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New Algorithm for Chromatic Number of Graphs and their Applications

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Abstract

In this paper, we have designed a new algorithm for calculating the exact value of the chromatic number of a graph and dubbed it as R- coloring algorithm. Using this R- coloring algorithm, we have proved some results. We have discussed some application in different fields of our life by using this new algorithm.

Keywords: Graph, Vertex-coloring, Chromatic Number, Adjacency Matrix.

2010 Mathematics subject classification: 05C15, 05C99, 05C90, 05C30.

1. Introduction:

In [1], Hermann and Hertz found the chromatic number by means of critical graphs. There are polynomials time algorithms to some classes of graphs. So, in general, evaluating the chromatic number of a graph is difficult. As has been mentioned in [2] a greedy method would be to iteratively pick, in a graph G, an uncolored vertex v, and to color it with the smallest color which is not yet used by its neighbors . Such a coloring will obviously stay proper until the whole vertex set is colored, and it never uses more than $\Delta(G) + 1$ different colors, where $\Delta(G)$ is the maximal degree of G, as in the procedure no vertex will ever exclude more than $\Delta(G)$ colors (for more details see [3-8]). Consider G is a graph. A k −coloring of G is a method of coloring the vertices by at most k colors provided that any adjacent vertices have different colors. The smallest number of colors needed to color the vertices of G such that any adjacent vertices have different colors is chromatic number of G, denoted by (G) .

From the above discussion we introduce a new algorithm to calculate the exact value of chromatic number of a graph.

2. The main results:

In this article we have designed a new algorithm, called R- coloring algorithm, to evaluate the exact value of the chromatic number of a graph. We introduce the following definition:

Definition 1. The associated adjacency matrix of a graph G, denoted by $A(G)$, is a matrix whose entry $a_{ij} = 1$ if the vertices v_i and v_j are adjacent such that $i \neq j$ and $a_{ij} = 0$ otherwise, where v_i and v_j are vertices in G.

Consider we have the graph G shown in Fig. (1)

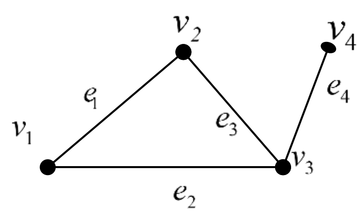


Fig. (1)

then the associated adjacency matrix of this graph is given by

$$AA(G) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

R- coloring algorithm will be introduced as follows:

Let G be a graph of order n with vertices as $v_1, v_2, v_3, \dots, v_n$

<u>Step1</u>: Evaluate the associated adjacency matrix of the graph G

$$AA(G) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}.$$

<u>Step2</u>: Confirm R- coloring matrix from the associated adjacency matrix as follows:

(a) In the first row of R- coloring matrix, put $a_{11}^* = 1$. This means that the vertex v_1 is colored by color 1. Then put $\boldsymbol{a_{12}^*} = \begin{cases} \mathbf{0} \ \ \boldsymbol{if} \ \ \boldsymbol{a_{12}} = \mathbf{0} \\ \mathbf{2} \ \ \boldsymbol{if} \ \ \boldsymbol{a_{12}} = \mathbf{1} \end{cases}$. After that put $\boldsymbol{a_{13}^*} = \begin{cases} \mathbf{0} & \boldsymbol{if} \ \ \boldsymbol{a_{13}} = \mathbf{0} \\ \mathbf{2} & \boldsymbol{if} \ \ \boldsymbol{a_{13}} = \mathbf{1}, \boldsymbol{a_{12}} = \mathbf{0} \\ \mathbf{3} & \boldsymbol{if} \ \ \boldsymbol{a_{13}} = \boldsymbol{a_{12}} = \boldsymbol{a_{23}} = \mathbf{1} \end{cases}$

In the same way, for entry a^*_{1j}, where $1 < j \le n$, put

$$\boldsymbol{a^*_{1j}} = \begin{cases} \mathbf{0} & \boldsymbol{if} \quad \boldsymbol{a_{1j} = 0} \\ \boldsymbol{min\{a^*_{1k^1}, a^*_{1k^2}, \dots, a^*_{1k^m}\}} & \boldsymbol{if} \quad \boldsymbol{a_{1j} = a_{1k^l} = 1, a_{k^l j} = 0 \forall l \in Z^+, 1 \le l \le m, 1 < k^l} < j \\ \boldsymbol{j} & \boldsymbol{if} \quad \boldsymbol{a_{12} = a_{13} = \dots = a_{1j} = a_{2j} = a_{3j} = \dots = a_{(j-1)j} = 1} \end{cases}$$

(b) For any column i has the entry $a^*_{1i} = h$, we put

$$\boldsymbol{a^*_{ji}} = \begin{cases} \boldsymbol{h} & \boldsymbol{if} \quad \boldsymbol{a_{ji} = 1, i \ne j} \\ \mathbf{0} & \boldsymbol{if} \quad \boldsymbol{a_{ji} = 0, i \ne j} \\ \boldsymbol{h} & \boldsymbol{if} \quad \boldsymbol{a_{ji} = 0, i = j} \end{cases},$$

where $1 < i, j \le n$.

(c) Now start from the row of the vertex which is colored after v_1 and repeat steps (a) and (b).

(d) Again repeat step (c) to complete R- coloring matrix

$$RC(G) = \begin{bmatrix} a^*_{11} & a^*_{12} & \dots & a^*_{1n} \\ a^*_{21} & a^*_{22} & \dots & a^*_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a^*_{n1} & a^*_{n2} & \dots & a^*_{nn} \end{bmatrix}$$

<u>Step3</u>: The greatest number in the diagonal of R- coloring matrix is the chromatic number of the graph G and the value of entry a^*_{ii} is the color of the vertex v_i.

Example1. Let G be a graph as shown in Fig. (2)

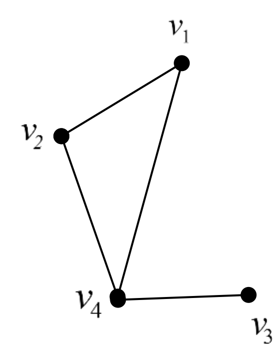


Fig. (2)

R- coloring algorithm is applied step by step as shown in the sequence of matrices below:

$$AA(G)=\begin{bmatrix}0&1&0&1\\1&0&0&1\\0&0&0&1\\1&1&1&0\end{bmatrix}\xrightarrow{Put\,a^*_{11}=1}\begin{bmatrix}1&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix}\xrightarrow[Put\;\boldsymbol{a^*_{12}}=2]{Since\;\boldsymbol{a_{12}}=\boldsymbol{1}}\begin{bmatrix}1&2&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix}$$

$$\xrightarrow[Put\;\boldsymbol{a^*_{13}}=0]{Since\;\boldsymbol{a_{13}}=\boldsymbol{0}}\begin{bmatrix}1&2&0&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix}\xrightarrow[Put\;\boldsymbol{a^*_{14}}=3]{Since\;\boldsymbol{a_{12}}=\boldsymbol{a_{14}}=\boldsymbol{a_{24}}=\boldsymbol{1}}\begin{bmatrix}1&2&0&3\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix}\xrightarrow[\boldsymbol{a^*_{21}}=\boldsymbol{a^*_{41}}=1]{Column\;1\;\;Put\;\boldsymbol{a^*_{31}}=0,}\begin{bmatrix}1&2&0&3\\1&\square&\square&\square\\0&\square&\square&\square\\1&\square&\square&\square\end{bmatrix}$$

$$\xrightarrow[\boldsymbol{a^*_{22}}=\boldsymbol{a^*_{42}}=2]{Column\;2\;\;Put\;\boldsymbol{a^*_{32}}=0,}\begin{bmatrix}1&2&0&3\\1&2&\square&\square\\0&0&\square&\square\\1&2&\square&\square\end{bmatrix}\xrightarrow[\boldsymbol{a^*_{24}}=\boldsymbol{a^*_{34}}=\boldsymbol{a^*_{44}}=3]{Column\;4\;\;Put}\begin{bmatrix}1&2&0&3\\1&2&\square&3\\0&0&\square&3\\1&2&\square&3\end{bmatrix}\xrightarrow[Put\;\boldsymbol{a^*_{23}}=0]{2^{nd}row\;\;Since\;\boldsymbol{a_{23}}=\boldsymbol{0}}\begin{bmatrix}1&2&0&3\\1&2&0&3\\0&0&\square&3\\1&2&\square&3\end{bmatrix}$$

$$\xrightarrow[Put\;\boldsymbol{a^*_{43}}=\boldsymbol{min}\{\boldsymbol{a^*_{41}},\boldsymbol{a^*_{42}}\}=1]{4^{th}row\;\;Since\;\boldsymbol{a_{43}}=\boldsymbol{1}}\begin{bmatrix}1&2&0&3\\1&2&0&3\\0&0&\square&3\\1&2&1&3\end{bmatrix}\xrightarrow{Put\,a^*_{33}=1}\begin{bmatrix}1&2&0&3\\1&2&0&3\\0&0&1&3\\1&2&1&3\end{bmatrix}$$

Then R- coloring matrix is

$$RC(G)=\begin{bmatrix}1&2&0&3\\1&2&0&3\\0&0&1&3\\1&2&1&3\end{bmatrix}.$$

Hence the chromatic number of the graph G is 3,i.e. $\chi(G)=3$, as shown from the diagonal of R-coloring matrix and the color of v_1 is 1, v_2 is 2, v_3 is 1 and v_4 is 3.

In the above example, if the associated adjacency matrix becomes:

$$AA(G)=\begin{bmatrix}0&1&1&1\\1&0&1&0\\1&1&0&0\\1&0&0&0\end{bmatrix}$$

where v_4 is in the first row, v_1 is in the second row, v_2 is in the third row and v_3 is in the fourth row, then R- coloring algorithm can be applied as follows :

$$AA(G)=\begin{bmatrix}0&1&1&1\\1&0&1&0\\1&1&0&0\\1&0&0&0\end{bmatrix}\xrightarrow{Put\,a_{11}^{*}=1}\begin{bmatrix}1&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix}\xrightarrow[Put\ \boldsymbol{a}_{12}^{*}=2]{Since\ \boldsymbol{a_{12}=1}}\begin{bmatrix}1&2&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix}$$

$$\xrightarrow[Put\ \boldsymbol{a}_{13}^{*}=3]{\substack{Since\\ \boldsymbol{a_{12}=a_{13}=a_{23}=1}}}\begin{bmatrix}1&2&3&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix}\xrightarrow[Put\ \boldsymbol{a}_{14}^{*}=2]{\substack{Since\\ \boldsymbol{a_{12}=a_{13}=a_{14}=1}\\ \boldsymbol{a_{24}=a_{34}=0}}}\begin{bmatrix}1&2&3&2\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix}\xrightarrow[=\boldsymbol{a}_{41}^{*}=1]{\substack{Column\ 1\\ Put\\ \boldsymbol{a}_{21}^{*}=\boldsymbol{a}_{31}^{*}}}\begin{bmatrix}1&2&3&2\\1&\square&\square&\square\\1&\square&\square&\square\\1&\square&\square&\square\end{bmatrix}$$

$$\xrightarrow[\boldsymbol{a}_{22}^{*}=\boldsymbol{a}_{32}^{*}=2]{\substack{Column\ 2\\ Put\ \boldsymbol{a}_{42}^{*}=0,}}\begin{bmatrix}1&2&3&2\\1&2&\square&\square\\1&2&\square&\square\\1&0&\square&\square\end{bmatrix}\xrightarrow[\boldsymbol{a}_{23}^{*}=\boldsymbol{a}_{33}^{*}=3]{\substack{Column\ 3\\ Put\,\boldsymbol{a}_{43}^{*}=0,}}\begin{bmatrix}1&2&3&2\\1&2&3&\square\\1&2&3&\square\\1&0&0&\square\end{bmatrix}\xrightarrow[\boldsymbol{a}_{24}^{*}=\boldsymbol{a}_{34}^{*}=0]{\substack{Column\ 4\\ Put\,\boldsymbol{a}_{44}^{*}=2,}}\begin{bmatrix}1&2&3&2\\1&2&3&0\\1&2&3&0\\1&0&0&2\end{bmatrix}$$

From the above example, we observe that R- coloring matrix was evaluated easily.

Note: To Evaluate R- coloring matrix easily, put the vertex with highest degree in the first row of the associated adjacency matrix.

Now we will make comparison between R- coloring algorithm and greedy coloring algorithm for calculating the chromatic number of a graph G as follows:

Consider we have a graph G in Fig. (3) (a). Fix two different orderings of its vertices as shown in Fig. (3) (b) and (c).

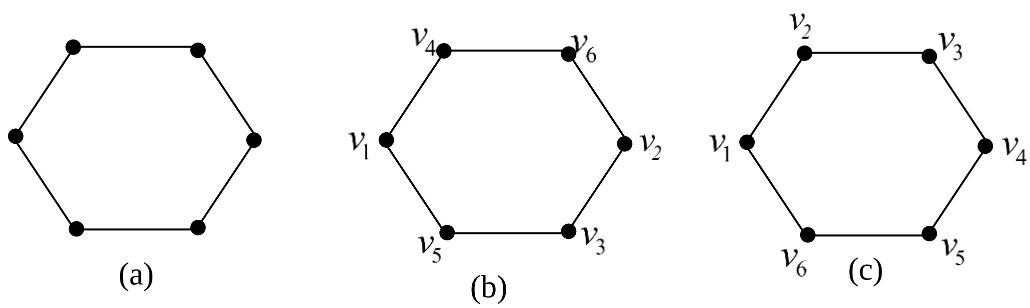


Fig. (3)

First, we will apply the greedy coloring algorithm to color graph G in the ordering shown in (b), observe the following sequence of diagrams:

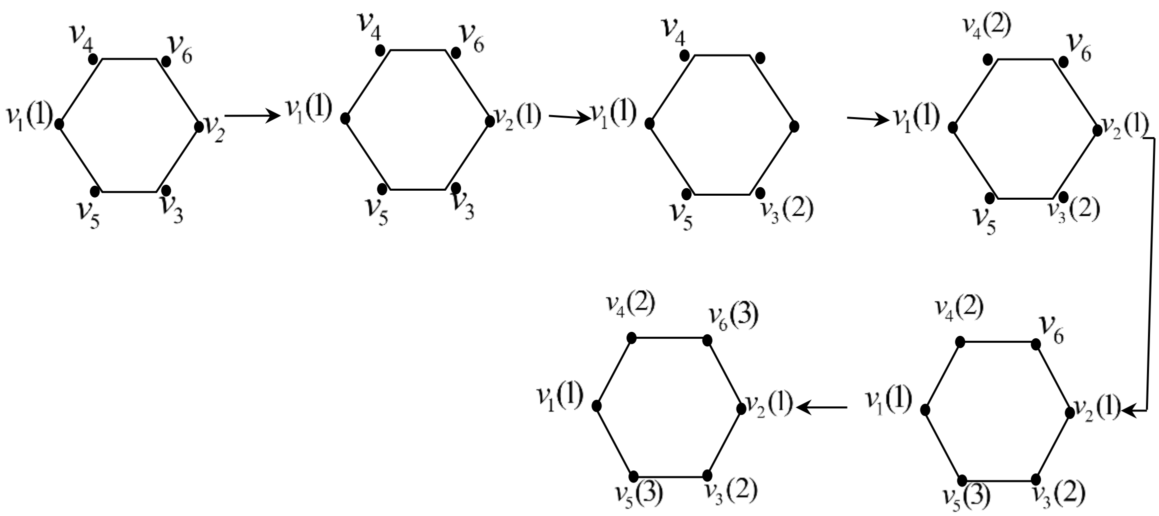


Fig. (4)

From the above diagrams we find that the chromatic number of the graph G is 3, i.e. $\chi(G) = 3$.

Now we will apply R- coloring algorithm to color graph G in the ordering shown in (b) as follows:

$$AA(G) = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{\textit{The first row}} \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \end{bmatrix} \xrightarrow{\textit{The columns } 1,4 \textit{ and } 5} \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & & & 0 & 0 & \\ 0 & & & 0 & 2 & \\ 1 & & & 2 & 0 & \\ 1 & & & 0 & 2 & \\ 0 & & & 2 & 0 & \end{bmatrix}$$

$$\xrightarrow[6^{th} column]{4^{th} row\ then} \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & & & 0 & 0 & 1 \\ 0 & & & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 & 0 & 1 \\ 1 & & & 0 & 2 & 0 \\ 0 & & & 2 & 0 & 1 \end{bmatrix} \xrightarrow[3^{rdth} column]{5^{th} row\ then} \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & & 1 & 0 & 0 & 1 \\ 0 & & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 & 2 & 0 \\ 0 & & 0 & 2 & 0 & 1 \end{bmatrix} \xrightarrow[2^{nd} column]{2^{nd} row\ then} RC(G) = \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 1 \end{bmatrix}$$

From R- coloring matrix we find that the chromatic number of the graph G is *2, i.e.* $\chi(G) = 2$.

By applying the greedy coloring algorithm to color graph G in the ordering shown in (c), the algorithm is shown in the following sequence of diagrams:

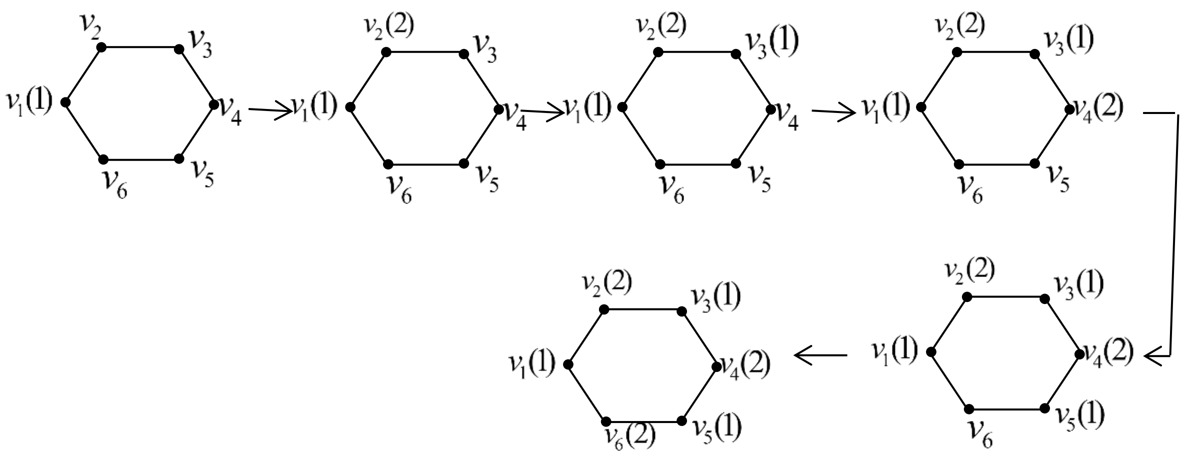
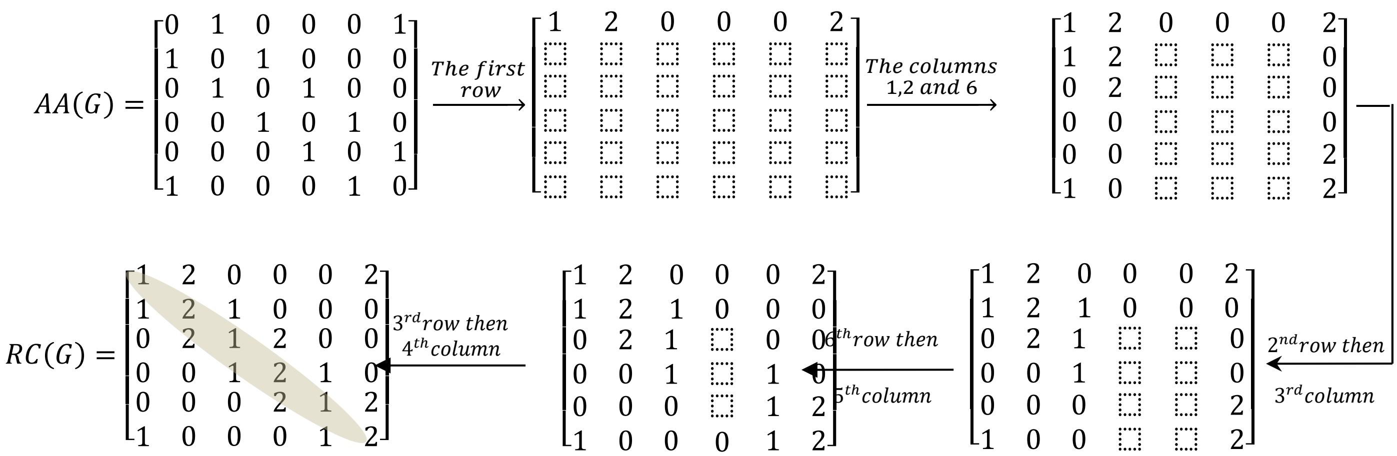


Fig. (5)

The above diagrams show that the chromatic number of the graph G is 2, i.e. $\chi(\mathrm{G}) = 2$.

We can apply R- coloring algorithm to color graph G in the ordering shown in (c) as follows:



Hence R- coloring matrix refers to the chromatic number of the graph G is also 2, i.e. $\chi(\mathrm{G}) = 2$.

We can deduct from the above that greedy coloring algorithm gives different chromatic numbers of the same graph depending on the order of the vertices. However, R- coloring algorithm gives only one chromatic number for all different orders of the vertices of the graph.

As a result, we can introduce the following theorem:

Theorem : R- coloring algorithm calculates the exact value of chromatic number of a graph G.

Proof. Suppose we have a graph G with more than one chromatic number calculated by R- coloring algorithm. This means that there exists more than one of R- coloring matrices with different greatest entry in their diagonals. Since R- coloring algorithm depends on the color of the neighbor vertices of the colored vertex such that the vertex colored by the smallest color is not used according to the order in the associated adjacency matrix which is fixed for the same graph (from property of matrices). Then, this is a contradiction and hence R- coloring algorithm calculates the exact value of chromatic number of the graph G.

We now prove some results by using R- coloring algorithm

Result 1: Let G be a graph of order n. Then $\chi(\mathrm{G}) = 1$ if and only if $\mathrm{G} \cong N_n$, where N_n is a null graph.

Proof: Let G be a graph of order n, $\chi(\mathrm{G}) = 1$ and $G \ncong N_n$. Since G is a graph of order n and $\chi(\mathrm{G}) = 1$ then every entry in the diagonal of R- coloring matrix equals 1 and the other entries equal 0, i.e.

$$RC(G) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix},$$

This means the entries in the associated adjacency matrix of the graph G equal 0, i.e. each vertex in G does not connect to any vertex. This is a contradiction. Hence $\mathrm{G} \cong N_n$.

To prove the converse, Let G be a graph of order n and $\mathrm{G} \cong N_n$. Then the associated adjacency matrix of the graph G is given by

$$AA(G) = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix},$$

Hence by using R- coloring algorithm, we find that R- coloring matrix is

$$RC(G) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}.$$

Then the chromatic number of the graph G equals 1 from R- coloring matrix, i.e. $\chi(G) = 1$.

Result 2. Let G be a graph of order $n \geq 2$. Then $\chi(G) = n$ if and only if $G \cong K_n$, where K_n is a complete graph of order *n*.

Proof: Let G be a graph of order $n \geq 2$ and $\chi(G) = n$. Then R- coloring matrix of the graph G is $n \times n$ matrix and each entry in its diagonal of it takes number from 1 to n such that $a_{ii}^{*} \neq a_{jj}^{*}$ $\forall i \neq j$. This means that, every vertex in G is connected to the others. Hence $G \cong K_n$.

To prove the converse, Let G be a graph of order n and $G \cong K_n$. Then the associated adjacency matrix of the graph G is given by

$$AA(G) = \begin{bmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 0 \end{bmatrix},$$

Hence by using R- coloring algorithm, we find that R- coloring matrix is

$$RC(G) = \begin{bmatrix} 1 & 2 & 3 & \dots & n \\ 1 & 2 & 3 & \cdots & n \\ 1 & 2 & 3 & \cdots & n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \dots & n \end{bmatrix}.$$

Then the chromatic number of the graph G equals n from R- coloring matrix, i.e. $\chi(G) = n$.

As a real-life application of coloring we take the traffic lights. Fig. (6) shows the intersection roads.

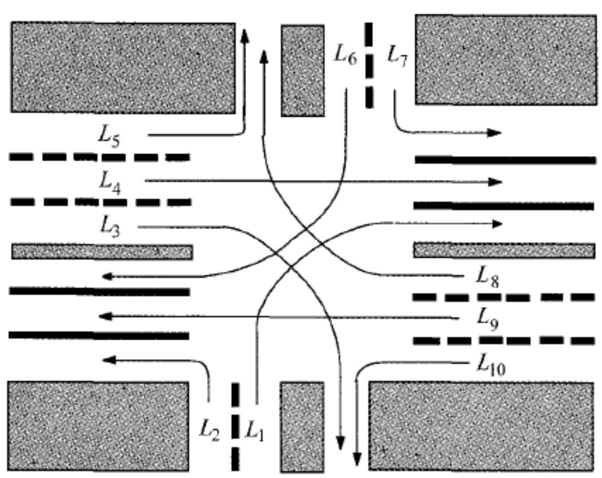


Fig. (6)

We find 10 traffic lanes, L_1 to L_{10} . A traffic light system includes phases. At every phase, vehicles in lanes for which the light is green may proceed safely through the intersection. Now, we want to calculate a minimum number of phases needed for the traffic light system so that all vehicles may proceed safely through the intersection.

Let us solve this problem by drawing the graph *G*, shown in Fig.(7), to model the situation

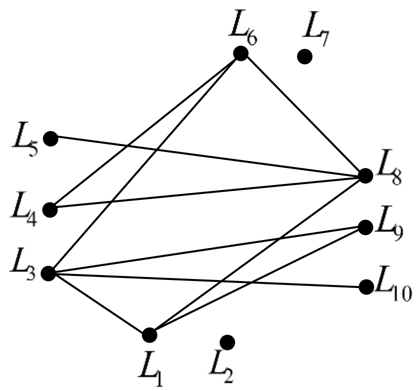


Fig. (7)

the associated adjacency matrix of the above graph is

$$AA(G) = \begin{array}{c} \\ L_3 \\ L_1 \\ L_2 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8 \\ L_9 \\ L_{10} \end{array} \begin{array}{c} \begin{matrix} L_3 & L_1 & L_2 & L_4 & L_5 & L_6 & L_7 & L_8 & L_9 & L_{10} \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

By applying R- coloring algorithm, we find that R- coloring matrix is

$$\boldsymbol{RC(G)} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{2} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{2} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} & \mathbf{0} & \mathbf{3} & \mathbf{2} & \mathbf{2} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{3} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{2} \end{bmatrix}.$$

Hence, three is the minimum number of phases in our problem.

Note: Also, we can write associated adjacency matrix of our graph without drawing the graph, i.e. $a_{ij} = 1$ if L_i intersect L_j and $a_{ij} = 0$ if L_i does not intersect L_j.

3. Conclusion

A new algorithm for calculating the exact value of the chromatic number of a graph has been designed and dubbed as R- coloring algorithm. Some basic results have been proved and the traffic light problem has been taken as a real life application.

4. References

[1] J. M. Harris, J. L. Hirst and M. J. Mossinghoff: combinatorial and graph theory, Springer science +Business Media, USA, 2008.

[2] C. Liu and E. Zhu: General Vertex-Distinguishing Total Coloring of Graphs, Journal of Applied Mathematics, Hindawi Publishing Corporation, 2014.

[3] J.A. Bondy and U.S.R. Murty: Graph Theory, Graduate Texts in Mathematics series, Springer, 2008.

[4] R. Diestel: Graph theory, Springer-Verlag, New York, 2000.

[5] F. Hermann and A.Hertz: Finding the chromatic number by means of critical graphs, Electronic Notes in Discrete Mathematics, 5, 174-176, (2000).

[6] I. Tomon: On the chromatic number of regular graphs of matrix algebras, Linear Algebra and its Applications, 475, 154-162, (2015) .

[7]G. Luna, J. R. M. Romero and Y. Moyao: An Approximate Algorithm for the Chromatic Number of Graphs, Electronic Notes in Discrete Mathematics, 46, 89-96, (2014).

[8] M. Molloy and B. Reed: Colouring graphs when the number of colours is almost the maximum degree, Journal of Combinatorial Theory, Series B, 109, 134-195, (2014).