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ON NRBUL CLASS OF LIFE DISTRIBUTIONS

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Abstract

In this paper, moment inequalities for a new class of life distributions called new renewal better than used in Laplace transform order ($NRBUL$) are proposed. For the new class $NRBUL$, the preservation under convolution and mixture are studied. A new test statistic for testing exponentiality versus $NRBUL$ is investigated based on these moment inequalities. Pitman asymptotic efficiencies of the test are proposed. The critical values of this test are tabulated. Some examples for censored and non-censored data are applied to the new test. Finally a new test for censored data is proposed.

Keywords: Convolution, Mixture, $NRBUL$,U-Statistic, Asymptotic Efficiency.
AMS Subject classification: 60K10,62E10,62N05.

1 Introduction

Certain classes of life distributions and their variations have been introduced in reliability theory, the applications of these classes of life distributions can be seen in engineering, social, biological science, maintenance and biometrics. Many statistician and reliability analysts proposed testing exponentiality versus some classes of life distribution. [1] studied the relation among the classes $NRBU, RNBU, NRBUE$ and $HNRBUE$. [2] studied the moment inequality for the $NRBU$ class. [3] proposed the U-statistic method for $RNBU$ class. [4] proposed moment inequality for the $RNBRU$ class. [5] studied the class $NBRUE$ based on Laplace transform.
The theme of this paper is to introduce a new class of life distributions, which is strictly larger than the new renewal better than used ($NRBU$) class, called new renewal better than used in Laplace transform order ($NRBUL$). Some properties of this class are studied and a test statistics for testing exponentiality versus this class is also proposed in cases of complete and right censored data.

1.1 Motivation and Definitions

Renewal survival function
Consider a component with life time X with distribution function $F(x)$, is put in operation. When the failure occurs, the component will be replaced by a sequence of mutually and identically components which are independent of the first component. In the long time, the remaining life distribution of a component in operation at time t is called the stationary renewal distribution. The corresponding renewal survival function is: $\bar{W}(t) = \frac{1}{\mu}\int_t^\infty \bar{F}(u)du$. where $\mu = \int_0^\infty \bar{F}(x)dx$. Now, we introduce the definitions of some of classes of life distributions.

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Definition 1.1. A non-negative random variable X with survival function $\bar{F}(x)$ is new renewal better (worse) than used $NRBU(NRWU)$ if:

$$\bar{F}(x+t) \leq (\geq)\bar{W}(t)\bar{F}(x).$$

Definition 1.2. A non-negative random variable X with survival function $\bar{F}(x)$ is renewal new better (worse) than used $RNBU(RNWU)$ if:

$$\bar{W}(x+t) \leq (\geq)\bar{W}(x)\bar{W}(t).$$

Definition 1.3. A non-negative random variable X with survival function $\bar{F}(x)$ is new renewal better (worse) than used in expectation $NRBUE(NRWUE)$ if:

$$\int_x^\infty \bar{F}(u)du \leq (\geq)\mu_w \bar{F}(x),$$

where

$$\mu_w = \int_0^\infty \bar{W}(u)du.$$

Definition 1.4. A non-negative random variable X with survival function $\bar{F}(x)$ is harmonic new renewal better (worse) than used in expectation $HNRBUE(HNRWUE)$ if:

$$\int_x^\infty \bar{W}(u)du \leq (\geq)\mu_w e^{-x/\mu_w}.$$

Definition 1.5. A non-negative random variable X with survival function $\bar{F}(x)$ is new renewal better (worse) than used in Laplace transform order $NRBUL(NRWUL)$ if:

$$\int_0^\infty e^{-sx}\bar{F}(x+t)dx \leq (\geq)\bar{W}(t)\int_0^\infty e^{-sx}\bar{F}(x)dx \quad (1.1)$$

In this paper, the properties of preservation for the $NRBUL$ class are introduced in Section 2. In Section 3, the moment inequalities for the $NRBUL$ class are derived. In Section 4, we test exponentiality versus $NRBUL$ class. In Section 5, Pitman asymptotic efficiencies (PAE) of our test are considered. In Section 6, critical values for the lower and upper percentiles of our test are calculated. In Section 7 our test is applied to sets of real examples. Finally, testing for censored data is developed in Section 8.

2 Some Properties of the $NRBUL$ *Class*

In this section some properties of $NRBUL$ class are discussed under convolution and mixture.

Theorem 2.1. *The $NRBUL$ class is preserved under convolution.*

Proof. The convolution of the two independent distribution functions F_1 and F_2 for the $NRBUL$ class is given by:

$$\bar{F}(z) = \int_0^\infty \bar{F}_1(z-u)dF_2(u),$$

so

$$\begin{aligned}\int_0^\infty e^{-sx}\bar{F}(x+t)dx &= \int_0^\infty \int_0^\infty e^{-sx}\bar{F}_1(x+t-u)dF_2(u)dx \\ &= \int_0^\infty \int_0^\infty e^{-sx}\bar{F}_1(x+t-u)dxdF_2(u).\end{aligned}$$

Since F_1 is $NRBUL$ then

$$\int_0^\infty e^{-sx}\bar{F}(x+t)dx \leq \int_0^\infty \int_0^\infty e^{-sx}\bar{W}(t)\bar{F}_1(x-u)dxdF_2(u),$$

and this leads to

$$\int_0^\infty e^{-sx}\bar{F}(x+t)dx \leq \bar{W}(t)\int_0^\infty e^{-sx}\bar{F}(x)dx,$$

which completes the proof. □

The following theorem is presented to show that the $NRBUL$ class is preserved under mixture.

Theorem 2.2. *The $NRBUL$ class is preserved under mixture.*

Proof. If F_α be a set of probability distributions, where the index α is governed by the distribution G, then the mixture F of F_α is

$$\bar{F}(x) = \int_{-\infty}^{\infty} \bar{F}_\alpha(x) dG(\alpha)$$

If F_α is $NRBUL$, then satisfies

$$\begin{aligned}\int_0^\infty e^{-sx}\bar{F}(x+t)dx &= \int_0^\infty \int_0^\infty e^{-sx}\bar{F}_\alpha(x+t)dG(\alpha)dx,\\ &= \int_0^\infty \int_0^\infty e^{-sx}\bar{F}_\alpha(x+t)dxdG(\alpha).\end{aligned}$$

Since F_α is $NRBUL$, then

$$\int_0^\infty \int_0^\infty e^{-sx}\bar{F}_\alpha(x+t)dxdG(\alpha) \leq \int_0^\infty \int_0^\infty e^{-sx}\bar{W}_\alpha(t)\bar{F}_\alpha(x)dxdG(\alpha).$$

Upon using Chebyschev inequality we get,

$$\int_0^\infty \int_0^\infty e^{-sx}\bar{F}_\alpha(x+t)dxdG(\alpha) \leq \int_0^\infty e^{-sx}\left\{\int_0^\infty \bar{W}_\alpha(t)dG(\alpha)\int_0^\infty \bar{F}_\alpha(x)dG(\alpha)\right\}dx,$$

and this leads to

$$\int_0^\infty e^{-sx}\bar{F}(x+t)dx \leq \bar{W}(t)\int_0^\infty e^{-sx}\bar{F}(x)dx,$$

which completes the proof. □

3 Moment Inequalities

In this section, the moment inequalities for the $NRBUL$ class are derived and all moments are assumed to be exist and finite.

Theorem 3.1. *If F is $NRBUL$, then for all integer $r \geq 0$*

$$\frac{\mu_{r+2}(1-\gamma(s))}{s\mu(r+1)(r+2)} \geq \frac{(-1)^{r+1}r!(1-\gamma(s))}{s^{r+2}} + \frac{r!}{s^{r+1}}\sum_{k=0}^{r}\frac{(-1)^k s^{r-k}\mu_{r-k+1}}{(r-k)!(r-k+1)}, \tag{3.1}$$

where $\gamma(s) = E(e^{-sX}) = \int_0^\infty e^{-sx}dF(x)$.

Proof. Since F is $NRBUL$, then

$$\int_0^\infty e^{-sx}\bar{F}(x)dx \int_0^\infty t^r\bar{W}(t)dt \geq \int_0^\infty \int_0^\infty t^r e^{-sx}\bar{F}(x+t)dxdt. \tag{3.2}$$

Since

$$\int_0^\infty e^{-sx}\bar{F}(x)dx = E\int_0^\infty e^{-sx}I(X>x)dx = \frac{1}{s}(1-\gamma(s)),$$

and

$$\int_0^\infty t^r\bar{W}(t)dt = \frac{1}{\mu}\int_0^\infty t^r\int_t^\infty \bar{F}(u)dudt = \frac{\mu_{(r+2)}}{\mu(r+1)(r+2)},$$

then left hand side of (3.2) is

$$\frac{\mu_{(r+2)}(1-\gamma(s))}{s\mu(r+1)(r+2)}. \tag{3.3}$$

The right hand side of (3.2) is equal to (see [6])

$$\frac{(-1)^{r+1}r!(1-\gamma(s))}{s^{r+2}}+\frac{r!}{s^{r+1}}\sum_{k=0}^{r}\frac{(-1)^k s^{r-k}\mu_{r-k+1}}{(r-k)!(r-k+1)}. \tag{3.4}$$

The result follows from (3.2), (3.3) and (3.4). □

Corollary 3.2. *Putting $r=1$, in (3.1) we get*

$$\frac{\mu_3(1-\gamma(s))}{s}\geq\frac{6\mu}{s^3}[(1-\gamma(s))+\frac{s^2\mu_2}{2}-s\mu].$$

4 Testing Exponentiality versus $NRBUL$ Class

In this section, we test $H_0 : F$ is exponential distribution versus $H_1 : F$ is $NRBUL$ and not exponential distribution. When $r=1$, we get the following measure of departure

$$\delta=\frac{s^2\mu_3-6\mu-3s^2\mu\mu_2+6s\mu^2+\gamma(s)(6\mu-s^2\mu_3)}{s^3}. \tag{4.1}$$

Note that under $H_0 : \delta = 0$, versus $H_1 : \delta > 0$, the empirical estimate of δ in (4.1) is

$$\hat{\delta}_n=\frac{1}{n^2s^3}\sum_{i=1}^{n}\sum_{j=1}^{n}[s^2X_i^3-6X_i-3s^2X_iX_j^2+6sX_iX_j+(6X_i-s^2X_i^3)e^{-sX_j}]. \tag{4.2}$$

Theorem 4.1. *(i) As $n\to\infty$, $\sqrt{n}\,(\hat{\delta}_n-\delta)$ is asymptotically normal with zero mean and variance σ^2 as given in (4.4).*

(ii) Under H_0 , the variance is reduced to σ_0^2 in (4.5).

Proof. Setting

$$\phi(X_1,X_2)=s^2X_1^3-6X_1-3s^2X_1X_2^2+6sX_1X_2+(6X_1-s^2X_1^3)e^{-sX_2},$$

define

$$\phi(X)=\phi_1(X)+\phi_2(X),$$

where

$$\begin{aligned}\phi_1(X)&=E[\phi(X_1,X_2)\,|X_1]\\&=s^2(X_1^3-6X_1)+\frac{(6X_1-s^2X_1^3)}{1+s}+6X_1(s-1),\end{aligned}$$

and

$$\begin{aligned}\phi_2(X)&=E[\phi(X_2,X_1)\,|X_1]\\&=s^2(-3X_1^2+6)+(6-6s^2)e^{-sX_1}+6(sX_1-1).\end{aligned}$$

Therefore

$$\begin{aligned}\phi(X)=s^2(X_1^3-3X_1^2-6X_1+6)+\frac{(6X_1-s^2X_1^3)}{1+s}\\+(6-6s^2)e^{-sX_1}+12sX_1-6(X_1+1).\end{aligned} \tag{4.3}$$

Using U-statistic theory (see [7]), we get the variance $\sigma^2=Var(\phi(X))$ of $\hat{\delta}_n$ is

$$\sigma^2=Var\left[\begin{aligned}&s^2(X_1^3-3X_1^2-6X_1+6)+\frac{(6X_1-s^2X_1^3)}{1+s}\\&+6(1-s^2)e^{-sX_1}+12sX_1-6(X_1+1)\end{aligned}\right]. \tag{4.4}$$

Under H_0, the variance σ^2 reduces to

$$\sigma_0^2=\frac{72s^8(7+5s)}{(1+s)^4(1+2s)}. \tag{4.5}$$

□

5 The Pitman Asymptotic Efficiency (PAE) of δ

In this section the Pitman asymptotic efficiencies (PAEs) are computed for the Linear failure rate family (LFR), Makeham and Weibull families. The PAE is defined by

$$PAE(\delta) = \frac{1}{\sigma_0} \left| \frac{d\delta}{d\theta} \right|_{\theta \to \theta_0}.$$

Where,

$$\frac{d\delta_\theta}{d\theta} = \frac{1}{s^3} \begin{bmatrix} s^2\mu'_{3\theta} - 6\mu'_\theta - 3s^2(\mu'_\theta\mu_{2\theta} + \mu_\theta\mu'_{2\theta}) + 12s\mu_\theta\mu'_\theta \\ + (6\mu'_\theta - s^2\mu'_{3\theta})\gamma_\theta(s) + (6\mu_\theta - s^2\mu_{3\theta})\gamma'_\theta(s) \end{bmatrix}.$$

Therefore,

$$PAE(\hat{\delta}, LFR) = \frac{1}{\sigma_0} \left| \frac{36 - 36s^2 - 12s^3}{s^2(1+s)^2} \right|,$$

$$PAE(\hat{\delta}, Makeham) = \frac{1}{\sigma_0} \left| \frac{-9s}{8 + 12s + 4s^2} \right|,$$

and

$$PAE(\hat{\delta}, Weibull) = \frac{1}{\sigma_0} \left| \frac{5.07 + 15.22s + 26.29s^2 + 7.14s^3 - 9s^4 - 6s(1-s^2)ln(1+s)}{s^3(1+s)^2} \right|.$$

We compare the Pitman asymptotic efficiencies PAEs at $s = 0.4$ of our test with some other tests. The results are shown in Table 5.1.

Table 5.1 The PAE's for LFR, Makeham and Weibull families

Distribution	**LFR**	**Makeham**	**Weibull**
[**8**]	0.8660	0.2886	1.2007
[**9**]	0.9184	0.2039	1.1316
δ	9.882	1.081	4.527

Table 5.1 shows that our class $NRBUL$ is more efficient for all used alternatives.
Fig. 5.1 shows the relation between s and efficiency of LFR, Makeham and Weibull families.

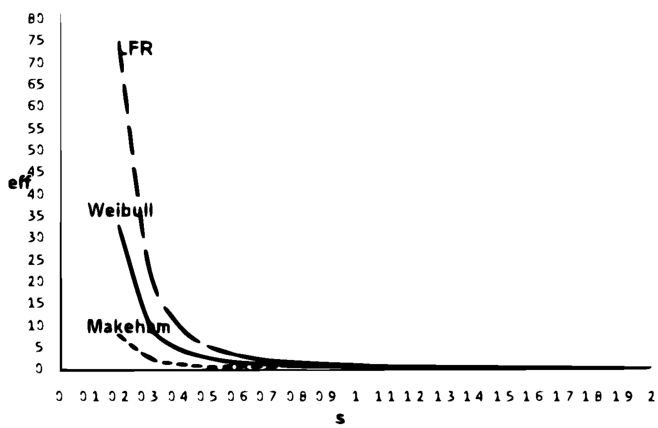

Figure 5.1. The relation between efficiencies and s

In view of Figure 5.1 it is noticed that the PAE's of δ are decreasing as s increasing and the PAE's for the LFR alternative is greater than the PAE's for Makeham and Weibull alternatives.

6 Monte Carlo Null Distribution Critical Points

In this section, we calculate the lower and upper percentiles of $\hat{\delta}_n$ given in (4.2) based on 10000 simulated samples of sizes $n = 25, 27, 30(5), 40, 43, 45(5), 90$. Table (6.1) gives these critical points of statistic $\hat{\delta}_n$ at $s = 0.4$.

Table 6.1 Critical values of statistic $\hat{\delta}_n$ at $s = 0.4$

n	0.02	0.05	0.10	0.90	0.95	0.98
25	-1.172	-0.824	-0.598	0.020	0.218	0.746
27	-1.165	-0.815	-0.567	0.025	0.222	0.785
30	-0.983	-0.708	-0.522	0.055	0.306	0.948
35	-0.867	-0.650	-0.490	0.072	0.307	0.913
40	-0.768	-0.574	-0.441	0.112	0.379	0.934
43	-0.777	-0.566	-0.426	0.117	0.375	0.906
45	-0.740	-0.539	-0.414	0.134	0.382	0.965
50	-0.689	-0.515	-0.394	0.129	0.381	0.906
55	-0.625	-0.480	-0.382	0.148	0.383	0.922
60	-0.602	-0.471	-0.365	0.178	0.406	0.917
65	-0.549	-0.427	-0.341	0.186	0.476	0.938
70	-0.533	-0.412	-0.328	0.192	0.435	0.935
75	-0.518	-0.399	-0.325	0.178	0.444	0.884
80	-0.494	-0.389	-0.312	0.200	0.469	0.970
85	-0.467	-0.375	-0.303	0.200	0.476	0.944
90	-0.466	-0.367	-0.299	0.201	0.463	0.899

7 Applications

In this section, $\hat{\delta}_n$ have been calculated for real examples to illustrate the application of our test.

Example 7.1. The data set of 40 patients suffering from blood cancer (Leukemia) from one of ministry of health hospitals in Saudi Arabia (see [10]). The ordered life times (in years) are

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025
2.036	2.162	2.211	2.370	2.532	2.693	2.805	2.910
2.912	3.192	3.263	3.348	3.348	3.427	3.499	3.534
3.767	3.751	3.858	3.986	4.049	4.244	4.323	4.381
4.392	4.397	4.647	4.753	4.929	4.973	5.074	4.381

It was found that $\hat{\delta}_n = -22.433$ which is less than the tabulated value in Table 6.1. Then we recognize that this data has exponential property.

Example 7.2. Consider the following data set of 27 observations and represent the time of successive failure (in hours) of the air conditioning systems of 7913 jet air planes of a fleet of Boeing 720 jet air planes in [11].

97	51	11	4	141	18	142	68
77	80	1	16	106	206	82	54
31	216	46	111	39	63	18	191
18	163	24					

It was found that $\hat{\delta}_n = -5.3835$, which is less than the critical value in Table 6.1. Then we recognize that this data has exponential property.

Example 7.3. Consider the following data set in Kotz and Johnson and represent the survival times (in years) after diagnosis of 43 patients with a certain kind of leukemia (see [12]). It was found that $\hat{\delta}_n = -12.4805$, which is less than the critical value in Table 6.1. Then we recognize that this data has exponential property.

8 Testing Hypothesis versus $NRBUL$ Alternative for Censored Data.

In this section, we propose a test statistic $\hat{\delta}_n$ for testing exponentiality versus $NRBUL$ class in case of randomly right censored samples. Suppose n objects are put on test, and $X_1, X_2, .., X_n$ denote their true life time. We assume that $X_1, X_2, .., X_n$ be independent, identically distributed (i.i.d.) according to a continuous life distribution F. Let $Y_1, Y_2, .., Y_n$ be (i.i.d.) according to a continuous life distribution G. Also we assume that X's and Y's are independent. Using the censored data (Z_i, δ_i), $i = 1, 2, 3, \cdots, n$, where $Z_i = min(X_i, Y_i)$ and

$$\delta_i = \begin{cases} 1 & if \quad Z_i = X_i \quad \text{(i-th observation is uncensored)} \\ 0 & if \quad Z_i = Y_i \quad \text{(i-th observation is censored)} \end{cases}$$

Let $Z_{(0)} = 0 < Z_{(1)} < Z_{(2)} ... < Z_{(n)}$ denote the ordered Z's and $\delta_{(i)}$ is δ_i corresponding to $Z_{(i)}$. Then the product limit estimator of the survival function $\bar{F}$ is given by:(see Kaplan and Meier [13])

$$\bar{F}_n(x) = 1 - F_n(x) = \prod_{j<Z_j<k} \left[\frac{n-j}{n-j+1}\right]^{\delta_j}, x \in [0\ , Z_{(n)}].$$

We propose the following test statistic

$$\hat{\delta}_n^c = \frac{1}{s^3}[s^2\Phi - 6\Omega - 3s^2\Omega\Theta + 6s\Omega^2 + (6\Omega - s^2\Phi)\Lambda], \tag{8.1}$$

where

$$\Omega = \sum_{k=1}^{n} \prod_{m=1}^{k-1} C_m^{\delta_m}(Z_k - Z_{k-1}),$$

$$\Theta = 2\sum_{i=1}^{n} \prod_{v=1}^{i-1} Z_i C_v^{\delta_v}(Z_i - Z_{i-1}),$$

$$\Phi = 3\sum_{r=1}^{n} \prod_{b=1}^{r-1} Z_r^2 C_b^{\delta_b}(Z_r - Z_{r-1}),$$

$$\Lambda = \sum_{j=1}^{n} e^{-sZ_j}\left(\prod_{p=1}^{j-2} C_p^{\delta_p} - \prod_{p=1}^{j-1} C_p^{\delta_p}\right),$$

$$d\bar{F}_n(Z_j) = \bar{F}_n(Z_{j-1}) - \bar{F}_n(Z_j)$$

and

$$C_k = \frac{n-k}{n-k+1}.$$

The percentile points of our test $\hat{\delta}_n^c$ in (8.1) are calculated based on 10000 simulated samples of size $n = 10(10)50, 51, 60, 70, 80, 81, 86$. Table 8.1 gives the critical values of statistic $\hat{\delta}_n^c$ at $s = 0.4$

Table 8.1. Critical values of statistic $\hat{\delta}_n^c$

n	0.01	0.05	0.10	0.90	0.95	0.99
10	-47.3	-35.3	-28.95	$5.0 * 10^{6}$	$7.1 * 10^{8}$	$6.0 * 10^{12}$
20	-31.0	-22.4	-17.71	$1.5 * 10^{16}$	$4.0 * 10^{20}$	$4.2 * 10^{27}$
30	-24.3	-16.3	-12.77	$7.2 * 10^{26}$	$6.8 * 10^{32}$	$1.5 * 10^{43}$
40	-19.8	-13.7	-10.31	$1.0 * 10^{38}$	$2.5 * 10^{46}$	$1.3 * 10^{60}$
50	-17.1	-11.5	-8.48	$8.8 * 10^{49}$	$4.7 * 10^{59}$	$6.9 * 10^{78}$
51	-16.9	-11.2	-8.11	$1.4 * 10^{50}$	$1.3 * 10^{60}$	$6.0 * 10^{78}$
60	-15.6	-9.8	-6.84	$4.1 * 10^{61}$	$6.7 * 10^{72}$	$3.1 * 10^{93}$
70	-13.9	-8.9	-6.31	$1.2 * 10^{72}$	$4.6 * 10^{85}$	$4.7 * 10^{110}$
80	-12.4	-7.8	-5.21	$1.2 * 10^{84}$	$7.8 * 10^{99}$	$3.5 * 10^{129}$
81	-12.6	-7.8	-5.26	$6.4 * 10^{86}$	$2.8 * 10^{102}$	$1.6 * 10^{128}$
86	-11.38	-7.24	-4.81	$1.6 * 10^{91}$	$9.4 * 10^{107}$	$2.1 * 10^{139}$

Next, we introduce three real examples for censored data at confidence level 95%.

Example 8.1. The data set of 81 survival times (in months) of patients melanoma. The ordered non-censored data are (see [14]):

3.25	3.5	4.75	4.75	5	5.25	5.75	5.75	6.25	6.5
6.5	6.75	6.75	7.78	8	8.5	8.5	9.25	9.5	9.5
10	11.5	12.5	13.25	13.5	14.25	14.5	14.75	15	16.25
16.25	16.5	17.5	21.75	22.5	24.5	25.5	25.75	27.5	29.5
31	32.5	34	34.5	35.25	58.5				

The ordered censored data are

4	5.25	11	12.5	13.75	16.75	18.25	19	20
20.25	21.5	23.25	25	27	28.5	30	31	31.25
32.25	32.5	33	33.5	35	36.75	37	37.75	38
38	39.5	45.25	47.5	48.25	48.5	53.25	53.75	

Here $\hat{\delta}_n^c = 1.91 \times 10^{337}$ which is more than the tabulated value in Table 8.1. Then we deduce that this data set has $NRBUL$ property.

Example 8.2. The following data represent 51 liver cancers patients taken from Elminia cancer center Ministry of Health - Egypt, which entered in (1999) (in days). Out of these 39 represents non-central data, and the others represents censored data (see [15]). It was found that $\hat{\delta}_n^c = 1.4 \times 10^{303}$ which is more than the tabulated value in Table 8.1. Then we deduce that this data set has $NRBUL$ property.

Example 8.3. On the basis of right-censored data for lung cancer patients from Pena (see [16]). These data consists of 86 survival times (in month) with 22 right censored. It was found that $\hat{\delta}_n^c = 1.45 \times 10^{225}$ which is more than the tabulated value in Table 8.1. Then we deduce that this data set has $NRBUL$ property.

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