Journal of the Egyptian Mathematical Society 25 (2017) 1–7

Contents lists available at ScienceDirect

Journal of the Egyptian Mathematical Society

journal homepage: www.elsevier.com/locate/joems

Original Article

Reduced differential transform method for nonlinear integral member of Kadomtsev–Petviashvili hierarchy differential equations

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ARTICLE INFO

Article history:
Received 10 November 2015
Revised 8 March 2016
Accepted 24 April 2016
Available online 15 September 2016

MSC:
14F35
26A33
34A08

Keywords:
Reduced differential transforms method
Partial differential equations
Integral KP hierarchy equation
Analytic approximate solutions

ABSTRACT

The main objective of this paper is to use the reduced differential to transform method (RDTM) for finding the analytical approximate solutions of two integral members of nonlinear Kadomtsev–Petviashvili (KP) hierarchy equations. Comparing the approximate solutions which obtained by RDTM with the exact solutions to show that the RDTM is quite accurate, reliable and can be applied for many other nonlinear partial differential equations. The RDTM produces a solution with few and easy computation. This method is a simple and efficient method for solving the nonlinear partial differential equations. The analysis shows that our analytical approximate solutions converge very rapidly to the exact solutions.

1. Introduction

Nonlinear partial differential equations (NLPDEs) are mathematical models that are used to describe complex phenomena arising in the world around us. The nonlinear equations appear in many applications of science and engineering such as fluid dynamics, plasma physics, hydrodynamics, solid state physics, optical fibers, acoustics and other disciplines [1]. On the other hand, there are many effective methods for obtaining the anaylatic approximate solutions and exact solutions of NPDEs among of these methods are the inverse scattering method [2], Hirota's bilinear method [3], Backlund transformation [4,5], Painlevé expansion [6] sine–cosine method [7], homogenous balance method [8], homotopy perturbation method [9–12], variation method [13,14], Adomian decomposition method [15,16], tang-function method [17–19], Jacobi elliptic function expansion method [20–23], F-expansion method [24–26] and exp-function method [27–29]. Recently Mahood et al. [30–33] used the optimal homotopy asymptotic to study the MHD slips flow over radiating sheet with heat transfer, the flow heat transfer viscoelastic fluid in an axisymetric channel with a porous wall and for the heat transfer in hollow sphere with the Robin boundary conditions. Also Mahood et al. [34] have discussed the analytical solutions for radiation effects on heat transfer in Blasius flow.

In the present article, we use the reduced differential to transform method (RDTM) which discussed in [35–38], to construct an appropriate solution of some highly nonlinear partial differential equations of mathematical physics. The reduced differential transforms technique is an iterative procedure for obtaining a Taylor series solution of differential equations. This method reduces the size of computational work and easily applicable to many nonlinear physical problems. In this paper, we discuss the analytic approximate solution for two members of the KP hierarchy were formally derived. Two members of the generalized KP hierarchy are given in the following form [39,40]

$$v_t = \frac{1}{2}v_{xxy} + \frac{1}{2}\partial_x^{-2}[v_{yyy}] + 2v_x\partial_x^{-1}[v_y] + 4vv_y, \tag{1}$$

and

$$v_t = \frac{1}{16}v_{xxxxx} + \frac{5}{4}\partial_x^{-1}[vv_{yy}] + \frac{5}{4}\partial_x^{-1}[v_y^2] + \frac{5}{16}\partial_x^{-3}v_{yyyy} + \frac{5}{4}v_x\partial_x^{-2}v_{yy} + \frac{5}{2}v\partial_x^{-1}[v_{yy}] + \frac{5}{2}v_y\partial_x^{-1}[v_y] + \frac{15}{2}v^2v_x + \frac{5}{2}v_xv_{xx} + \frac{5}{4}vv_{xxx} + \frac{5}{8}v_{xyy} \tag{2}$$

where $\partial_x^{-1} = \int_0^x dx$.

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http://dx.doi.org/10.1016/j.joems.2016.04.007

The two members of the hierarchy (1) and (2) were derived in [39] and [40]. The paper has been organized as follows. Notations and basic definitions are given in Section 2. In Section 3, we apply the RDTM to solve two types of NLPDEs. Conclusions are given in Section 4.

2. Preliminaries and notations

In this section, we give some basic definitions and properties of the reduced differential transform method which are further used in this paper. Consider a function of three variables $u(x, y, t)$ and suppose that it can be represented as a product of three single-variable functions, i.e., $u(x, y, t) = f(x)h(y)g(t)$. Based on the properties the $(2+1)$ of -dimensional differential transform, the function $u(x, y, t)$ can be represented as follows:

$$u(x,y,t) = \left(\sum_{i=0}^{\infty} F(i)x^i\right)\left(\sum_{j=0}^{\infty} H(j)y^j\right)\left(\sum_{l=0}^{\infty} G(l)t^l\right) = \sum_{k=0}^{\infty} U_k(x,y)t^k \tag{3}$$

where $U_k(x, y)$ is called t-dimensional spectrum function of *u(x, y,t).* The basic definitions of RDTM are introduced as follows [35–38].

Definition 2.1. [35–38] If the function $u(x, y, t)$ is analytic and differentiated continuously with respect to time t and space in the domain of interest, then let

$$U_k(x,y) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,y,t)\right]_{t=0}, \tag{4}$$

where the $t-$ dimensional spectrum function $U_k(x, y)$, is the transformed function. In this paper the lowercase $u(x, y, t)$ represents the original function while the uppercase $U_k(x, y)$ stands for the transform function.

Definition 2.2. [35–38]. The differential inverse transform $U_k(x, y)$ is defined as follows

$$u(x,y,t) = \sum_{k=0}^{\infty} U_k(x,y)\,t^k. \tag{5}$$

Then, combining Eqs. (4) and (5) we have

$$u(x,y,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,y,t)}{\partial t^k}\right]_{t=0} t^k. \tag{6}$$

From the above definitions, it can be found that the concept of the RDTM is derived from the power series expansion. To illustrate the basic concepts of the RDTM, consider the following nonlinear partial differential equation written in an operator form

$$L[u(x,y,t)] + R[u(x,y,t)] + N[u(x,y,t)] = g(x,y,t), \tag{7}$$

with initial condition

$$u(x,y,0) = f(x,y), \tag{8}$$

where $L = \frac{\partial}{\partial t}$, R is a linear operator which has partial derivatives, N is a nonlinear operator and $g(x, y, t)$ is an inhomogeneous term. According to the RDTM, we can construct the following iteration formula:

$$(k+1)U_{k+1}(x,y,t) = G_k(x,y) - R[U_k(x,y)] - N[U_k(x,y)] \tag{9}$$

where $U_k(x, y)$, $R[U_k(x, y)]$,$N[U_k(x, y)]$ and $G_k(x, y)$ are the transformations of the functions $u(x, y, t)$, $R[u(x, y, t)]$,$N[u(x, y, t)]$ and $g(x, y, t)$ respectively. From the initial condition (8), we write

$$U_0(x,y) = f(x,y). \tag{10}$$

Table 1
The fundamental operations of RDTM.

Functional form	Transformed form
$u(x,t)$	$\frac{1}{k!}[\frac{\partial^k}{\partial t^k}u(x,y,t)]_{t=0}$,
$w(x,t) = u(x,t) \pm v(x,t)$	$W_k(x) = U_k(x) \pm V_k(x)$
$w(x,t) = \alpha u(x,t)$	$W_k(x) = \alpha U_k(x,t)$ (α is a constant)
$w(x,t) = x^m t^n$	$W_k(x) = x^m\delta(k-n)$, $\delta(k) = \{\begin{matrix}1k=0\\0k\neq 0\end{matrix}$
$w(x,t) = x^m t^n u(x,t)$	$W_k(x) = x^m U(k-n)$
$w(x,t) = u(x,t)v(x,t)$	$W_k(x) = \sum_{r=0}^{k} U_r(x)V_{k-r}(x) = \sum_{r=0}^{k} V_r(x)W_{k-r}(x)$
$w(x,t) = \frac{\partial^r}{\partial t^r}u(x,t)$	$W_k(x) = (k+1)...(k+r)W_{k+r}(x) = \frac{(k+r)!}{k!}W_{k+r}(x)$
$w(x,t) = \frac{\partial}{\partial x}u(x,t)$	$W_k(x) = \frac{\partial}{\partial x}U_k(x)$
$w(x,t) = \frac{\partial^2}{\partial x^2}u(x,t)$	$W_k(x) = \frac{\partial^2}{\partial x^2}U_k(x)$.

Substituting (10) into (9) and by straightforward iterative calculation, we get the following $U_k(x, y)$values. Then, the inverse transformation of the set of $U_k(x, y)$, $k = 1, 2, 3, \ldots$ is giving the n- terms approximation solution as follows

$$u_n(x,y,t) = \sum_{k=0}^{n} U_k(x,y)\,t^k. \tag{11}$$

Therefore, the exact solution of the problem is given by

$$u(x,y,t) = \lim_{n\to\infty} u_n(x,y,t). \tag{12}$$

The fundamental mathematical operations performed by RDTM can be readily obtained and are listed in Table 1 [35–38].

3. Numerical results

To demonstrate the effectiveness of the reduced differential transform method (RDTM) algorithm this discussed in the above section. We use this method to construct the analytic approximate solutions for integral member of the generalized KP hierarchy differential equations which have a great attention by many researchers in physics and engineering. The results have been provided by software packages such as Mathematica 9.

Example 3.1. We first consider the KP hierarchy (1) reads:

$$v_t = \frac{1}{2}v_{xxy} + \frac{1}{2}\partial_x^{-2}[v_{yyy}] + 2v_x\partial_x^{-1}[v_y] + 4vv_y. \tag{13}$$

Wazwaz et al have studied the n soliton solution and distinct dispersion of the Kadomtsev–Petviashvili hierarchy in [39, 40]. To get rid of the one and two folds integral operators, we use the transformation

$$v(x,y,t) = u_{xx}(x,y,t), \tag{14}$$

that carries (13) to

$$u_{xxt} = \frac{1}{2}u_{xxxxy} + \frac{1}{2}u_{yyy} + 2u_{xxx}u_{xy} + 4u_{xx}u_{xxy}, \tag{15}$$

with the initial condition

$$u(x,y,0) = Ln(1+e^{kx+ry}), \tag{16}$$

Or

$$v(x,y,0) = \frac{k^2e^{kx+ry}}{(1+e^{kx+ry})^2}, \tag{17}$$

where k and r are an arbitrary constant.

Applying the reduced differential transform to the Eq. (13), we obtain the following iteration relation,

$$(k+1)V_{k+1}(x,y) = \frac{1}{2}\frac{\partial^3 V_k(x,y)}{\partial x^2\partial y} + \frac{1}{2}\partial_x^{-2}\left[\frac{\partial^3 V_k(x,y)}{\partial y^3}\right] + 2B_k(x,y) + 4C_k(x,y), \tag{18}$$

where

$$B_k(x,y) = \sum_{j=0}^{k} \frac{\partial V_j}{\partial x}\left[\partial_x^{-1}\left(\frac{\partial V_{k-j}}{\partial y}\right)\right], \tag{19}$$

and

$$C_k(x,y) = \sum_{j=0}^{k} V_i\left[\frac{\partial V_{k-j}}{\partial y}\right]. \tag{20}$$

Using the initial condition (17), we have

$$V_0(x,y) = \frac{k^2 e^{kx+ry}}{[1+e^{kx+ry}]^2}. \tag{21}$$

Now, substituting (21) into (18)–(20), we obtain the following $V_{k+1}(x,y)$ values successively as follows

$$\begin{aligned}
V_1(x,y) &= -\frac{r(k^4+r^2)e^{kx+ry}(-1+e^{kx+ry})}{2(1+e^{kx+ry})^3},\\
V_2(x,y) &= -\frac{r^2(k^4+r^2)^2 e^{kx+ry}(4e^{kx+ry}-e^{2kx+2ry}-1)}{8k^2(1+e^{kx+ry})^4},\\
V_3(x,y) &= -\frac{r^3(k^4+r^2)^3 e^{kx+ry}(11e^{kx+ry}-11e^{2kx+2ry}+e^{3kx+3ry}-1)}{48k^4(1+e^{kx+ry})^5},\\
V_4(x,y) &= -\frac{r^4(k^4+r^2)^4 e^{kx+ry}(26e^{kx+ry}-66e^{2kx+2ry}+26e^{3kx+3ry}-e^{4kx+4ry}-1)}{384k^6(1+e^{kx+ry})^6},\\
V_5(x,y) &= -\frac{r^5(k^4+r^2)^5 e^{kx+ry}(57e^{kx+ry}-302e^{2kx+2ry}+302e^{3kx+3ry}-57e^{4kx+4ry}+e^{5kx+5ry}-1)}{3840k^8(1+e^{kx+ry})^7},\\
V_6 &= -\frac{r^6(k^4+r^2)^6 e^{kx+ry}(120e^{kx+ry}-1191e^{2kx+2ry}+2416e^{3kx+3ry}-1191e^{4kx+4ry}+120e^{5kx+5ry}-e^{6kx+6ry}-1)}{46080k^{10}(1+e^{kx+ry})^8},
\end{aligned} \tag{22}$$

and so on. In the same manner, the rest of components can be obtained by using Mathematica software. Taking the inverse transformation of the set of values $[V_k(x,y)]_{k=0}^{n}$ gives n-terms approximation solutions. Finally the differential inverse transform of $V_k(x,y)$ give

$$\begin{aligned}
v_n(x,y,t) &= \sum_{k=0}^{n} V_k(x,y)t^n = V_0+V_1t+V_2t^2+V_3t^3+V_4t^4\\
&\quad +V_5t^5+V_6t^6+\ldots.\\
&= \frac{k^2e^{kx+ry}}{[1+e^{kx+ry}]^2} - \frac{r(k^4+r^2)e^{kx+ry}(-1+e^{kx+ry})}{2(1+e^{kx+ry})^3}t\\
&\quad -\frac{r^2(k^4+r^2)^2 e^{kx+ry}(4e^{kx+ry}-e^{2kx+2ry}-1)}{8k^2(1+e^{kx+ry})^4}t^2\\
&\quad -\frac{r^3(k^4+r^2)^3 e^{kx+ry}(11e^{kx+ry}-11e^{2kx+2ry}+e^{3kx+3ry}-1)}{48k^4(1+e^{kx+ry})^5}t^3\\
&\quad -\frac{r^4(k^4+r^2)^4 e^{kx+ry}(26e^{kx+ry}-66e^{2kx+2ry}+26e^{3kx+3ry}-e^{4kx+4ry}-1)}{384k^6(1+e^{kx+ry})^6}t^4\\
&\quad -\frac{r^5(k^4+r^2)^5 e^{kx+ry}(57e^{kx+ry}-302e^{2kx+2ry}+302e^{3kx+3ry}-57e^{4kx+4ry}+e^{5kx+5ry}-1)}{3840k^8(1+e^{kx+ry})^7}t^5\\
&\quad -\frac{r^6(k^4+r^2)^6 e^{kx+ry}(120e^{kx+ry}-1191e^{2kx+2ry}+2416e^{3kx+3ry}-1191e^{4kx+4ry}+120e^{5kx+5ry}-e^{6kx+6ry}-1)}{46080k^{10}(1+e^{kx+ry})^8}t^6+\ldots
\end{aligned} \tag{23}$$

Table 2
The exact solution (24) is compared with the approximate solution (23) for the k =0.5, r= 0.1.

t	x	y	v_5	Exact solution	$\lvert v_5 - Exact\rvert$
0.5	0.2	0.2	0.062313877	0.062313877	4.8572×10^{-17}
	0.4	0.4	0.061807863	0.061807863	6.9389×10^{-17}
	0.6	0.6	0.060991518	0.060991518	4.8572×10^{-17}
	0.8	0.8	0.059881321	0.059881321	6.9389×10^{-18}
	1	1.	0.058499153	0.058499153	6.9389×10^{-18}

Therefore, the exact solution of the problem is readily obtained as follows

$$\begin{aligned}
v(x,y,t) &= \lim_{n\to\infty} v_n(x,y,t)\\
&= \frac{k^2 \exp[\frac{1}{2k^2}(2k^3x+2rk^2y+rt[k^4+r^2])]}{\{1+\exp[\frac{1}{2k^2}(2k^3x+2rk^2y+rt[k^4+r^2])]\}^2}.
\end{aligned} \tag{24}$$

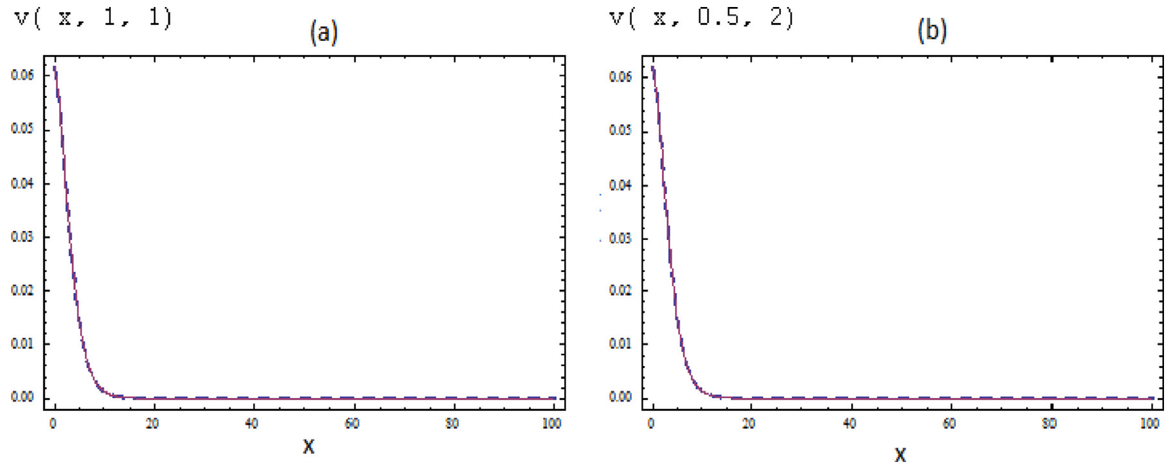

Fig. 1. (red color) The exact solution (24) is compared with the (blue color) approximate solution (23) for the k=0.5, r=0.2 when (a) y=1, t=1 and (b) y=0.5 and t=2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

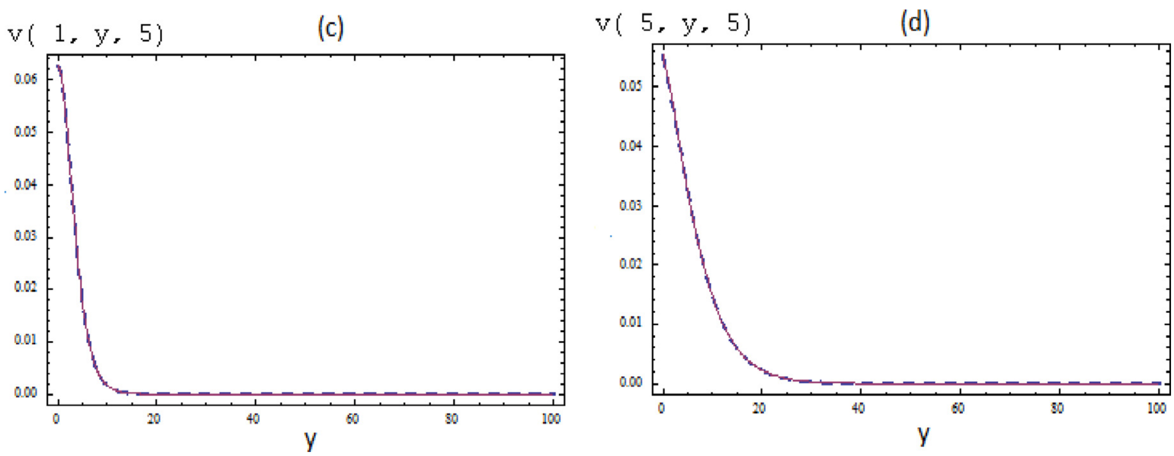

Fig. 2. (red color) The exact solution (24) is compared with the (blue color) approximate solution (23) for the k=0.5, r=0.2 when (c) x=1, t=5 and (d) x=5 and t=5. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

To examine the accuracy of the RDTM solution, the absolute errors of the 7-terms approximate solution is listed in Table 2 and plotted in Figs. 1–3.

From Figs. 1–3 and Table 2 the approximate solution is rapidly convergence to the exact solution.

Example 3.2. Consider the second integral nonlinear KP equation [39,40]:

$$v_t = \frac{1}{16}v_{xxxxx} + \frac{5}{4}\partial_x^{-1}(vv_{yy}) + \frac{5}{4}\partial_x^{-1}(v_y)^2 + \frac{5}{16}\partial_x^{-3}v_{yyyy} + \frac{5}{4}v_x\partial_x^{-2}v_{yy} + \frac{5}{2}v\partial_x^{-1}v_{yy} + \frac{5}{2}v_y\partial_x^{-1}v_y + \frac{15}{2}v^2v_x + \frac{5}{2}v_xv_{xx} + \frac{5}{4}vv_{xxx} + \frac{5}{8}v_{xyy}. \quad (25)$$

Notice that the equation contains one, two, and three, fold integral operators. However, differentiating both sides of (25) with respect to x, and using the transformation

$$v(x, y, t) = u_{xx}(x, y, t), \quad (26)$$

Consequently, we have

$$u_{xxxt} = \frac{1}{16}u_{8x} + \frac{15}{4}u_{xx}u_{xxyy} + \frac{15}{4}(u_{xxy})^2 + \frac{5}{16}u_{yyyy} + \frac{15}{4}u_{xxx}v_{xyy} + \frac{5}{4}u_{xxxx}u_{yy} + \frac{5}{2}u_{xxxy}u_{xy} + \frac{15}{2}(u^2{}_{xx}u_{xxx})_x + \frac{5}{2}(u_{xxx}u_{xxxx})_x + \frac{5}{4}(u_{xx}u_{xxxxx})_x + \frac{5}{8}u_{xxxxyy}. \quad (27)$$

with the initial condition

$$u(x, y, 0) = Ln(1 + e^{kx+ry}), \quad (28)$$

Or

$$v(x, y, 0) = \frac{k^2e^{kx+ry}}{(1 + e^{kx+ry})^2}, \quad (29)$$

where k and r are an arbitrary constant. Applying the reduced differential transform to the Eq. (25), we obtain the following iteration relation,

$$(k+1)V_{k+1} = \frac{1}{16}\frac{\partial^5V_k}{\partial x^5} + \frac{5}{4}\partial_x^{-1}[A_k(x, y)] + \frac{5}{4}\partial_x^{-1}[B_k(x, y)] + \frac{5}{16}\partial_x^{-3}\left[\frac{\partial^3V_k}{\partial y^3}\right] + \frac{5}{4}C_k(x, y) + \frac{5}{2}D_k(x, y) + \frac{5}{2}H_k(x, y) + \frac{15}{2}S_k(x, y) + \frac{5}{2}L_k(x, y) + \frac{5}{4}M_k(x, y) + \frac{5}{8}\frac{\partial^3V_k}{\partial x\partial y^2} \quad (30)$$

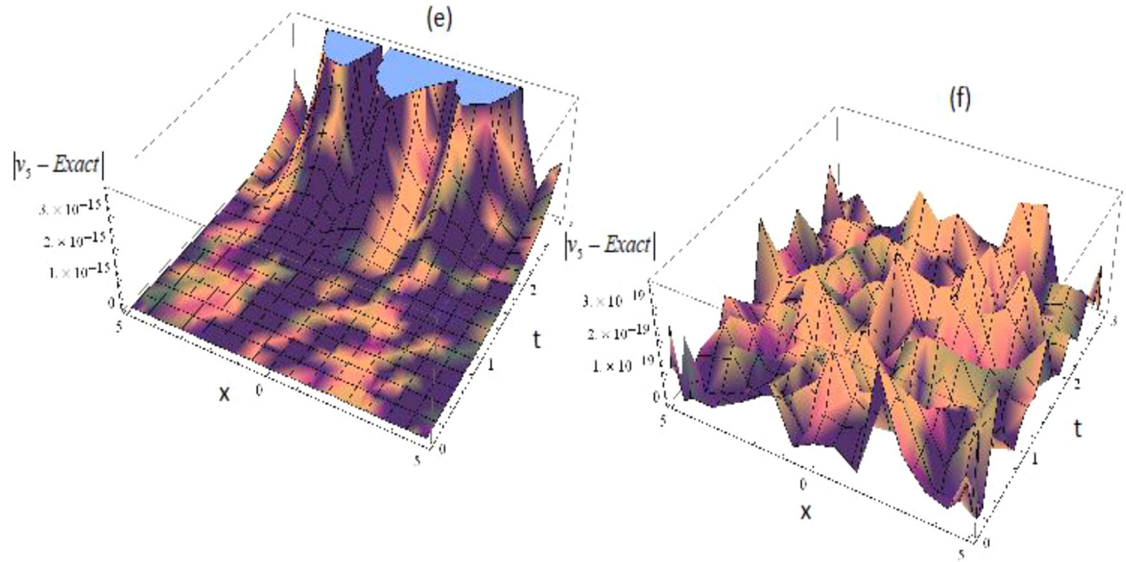

Fig. 3. The absolute error of u_5 and exact solution of (24) at (e) k = 0.5, r = 0.05, y = 1 and (f) k = 0.05, r = 0.01, y = 1.

and

$$A_k(x,y)=\sum_{i=0}^{k} V_i\partial_{yy}^2(V_{k-i});\quad B_k(x,y)=\sum_{i=0}^{k}\partial_y(V_i)\ \partial_y(V_{k-i}),$$
$$C_k(x,y)=\sum_{i=0}^{k}\partial_x(V_i)\ \partial_x^{-2}[\partial_{yy}^2 V_{k-i}],\quad D_k(x,y)=\sum_{i=0}^{k}V_i\ \partial_x^{-1}[\partial_{yy}^2 V_{k-i}],$$
$$H_k(x,y)=\sum_{i=0}^{k}\partial_y(V_i)\ \partial_x^{-1}[\partial_y V_{k-i}],\ S_k(x,y)=\sum_{i=0}^{k}[\sum_{j=0}^{i}V_jV_{i-j}\ \partial_x(V_{k-i})],$$
$$L_k(x,y)=\sum_{i=0}^{k}\partial_x(V_i)\ \partial_{xx}(V_{k-i}),\quad M_k(x,y)=\sum_{i=0}^{k}V_i\ \partial_{xxx}(V_{k-i}). \tag{31}$$

We can assume the initial condition has the following form:

$$V_0(x,y)=\frac{k^2e^{kx+ry}}{(1+e^{kx+ry})^2} \tag{32}$$

Now, substituting (32) into (30)–(31), we obtain the following $V_{k+1}(x,y)$ values successively as follows:

$$V_1(x,y)=-\frac{(k^8+5r^2+10k^4r^2)e^{kx+ry}(-1+e^{kx+ry})}{16k(1+e^{kx+ry})^3},$$
$$V_2(x,y)=-\frac{(k^8+5r^2+10k^4r^2)^2e^{kx+ry}(-1+4e^{kx+ry}-e^{2kx+2ry})}{512k^4(1+e^{kx+ry})^4},$$
$$V_3(x,y)=-\frac{(k^8+5r^2+10k^4r^2)^3e^{kx+ry}(-1+11e^{kx+ry}-11e^{2kx+2ry}+e^{3kx+3ry})}{24576k^7(1+e^{kx+ry})^5},$$
$$V_4(x,y)=-\frac{(k^8+5r^2+10k^4r^2)^4e^{kx+ry}(-1+26e^{kx+ry}-66e^{2kx+2ry}+26e^{3kx+3ry}-e^{4kx+4ry})}{1572864k^{10}(1+e^{kx+ry})^6}, \tag{33}$$

and so on. We calculate the 5th iteration, but write the five first terms for convenience to the reader. In the same manner, the rest of components can be obtained by using Mathematica software. Taking the inverse transformation of the set of values $[V_k(x,y)]_{k=0}^{n}$ gives n-terms approximation solutions. Finally the differential inverse transform of $V_k(x,y)$ give

$$\begin{aligned}v_n(x,y,t) &= \sum_{k=0}^{n}V_k(x,y)t^n=V_0+V_1t+V_2t^2+V_3t^3+V_4t^4\\&\quad+V_5t^5+V_6t^6+V_7t^7+\ldots\\&=\frac{k^2e^{kx+ry}}{(1+e^{kx+ry})^2}-\frac{(k^8+5r^2+10k^4r^2)e^{kx+ry}(-1+e^{kx+ry})t}{16k(1+e^{kx+ry})^3}\\&\quad-\frac{(k^8+5r^2+10k^4r^2)^2e^{kx+ry}(-1+4e^{kx+ry}-e^{2kx+2ry})}{512k^4(1+e^{kx+ry})^4}t^2\\&\quad-\frac{(k^8+5r^2+10k^4r^2)^3e^{kx+ry}(-1+11e^{kx+ry}-11e^{2kx+2ry}+e^{3kx+3ry})}{24576k^7(1+e^{kx+ry})^5}t^3+\ldots\end{aligned} \tag{34}$$

Therefore, the exact solution of the problem is readily obtained as follows

$$\begin{aligned}v(x,y,t)&=\underset{n\to\infty}{lin}\ v_n(x,y,t)\\&=\frac{k^2\exp[\frac{1}{16k^3}(16k^4x+16rk^3y+k^8t+5r^4t+10tr^2k^4)]}{1+\exp[\frac{1}{16k^3}(16k^4x+16rk^3y+k^8t+5r^4t+10tr^2k^4)]}.\end{aligned} \tag{35}$$

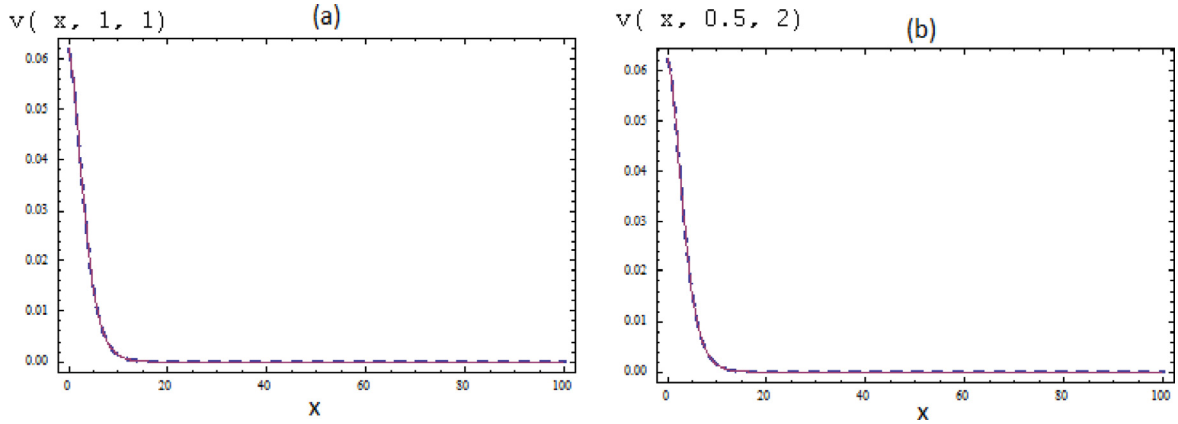

Fig. 4. (red color) The exact solution (35) is compared with the (blue color) approximate solution (34) for the k=0.5, r=0.2 when (a) y=1, t=1 and (b) y=0.5 and t=2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

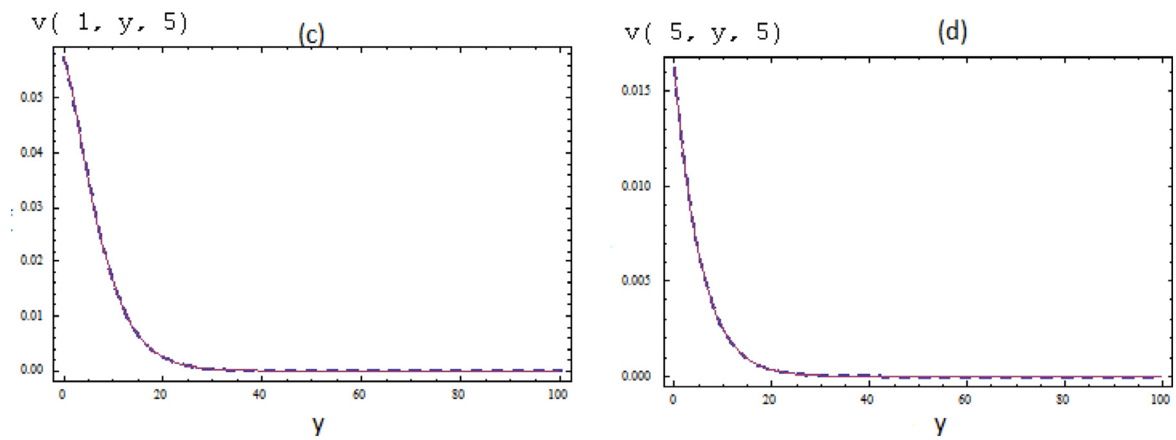

Fig. 5. (red color) The exact solution (35) is compared with the (blue color) approximate solution (34) for the k=0.5, r=0.2 when (c) x=1, t=5 and (d) x=5 and t=5. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

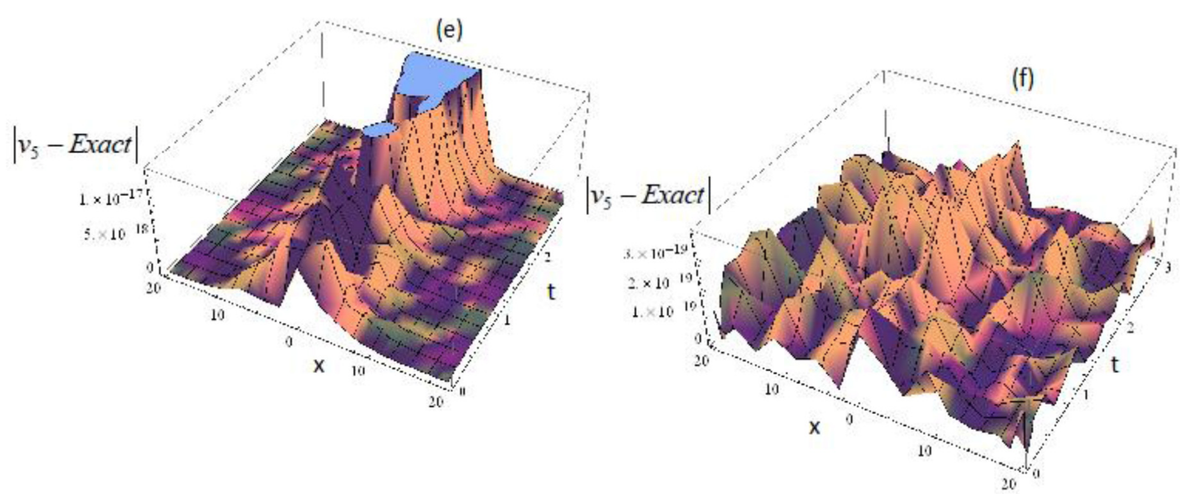

Fig. 6. The absolute error of u_5 in (35) and exact solution of (30) at (e) k=0.5, r=0.05, y=1 and (f) k=0.05, r=0.01, y=1.

To examine the accuracy of the RDTM solution, the absolute errors of the 5-terms approximate solution is listed in Table 3 and plotted in Figs. 4–6.

From Figs. 4–6 and Table 3 the approximate solution is rapidly convergence to the exact solution.

Table 3
The exact solution (35) is compared with the approximate solution (34) for the k =0.55, r= 0.22.

t	x	Y	v_5	Exact solution	$\lvert v_5 - Exact\rvert$
0.5	0.2	0.2	0.075323303	0.075323303	1.3461×10^{-15}
	0.4	0.4	0.074538419	0.074538419	1.1102×10^{-15}
	0.6	0.6	0.073287752	0.073287752	8.3267×10^{-16}
	0.8	0.8	0.071602789	0.071602789	4.9964×10^{-16}
	1	1	0.069524675	0.069524675	1.5266×10^{-16}

4. Conclusions

In this work, we present new applications of the reduced differential transform method (RDTM) by handling two nonlinear physical models, namely, generalized KP hierarchy equations. This method is an alternative approach to overcome the demerit of complex calculation of differential transform method (DTM). The proposed technique, which does not require linearization, discretization or perturbation, gives the solution in the form of a convergent power series with elegantly computed components. Therefore, the solution procedure of the RDTM is simpler than other traditional methods. The main advantage of the proposed method is that it requires less amount of computation. The results show that the RDTM is a powerful mathematical tool for handling nonlinear PDEs. The approximate solutions are rabidly convergence to the exact solutions. Our results show that RDTM can be applied to many complicated and strongly nonlinear partial differential equations.

Acknowledgments

The authors express our sincere thanks to the referees for their valuable suggestions and comments.

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