_1(X)$ is a constant and $g(X)=0$. For simplicity, we take $a_1(X)=1$, and balancing the degrees of $h(X)$ and $a_0(X)$ we conclude that $\deg(h(X))=1$, only.

Now, suppose that $h(X) = AX + B$,and $A\neq 0$, then we find $a_0(X)$

$$a_0(X) = \frac{1}{2}AX^2 + BX + D, \tag{19}$$

where D is an arbitrary integration constant.

Substituting $a_0(X)$, $a_1(X)$ and $h(X)$ into (18c), and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it via Mathematica 9, we obtain

$$D = \frac{1}{2}k^2\sqrt{\beta^2-(1+\lambda)^2},\quad A = -\frac{2}{\sqrt{\beta^2-(1+\lambda)^2}},\quad B=0 \tag{20a}$$

$$D = -\frac{1}{2}k^2\sqrt{\beta^2-(1+\lambda)^2}, A = \frac{2}{\sqrt{\beta^2-(1+\lambda)^2}},\quad B=0 \tag{20b}$$

Setting (20a) in (16) leads to

$$Y(\xi) = \frac{1}{\sqrt{\beta^2-(1+\lambda)^2}}X^2(\xi) - \frac{1}{2}k^2\sqrt{\beta^2-(1+\lambda)^2} \tag{21}$$

Combining (21) with (14a), a first-order ODE is derived, then by solving this derived equation, we obtain the exact solution to (13). Thus, the traveling wave solution of the CNLST Eq. (1) is obtained and can be written as

$$\begin{cases} u_1(x,t) = -(kR_1/\sqrt{2})\tanh((k/\sqrt{2})[x-\lambda t+\xi_0-2R_1\varsigma]) \\ \quad \times \exp[i(kx-\omega t)] \\ v_1(x,t) = -(\lambda k^2R_1^2/2S_1)\tanh^2((k/\sqrt{2})[x-\lambda t+\xi_0-2R_1\varsigma]) \\ w_1(x,t) = (\lambda k^2R_1^2/2N)\tanh^2((k/\sqrt{2})[x-\lambda t+\xi_0-2R_1\varsigma]) \end{cases} \tag{22}$$

where ς is an arbitrary constant.

Therefore, by using Liu's Theorem 3, we get another traveling wave solutions as

$$\begin{cases} u_2(x,t) = -(kR_1/\sqrt{2}) \\ \quad \times \begin{bmatrix} \tanh((2k/\sqrt{2})[x-\lambda t+\xi_0-2R_1\varsigma]) \\ \pm i\,\mathrm{sech}((2k/\sqrt{2})[x-\lambda t+\xi_0-2R_1\varsigma]) \end{bmatrix} \\ \quad \times \exp[i(kx-\omega t)] \\ v_2(x,t) = -(\lambda k^2{R_1}^2/2S_1) \\ \begin{bmatrix} \tanh((2k/\sqrt{2})[x-\lambda t+\xi_0-2R_1\varsigma]) \\ \pm i\,\mathrm{sech}((2k/\sqrt{2})[x-\lambda t+\xi_0-2R_1\varsigma]) \end{bmatrix}^2 \\ w_2(x,t) = (\lambda k^2{R_1}^2/2N) \\ \quad \times \begin{bmatrix} \tanh((2k/\sqrt{2})[x-\lambda t+\xi_0-2R_1\varsigma]) \\ \pm i\,\mathrm{sech}((2k/\sqrt{2})[x-\lambda t+\xi_0-2R_1\varsigma]) \end{bmatrix}^2 \end{cases} \tag{23}$$

where,

$$R_1 = \sqrt{\beta^2-(1+\lambda)^2}, S_1 = (\beta+1)(\beta+\lambda+1), \\ N = (\beta-1)(\beta-\lambda-1), \tag{24}$$

and ς is an arbitrary integration constant.

Similarly, in the case of (20b), from (16), we obtain

$$Y(\xi) = -\frac{1}{\sqrt{\beta^2-(1+\lambda)^2}}X^2(\xi) + \frac{1}{2}k^2\sqrt{\beta^2-(1+\lambda)^2} \tag{25}$$

Combining (25) with (14a), we obtain the exact solution to (13) and therefore, the traveling wave solution of the CNLST Eq. (1) are found as

$$\begin{cases} u_3(x,t) = -(kR_1/\sqrt{2})\tanh((k/\sqrt{2})[x-\lambda t+\xi_0+2R_1\varsigma]) \\ \quad \times \exp[i(kx-\omega t)] \\ v_3(x,t) = -(\lambda k^2R_1^2/2S_1)\tanh^2((k/\sqrt{2})[x-\lambda t+\xi_0+2R_1\varsigma]) \\ w_3(x,t) = (\lambda k^2R_1^2/2N)\tanh^2((k/\sqrt{2})[x-\lambda t+\xi_0+2R_1\varsigma]) \end{cases} \tag{26}$$

Hence, via Liu's Theorem 3, we also have

$$\begin{cases} u_4(x,t) = -(kR_1/\sqrt{2})[\tanh((2k/\sqrt{2})[x-\lambda t+\xi_0+2R_1\varsigma]) \\ \quad \pm i\,\mathrm{sech}((2k/\sqrt{2})[x-\lambda t+\xi_0+2R_1\varsigma])] \\ \quad \times \exp[i(kx-\omega t)] \\ v_4(x,t) = -(\lambda k^2{R_1}^2/2S_1)[\tanh((2k/\sqrt{2})[x-\lambda t+\xi_0+2R_1\varsigma]) \\ \quad \pm i\,\mathrm{sech}((2k/\sqrt{2})[x-\lambda t+\xi_0+2R_1\varsigma])]^2 \\ w_4(x,t) = (\lambda k^2{R_1}^2/2N)[\tanh((2k/\sqrt{2})[x-\lambda t+\xi_0+2R_1\varsigma]) \\ \quad \pm i\,\mathrm{sech}((2k/\sqrt{2})[x-\lambda t+\xi_0+2R_1\varsigma])]^2 \end{cases} \tag{27}$$

where ς is an arbitrary constant and R_1, S_1 and N are given as in (24).

Case II: Suppose that $m=2$, by equating the coefficients of $Y^i(i=3,2,1,0)$ on both sides of (17), we have

$$a_2'(X) = g(X)a_2(X), \tag{28a}$$

$$a_1'(X) = h(X)a_2(X) + g(X)a_1(X) \tag{28b}$$

$$a'_0(X) + 2a_2(X)\left[(-k^2)X(\xi) - \left(\frac{2}{(\lambda+1+\beta)\ (\lambda+1-\beta)}\right)X^3(\xi)\right] = h(X)a_1(X) + g(X)a_0(X) \tag{28c}$$

$$a_1(X)\left[(-k^2)X(\xi)-\left(\frac{2}{(\lambda+1+\beta)(\lambda+1-\beta)}\right)X^3(\xi)\right]$$
$$=h(X)a_0(X) \tag{28d}$$

Since, $a_i(X)(i=0,1,2)$ are polynomials, then from (28a) we deduce that $a_2(X)$ is a constant and $g(X)=0$. For simplicity, we take $a_2(X)=1$, and balancing the degrees of $h(X), a_1(X)$ and $a_2(X)$ we conclude that $\deg(h(X))=1$, only. In this case, we assume that $h(X)=AX+B$, and $A\neq 0$, then we find $a_1(X)$ and $a_0(X)$ as follows

$$a_1(X)=\frac{1}{2}AX^2+BX+D \tag{29a}$$

$$a_0(X)=\left(\frac{A}{8}+\frac{1}{(1+\lambda+\beta)(1+\lambda-\beta)}\right)X^4+\frac{1}{2}ABX^3+\left(\frac{AD+B^2}{2}+k^2\right)X^2+BDX+F, \tag{29b}$$

where D and F is an arbitrary integration constants.

Substituting $a_0(X)$, $a_1(X)$, $a_2(X)$ and $h(X)$ in (28d), and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$D=-2\sqrt{F}, B=0, \lambda=-1-(\sqrt{-4F+k^4\beta^2})/k^2, A=2k^2/\sqrt{F}, \tag{30a}$$

$$D=-2\sqrt{F}, B=0, \lambda=-1+(\sqrt{-4F+k^4\beta^2})/k^2, A=2k^2/\sqrt{F}, \tag{30b}$$

$$D=2\sqrt{F}, B=0, \lambda=-1-(\sqrt{-4F+k^4\beta^2})/k^2, A=-2k^2/\sqrt{F}, \tag{30c}$$

$$D=2\sqrt{F}, B=0, \lambda=-1+(\sqrt{-4F+k^4\beta^2})/k^2, A=-2k^2/\sqrt{F}, \tag{30d}$$

Using the conditions (30a) and (30b) in (16), we obtain

$$Y(\xi)=[2F-k^2X^2(\xi)]/2\sqrt{F} \tag{31}$$

Combining (31) with (14a), then we obtain the exact solution to (13) and thus, the traveling wave solutions of the CNLST Eq. (1) can be written as

$$\begin{cases} u_5(x,t)=(\sqrt{2F}/k) \\ \quad\times\ \tanh[(k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma]] \\ \times\ \exp[i(kx-\omega t)] \\ v_5(x,t)=R_2\times\tanh^2 \\ [(k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma]] \\ w_5(x,t)=S_2 \\ \times\ \tanh^2[(k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma]] \end{cases}, \tag{32}$$

$$\begin{cases} u_6(x,t)=(\sqrt{2F}/k) \\ \quad\times\ \tanh[(k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma]] \\ \quad\times\ \exp[i(kx-\omega t)] \\ v_6(x,t)=R_3 \\ \quad\times\tanh^2[(k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\ \ \varsigma]] \\ w_6(x,t)=S_3 \\ \quad\times\ \tanh^2[(k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma]] \end{cases} \tag{33}$$

respectively. Therefore, we also get the extra exact solutions of the considered Eq. (1) as

$$\begin{cases} u_7(x,t)=(\sqrt{2F}/k) \\ \times\left[\begin{matrix}\tanh((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\end{matrix}\right] \\ \times\ \exp[i(kx-\omega t)] \\ v_7(x,t)=R_2 \\ \times\left[\begin{matrix}\tanh((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\end{matrix}\right]^2 \\ w_7(x,t)=S_2 \\ \times\left[\begin{matrix}\tanh((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\end{matrix}\right]^2 \end{cases} \tag{34}$$

$$\begin{cases} u_8(x,t)=(\sqrt{2F}/k) \\ \times\left[\begin{matrix}\tanh((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\end{matrix}\right] \\ \times\exp[i(kx-\omega t)] \\ v_8(x,t)=R_3 \\ \times\left[\begin{matrix}\tanh((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\end{matrix}\right]^2 \\ w_8(x,t)=S_3 \\ \times\left[\begin{matrix}\tanh((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0+2\sqrt{F}\varsigma])\end{matrix}\right]^2 \end{cases} \tag{35}$$

respectively, where,

$$R_2=\frac{2F(k^2+\sqrt{-4F+k^4\beta^2})}{k^2(1+\beta)(k^2\beta-\sqrt{-4F+k^4\beta^2})},$$
$$S_2=\frac{2F(k^2+\sqrt{-4F+k^4\beta^2})}{k^2(-1+\beta)(k^2\beta+\sqrt{-4F+k^4\beta^2})}$$
$$R_3=\frac{2F(k^2-\sqrt{-4F+k^4\beta^2})}{k^2(1+\beta)(k^2\beta+\sqrt{-4F+k^4\beta^2})},$$
$$S_3=\frac{2F(k^2-\sqrt{-4F+k^4\beta^2})}{k^2(1+\beta)(k^2\beta+\sqrt{-4F+k^4\beta^2})} \tag{36}$$

Similarly, as in the case of (30c) and (30d), from (16), we get

$$Y(\xi) = [-2F + k^2X^2(\xi)]/2\sqrt{F}. \tag{37}$$

and therefore, the traveling wave solutions of the CNLST Eq. (1) are thus obtained as

$$\begin{cases} u_9(x,t) = -(\sqrt{2F}/k) \\ \times \tanh[(k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]] \\ \times \exp[i(kx-\omega t)] \\ v_9(x,t) = R_2 \\ \times \tanh^2[(k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]] \\ w_9(x,t) = S_2 \\ \times \tanh^2[(k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]], \end{cases} \tag{38}$$

$$\begin{cases} u_{10}(x,t) = -(\sqrt{2F}/k) \\ \times \tanh[(k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]] \\ \times \exp[i(kx-\omega t)] \\ v_{10}(x,t) = R_3 \\ \times \tanh^2[(k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]] \\ w_{10}(x,t) = S_3 \\ \times \tanh^2[(k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]], \end{cases} \tag{39}$$

respectively.

As the previous obtained solutions, we therefore have the following extra exact solutions

$$\begin{cases} u_{11}(x,t) = -(\sqrt{2F}/k) \\ \times \begin{bmatrix} \tanh((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]) \end{bmatrix} \\ \times \exp[i(kx-\omega t)] \\ v_{11}(x,t) = R_2 \\ \times \begin{bmatrix} \tanh((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]) \end{bmatrix}^2 \\ w_{11}(x,t) = S_2 \\ \times \begin{bmatrix} \tanh((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t+(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]) \end{bmatrix}^2, \end{cases} \tag{40}$$

$$\begin{cases} u_{12}(x,t) = -(\sqrt{2F}/k) \\ \times \begin{bmatrix} \tanh((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]) \end{bmatrix} \\ \times \exp[i(kx-\omega t)] \\ v_{12}(x,t) = R_3 \\ \times \begin{bmatrix} \tanh((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]) \end{bmatrix}^2 \\ w_{12}(x,t) = S_3 \\ \times \begin{bmatrix} \tanh((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma])\pm \\ i\,\mathrm{sec}\,h((2k/\sqrt{2})[x+t-(\sqrt{-4F+k^4\beta^2}/k^2)t+\xi_0-2\sqrt{F}\varsigma]) \end{bmatrix}^2, \end{cases} \tag{41}$$

respectively. Where ς is an arbitrary constant and R_2, R_3, S_2, and S_3 are given as in (36). Comparing these results with the results obtained in [27], it can be seen that the solutions here are new.

4. Algorithm of the SIV method

Jabbari et al. in [33] have been written the He's semi – inverse variational (SIV) method in the following steps:

Step 1. If possible, integrate Eq. (5) term by term one or more times, this yields constant(s) of integration. For simplicity, the integration constant(s) can be set to zero.

Step 2. According to He's semi – inverse method, we construct the following trial – functional.

$$J(u) = \int L\, d\xi\ , \tag{42}$$

where L is an unknown function of u and its derivatives.

Step 3. By the Ritz method, we can obtain different forms of solitary wave solutions, such as

$$\begin{aligned} &u(\xi) = H\,\mathrm{sec}\,h(K\xi), \quad u(\xi) = H\,\mathrm{csc}\,h(K\xi), \\ &u(\xi) = H\tanh(K\xi), \quad u(\xi) = H\coth(K\xi), \end{aligned} \tag{43}$$

and so on. For example in this paper,we search a solitary wave solution in the form

$$u(\xi) = H\,\mathrm{sec}\,h(K\xi), \tag{44}$$

where H and K are constants to be further determined. Substituting Eq. (44) into Eq. (5) and making J stationary with respect to H and K results in

$$\partial J/\partial H = 0, \tag{45a}$$

$$\partial J/\partial K = 0. \tag{45b}$$

Solving Eqs. (45a) and (45b), we obtain values of H and K. Hence the solitary wave solution (44) is well determined.

5. Application

By He's semi – inverse method [26,34,35], we can obtain the following variational formulation

$$\begin{aligned} J = \int_0^\infty &[-(1/2)(\phi')^2 + (k^2/2)\phi^2(\xi) \\ &+ (1/2(\lambda+1+\beta)(\lambda+1-\beta)\alpha k)\phi^4(\xi)]d\xi. \end{aligned} \tag{46}$$

By a Ritz – like method, we search for a solitary wave solution in the form

$$\phi(\xi) = H\,\mathrm{sec}\,h(K\xi), \tag{47}$$

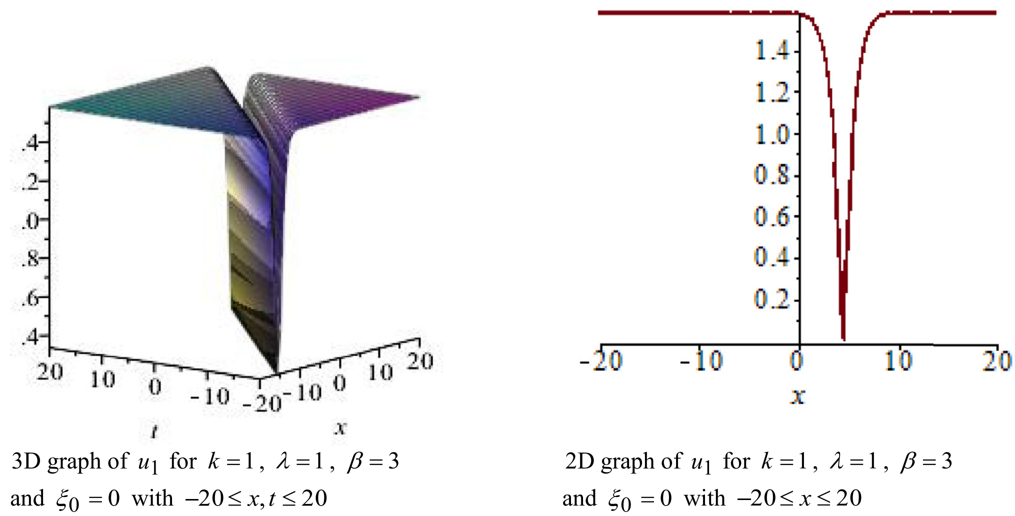


Fig. 1. (a) 3D graph of u_1 for $k = 1$, $\lambda = 1$, $\beta = 3$ and $\xi_0 = 0$ with $-20 \le x,\ t \le 20$. (b) 2D graph of u_1 for $k = 1$, $\lambda = 1$, $\beta = 3$ and $\xi_0 = 0$ with $-20 \le x \le 20$.

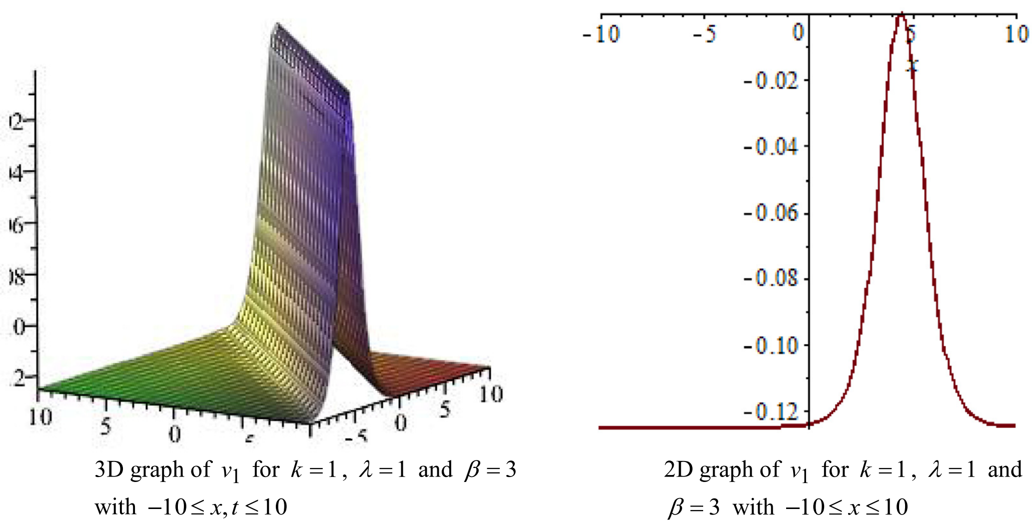


Fig. 2. (a) 3D graph of v_1 for $k = 1$, $\lambda = 1$ and $\beta = 3$ with $-10 \le x,\ t \le 10$ (b) 2D graph of v_1 for $k = 1$, $\lambda = 1$ and $\beta = 3$ with $-10 \le x \le 10$.

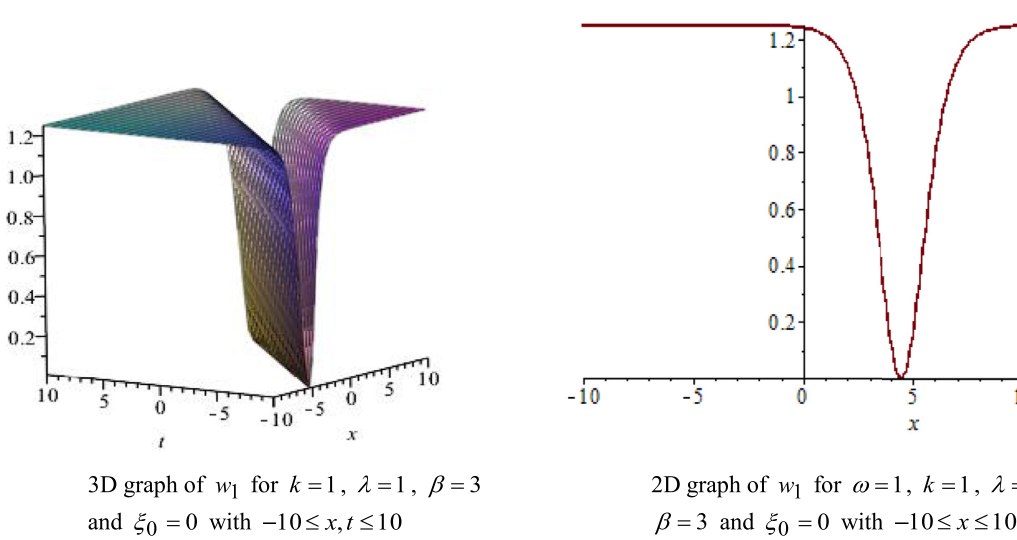
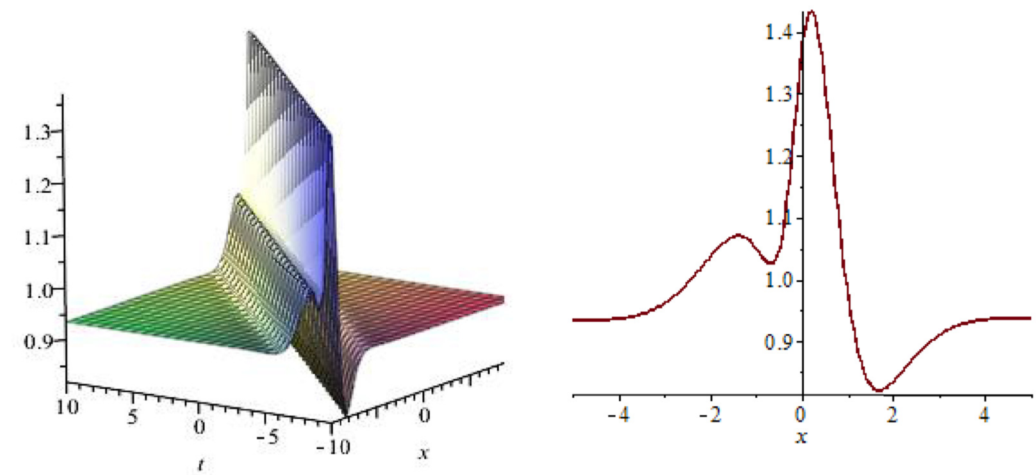
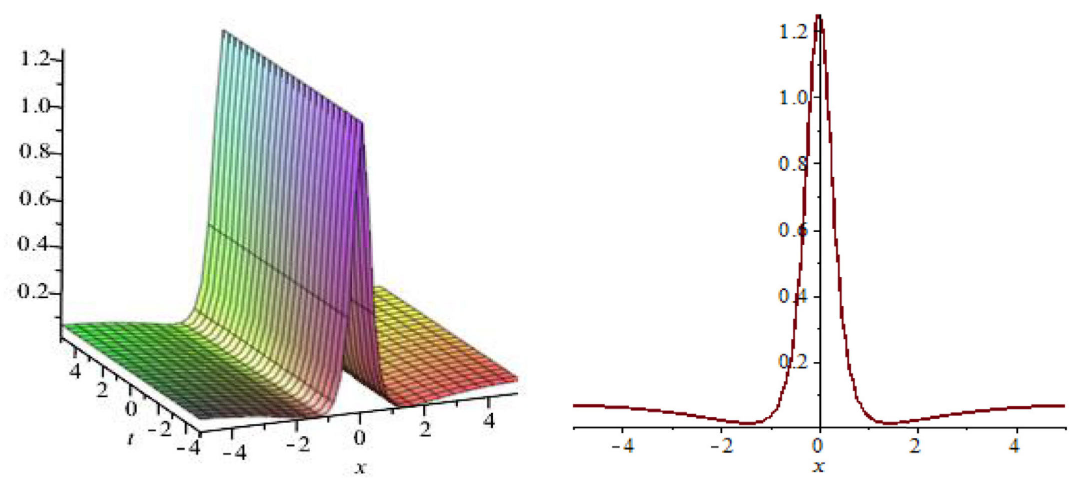


Fig. 3. (a) 3D graph of w_1 for $k = 1$, $\lambda = 1$, $\beta = 3$ and $\xi_0 = 0$ with $-10 \le x,\ t \le 10$ (b) 2D graph of w_1 for $\omega = 1$, $k = 1$, $\lambda = 1$, $\beta = 3$ and $\xi_0 = 0$ with $-10 \le x \le 10$.



3D graph of u_2 for $\omega=1,\ k=1,\ \lambda=1$ and $\beta=1.5$ with $-10\le x,t\le 10$.
2D graph of u_2 for $\omega=1,\ k=1,\ \lambda=1$ and $\beta=1.5$ with $-10\le x\le 10$.

Fig. 4. (a) 3D graph of u_2 for $\omega = 1$, $k = 1$, $\lambda = 1$ and $\beta = 1.5$ with $-10 \le x,\ t \le 10$ (b) 2D graph of u_2 for $\omega = 1$, $k = 1$, $\lambda = 1$ and $\beta = 1.5$ with $-10 \le x \le 10$.



3D graph of v_2 for $k=1,\ \lambda=1$ and $\beta=1.5$ with $-10\le x,t\le 10$.
2D graph of v_2 for $k=1,\ \lambda=1$ and $\beta=1.5$ with $-10\le x\le 10$.

Fig. 5. (a) 3D graph of v_2 for$k = 1$, $\lambda = 1$ and $\beta = 1.5$ with $-10 \le x,\ t \le 10$ (b) 2D graph of v_2 for $k = 1$, $\lambda = 1$ and $\beta = 1.5$ with $-10 \le x \le 10$.

where H and K are unknown constants to be determined later. Substituting Eq. (47) into Eq. (13), we have

$$J = \int_0^{\infty} [-(H^2K^2/2)\sec h^2(K\xi)\tanh^2(K\xi) + (k^2/2)H^2 \sec h^2(K\xi) + (1/2(\lambda+1+\beta)(\lambda+1-\beta))A^4 \sec h^4(K\xi)]d\xi$$
$$= -(H^2 K/6) + (H^4/3K(\lambda+1+\beta)(\lambda+1-\beta)) + (H^2k^2/2K). \quad (48)$$

Making J stationary with H and K yields

$$\partial J/\partial H = -(HK/3) + (Hk^2/K) + [4H^3/3K(\lambda+1+\beta)(\lambda+1-\beta)], \quad (49a)$$

$$\partial J/\partial K = -(H^2/6) - (H^2 k^2/2K^2) - [H^4/3K^2(\lambda+1+\beta)(\lambda+1-\beta)]. \quad (49b)$$

From Eqs. (49a) and (49b), we get

$$H = k\sqrt{-1+\beta^2-2\lambda-\lambda^2}, \quad K = ik. \quad (50)$$

The soliton solutions are, therefore, obtained for Eq. (1) and can be written as

$$\begin{cases} u_{13}(x,t) = kR_1 \sec h[ik(x-\lambda t+\xi_0)] \times \exp[i(kx-\omega t)] \\ v_{13}(x,t) = -(\lambda k^2 {R_1}^2/S_1)\sec h^2[ik(x-\lambda t+\xi_0)] \\ w_{13}(x,t) = (\lambda k^2 {R_1}^2/N)\sec h^2[ik(x-\lambda t+\xi_0)], \end{cases} \quad (51)$$

where ξ_0 is an arbitrary constant and R_1, $S_1$and N are given as in (24).

These solutions are all new exact solutions.

Remark 1. If we seek a solitary wave solution in the form $u(\xi) = A\tanh(B\xi)$ or $u(\xi) = A\coth(B\xi)$, we can further apply Liu's Theorem 3 to get more exact traveling wave solutions to the new coupled nonlinear Schrodinger type (CNLST) Eq. (1), and this is left for the reader.

Remark 2. First integrals play an important role in studing the nonlinear ODEs., since they allow to find solutions of a nonlinear differential equation by quadratures. We also note that our results for using the FI method were based on the assumptions $m = 1$

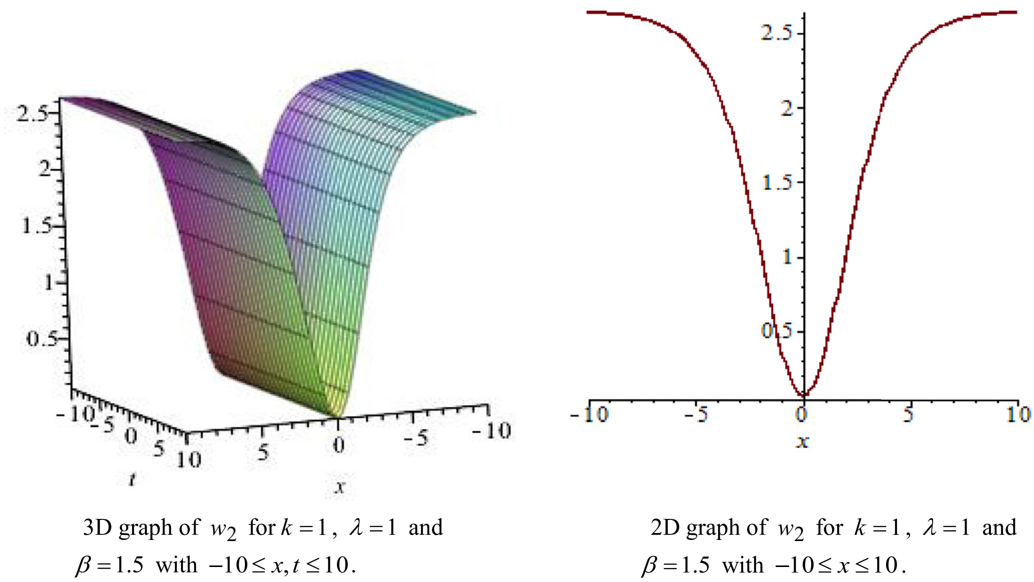


Fig. 6. (a) 3D graph of w_2 for$k = 1$, $\lambda = 1$ and $\beta = 1.5$ with $-10 \le x, t \le 10$ (b) 2D graph of w_2 for $k = 1$, $\lambda = 1$ and $\beta = 1.5$ with $-10 \le x \le 10$.

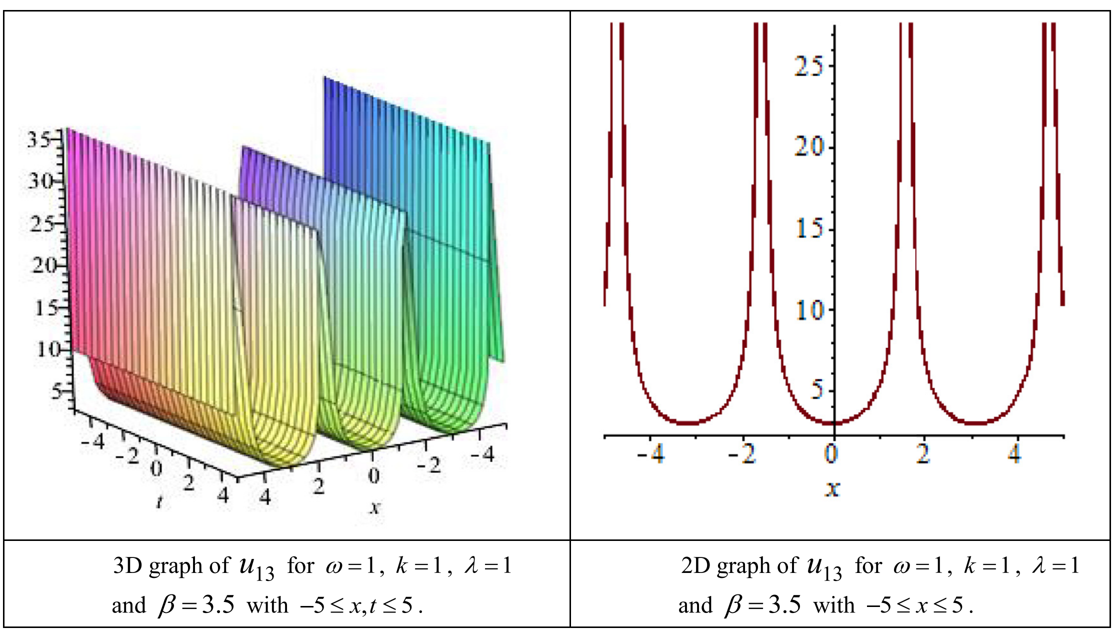


Fig. 7. (a) 3D graph of u_{13} for $\omega = 1$, $k = 1$, $\lambda = 1$ and $\beta = 3.5$ with $-5 \le x, t \le 5$ (b) 2D graph of u_{13} for $\omega = 1$, $k = 1$, $\lambda = 1$ and $\beta = 3.5$ with $-5 \le x \le 5$.

and $m = 2$.The discussion becomes more complicated for the cases $m = 3$ and $m = 4$ because the hyper-elliptic integrals, the irregular singular point theory and the elliptic integrals of the second kind are involved. Also, we do not need to consider the cases $m \ge 5$ because it is known that an algebraic equation with degree greater than or equal to 5 is generally not solvable.

6. Graphical representation of the solutions

Solutions u_1 and w_1 represent squeezed anti-bell shape soliton solution, on the other side v_1 represent the bell shape soliton solution. A soliton is a solitary wave which retains its shape and amplitude after the collision with another solitary wave in the course of propagation with a constant velocity. Its form is usual and stable. Solitons are due to a skilled balance of nonlinear and dispersive effects of the medium. Solitons are the solutions of a comprehensive group of weakly nonlinear dispersive partial differential equations recounting physical systems.

The modulus of the solutions u_2, u_4, u_6,..., u_{12} represent multi-soliton solution whereas solutions v_2, v_4, v_6,..., v_{12} represent the squeezed bell shape soliton solution. On the other hand w_2, w_4, w_6,..., w_{12} represent anti-bell shape soliton solution. For minimalism, the rest of the graphs of the solutions have not been depicted.

Solutions u_{13}, v_{13} and w_{13} illustrate the singular periodic solution. Periodic traveling waves play a significant role in various physical problems, including reaction-diffusion-advection impulsive systems, systems, self-reinforcing systems, etc. The mathematical formulation of abundant intricate phenomena, for instance, chemistry, physics, biology, mathematical physics and many more phenomena leads to periodic traveling wave solutions. Here we have sketched only the graph of the solution u_{13}. As the shape of the other two solutions v_{13} and w_{13} are similar to the shape of the solution u_{13}, therefore for succinctness, the other figures have not been plotted.

7. Conclusion

In the present work, we have succeeded in extracting new explicit and exact traveling wave solutions of the new coupled nonlinear Schrodinger type (CNLST) Eq. (1) owing to the effective combination of the FI method and Liu's Theorem 3. The SIV principle is a very dominant approach to find solitary solutions (solitons) for the new coupled nonlinear Schrodinger type (CNLST) Eq. (1),where we have searched for these kinds of solutions in the forms of Eq. (47). Another soliton solutions of the considered equation can be obtained via the assumption $u(\xi) = \Omega \sec h^2(\Gamma\xi)$, where Ω and Γ are constants to be determined, and we left this work also for the reader.

The obtained solutions may be important for the explanation of some practical physical problems. The first integral method described herein is not only efficient but also has the merit of being widely applicable. Therefore, all the used methods in this paper can be extended for applications to other nonlinear PDEs. with power laws nonlinearities and this will be conducted in a future work.

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References

[1] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform, Cambridge University Press, London, 1990.
[2] M.R. Miura, Backlund Transformation, Springer-Verlag, Berlin, 1978.
[3] R. Hirota, Exact solutions of the Korteweg – de Vries equation for multiple collision of solitons, Phys. Rev. Lett. 27 (1971) 1192–1194.
[4] R. Hirota, The Direct Method in Soliton Theory, Cambridge University Press, London, 2004.
[5] W.X. Ma, Travelling wave solutions to a seventh order generalized KdV equation, Phys. Lett. A 180 (1993) 221–224.
[6] A.H. Hhater, W. Malfliet, D.K. Callebaut, E.S. Kamel, The tanh method, a simple transformation and exact analytical solutions for nonlinear reaction – diffusion equations, Chaos Soliton Fract. 14 (3) (2002) 513–522.
[7] A.M. Wazwaz, Two reliable methods for solving variants of the KdV equation with compact and noncompact structures, Chaos Soliton Fract. 28 (2) (2006) 454–462.
[8] S.A El-Wakil, M.A. Abdou, New exact traveling wave solutions using modified extended tanh-function method, Chaos Soliton Fract. 31 (4) (2007) 840–852.
[9] E. Fan, Extended tanh – function method and its applications to nonlinear equations, Phys. Lett. A 277 (4–5) (2000) 212–218.
[10] A.M. Wazwaz, The tanh-function method: solitons and periodic solutions for the Dodd- Bullough- Mikhailov and the Tzitzeica- Dodd –Bullough equations, Chaos Soliton Fract. 25 (1) (2005) 55–63.
[11] S. Demiray, O. Unsal, A. Bekir, Exact solutions of nonlinear wave equation using $(G'/G,1/G)$ -expansion method, J. Egypt. Math. Soc. 23 (1) (2015) 78–84.
[12] S. El-Ganaini, Solutions of some class of nonlinear PDEs in mathematical physics, J. Egypt. Math. Soc. (2015). http://dx.doi.org/10.1016/j.joems2015.02.005.
[13] M. Eslami, M. Mirzazadeh, A. Biswas, Soliton solutions of the resonant nonlinear Schrodinger equation in optical fibres with time-dependent coefficients by simplest equation approach, J. Mod. Optic 60 (19) (2013) 1627–1636.
[14] E. Fan, Two new applications of the homogeneous balance method, Phys. Lett. A 265 (5-6) (2000) 353–357.
[15] M. Senthilvelan, On the extended applications of homogeneous balance method, Appl. Math. Comput. 123 (2001) 381–388.
[16] M. Inc, M. Ergut, Periodic wave solutions for the generalized shallow water wave equation by the improved jacobi elliptic function method, Appl. Math. E-Notes 5 (2005) 89–96.
[17] S. Zhang, The periodic wave solutions for the (2+1)dimensional Konopelchenko–Dubrovsky equations, Chaos Soliton Fract. 30 (2006) 1213–1220.
[18] W.X. Ma, J.-H. Lee, A transformed rational function method and exact solutions to the (3+1)-dimensional Jimbo-Miwa equation, Chaos Solitons Fract. 42 (2009) 1356–1363.
[19] Z.S. Feng, The first integral method to study the Burgers-Korteweg –deVries equation, J. Phys. A35 (2) (2002) 343–349.
[20] Z.S. Feng, X.H. Wang, The first integral method to the two-dimensional Burgers-KdV equation, Phys. Lett. A 308(2002)173-178.
[21] S.I.A. EL-Ganaini, M. Mirzazdeh, A. Biswas, Solitons and other solutions to Long-Short wave resonance equation, Appl. Comput. Math. 14 (3) (2015) 248–259 Special Issue.
[22] S.I. A El-Ganaini, New exact solutions of some nonlinear systems of partial differential equations using the first integral method method, Abstract Appl. Anal., Volume 2013, Article ID 693076, 13 pages, http://dx.doi.org/10.1155/2013/693076.
[23] S.I. A El-Ganaini, The first integral method to the nonlinear Schrodinger equations in higher dimensions, Abstract Appl. Anal., Volume 2013, Article ID 349173, 10 pages, http://dx.doi.org/10.1155/2013/349173.
[24] F. Tascan, A. Bekir, M. Koparan, Traveling wave solutions of nonlinear evolution equations by using the first integral method, Commun, Nonlinear Sci. Num. Simulat. 14 (5) (2009) 1810–1815.
[25] A. Bekir, O. Unsal, Analytic treatment for nonlinear evolution equations by using first integral method, Pramana-J. Phys. 79 (1) (2012) 3–17.
[26] J.H. He, Variational principles for some nonlinear partial differential equations with variable coefficients, Chaos Solitons Fract. 19 (4) (2004) 847–851.
[27] X. Yong, J. Gao, Z. Zhang, Singularity analysis and explicit solutions of a new coupled nonlinear Schrodinger type equation, Commun. Nonlinear Sci. Numer. Simul. 16 (2011) 2513–2518.
[28] Y.L. Ma, X.G. Geng, A coupled nonlinear Schrodinger type equation and its explicit solutions, Chaos Solitons Fract. 42 (2009) 2949–2953.
[29] T.R. Ding, C.Z. Li, Ordinary Differential Equations, Peking University Press, Peking, 1996.
[30] N. Bourbaki, Commutative Algebra, Addison- Wesley, Paris, 1972.
[31] C.P. Liu, The relation between the kink-type solution and the kink-bell-type solution of nonlinear evolution equations, Phys. Lett. A 312 (1–2) (2003) 41–48.
[32] Z.Y. Yan, K.W. Chow, B.A. Malomed, Exact stationary wave patterns in three coupled nonlinear Schrodinger/Gross-Pitaevskii equation, Chaos Solitons Fract. 42 (2009) 3013–3019.
[33] A. Jabbari, H. Kheiri, A. Bekir, Exact solutions of the coupled Higgs equation and the Maccari system using He's semi-inverse method and (G'/G)- expansion method, Comput. Math. Appl. 62 (2011) 2177–2186.
[34] M.F. El-Sabbagh, S.I. El-Ganaini, New exact travelling wave solutions of the generalized Zakharov system via distinct methods, Int. Math. Forum 7 (2012) 2191–2204.
[35] S.I. El-Ganaini, New Soliton Solutions of Some Important Nonlinear Systems via He's Variational Principle, Int. J. Math. Anal. 6 (2012) 2277–2286.