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GENERALIZED TOPOLOGICAL APPROXIMATION SPACES AND THEIR MEDICAL APPLICATIONS

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Abstract: We generalize Pawlak's approximation space to topological approximation spaces using some topological near open sets, such as regular open sets, semi-open sets, pre-open sets, γ-open sets, α-open sets and β-open sets and others. The properties of the topological approximations space will be studied. Finally, We applied our results on medical data.

Keywords: Topological Spaces; Rough Sets; Rough Approximations; Information System; Data Reduction.
MSC: 54A05; 54B05; 54D35; 03B70.

1. Introduction

Rough set theory, proposed by Pawlak in [1], is an extension of set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. Many suggestions have been made for generalizing and interpreting rough sets [2-4]. Moreover, this theory may serve as a new mathematical tool to soft computing and has been successfully applied in machine learning, information sciences, expert systems, data reduction, and so on. Recently, lots of researchers are interested to generalize this theory in many fields of applications [5-9]. Several interesting and meaningful generalizations to equivalence relation have been proposed in the past, such as topological bases and subbases [10, 11, 12]. Particularly, some researchers have used coverings of the universe of discourse for establishing the generalized rough sets by coverings [13,14].

Rough set theory is a recent approach for reasoning about data. This theory depends basically on a certain topological structure and has achieved a great success in many fields of real life applications. The concept of a topological rough set given by Wiweger [15] in 1989 is one of the most important topological generalization of rough sets. In 1983 Abd El-Monsef in [16] introduced the concept of β-open sets. This paper is organized as follows:

In Section 2 we give topological basic concepts. Also, Section 3 discussed the fundamentals of rough sets and investigated the concept of topological approximation space. Section 4 is devoted to introduce medical application example. The paper's conclusion is given in Section 5.

2. Topological Basic Concepts

More details about the following topological near open sets found in [17-20].

Definition 2. 1 [17,18] A subset A of a topological space (U,τ) is called:

(1) semi-open set if $A \subseteq cl(\text{int}(A))$ and it is called a semi-closed set if $\text{int}(cl(A)) \subseteq A$

(2) pre-open set if $A \subseteq \text{int}(cl(A))$ and it is called a pre-closed set if $cl(\text{int}(A)) \subseteq A$

(3) $\alpha-$open set if $A \subseteq \text{int}(cl(\text{int}(A)))$ and it is called a $\alpha-$closed set if $cl(\text{int}(cl(A))) \subseteq A$.

(4) semi-pre-open set ($\beta-$open [1]) if $A \subseteq cl(\text{int}(cl(A)))$ and it is called a semi-pre-closed set ($\beta-$ closed) if $\text{int}(cl(\text{int}(A))) \subseteq A$.

(5) regular-open set if $A = \text{int}(cl(A))$ and it is called a regular-closed set if $cl(\text{int}(A)) = A$.

(6) semi-regular set if it both semi-open and semi-closed in (U,τ).

(7) $\delta-$closed set if $A = cl_{\delta}(A)$, where $cl_{\delta}(A) = \{x \in U : \text{int}(cl(G)) \cap A \neq \phi, x \in G, G \in \tau\}$.

The semi-closure (resp. α-closure, semi-pre-closure) of a subset A of (U,τ) is the intersection of all semi-closed (resp. α–closed, semi-pre-closed) sets that contains A and is denoted by $scl(A)$ (resp. $\alpha-cl(A)$, $spcl(A)$). The union of all semi-open subsets of U is called the semi-interior of A and is denoted by $s\text{int}(A)$.

Definition 2.2 [19,20] A subset A of a topological space (U,τ) is called:

(1) generalized closed (briefly $g-$ closed) set if $cl(A) \subseteq G$ whenever $A \subseteq G$ and $G \in \tau$.

(2) semi-generalized closed set (briefly $sg-$ closed) if $scl(A) \subseteq G$ whenever $A \subseteq G$ and G is semi-open set in (U,τ). The complement of a $sg-$closed set is called a $sg-$open set.

(3) generalized semi-closed set (briefly $gs-$ closed) if $scl(A)\subseteq U$ whenever $A\subseteq G$ and $G\in\tau$.

(4) $\alpha-$ generalized closed set (briefly $\alpha g-$ closed) if $\alpha cl(A)\subseteq G$ whenever $A\subseteq G$ and $G\in\tau$

(5) generalized $\alpha-$ closed set (briefly $g\alpha-$ closed) if $\alpha cl(A)\subseteq G$ whenever $A\subseteq G$ and G is $\alpha-$open in (U,τ) .

(6) $g\alpha^{**}$ -closed set if $cl(A)\subseteq \operatorname{int}(cl(G))$ whenever $A\subseteq G$ and G is $\alpha-$open in (U,τ).

(7) generalized semi-pre-closed (briefly $gsp-$ closed) set if $spcl(A)\subseteq G$ whenever $A\subseteq G$ and $G\in\tau$.

(8) $\delta-$ generalized closed (briefly $\delta g-$ closed) set if $cl\delta(A)\subseteq U$ whenever $A\subseteq G$ and $G\in\tau$.

(9) Q -set if $\operatorname{int}(cl(A))=cl(\operatorname{int}(A))$.

The equivalence classes of R are also known as the granules, elementary sets or blocks; we will use $[x]_R\subseteq U$ to denote the equivalence class containing $x\in U$. In the approximation space, we consider two operators $\underline{R}(X)=\{x\in U:[x]_R\subseteq X\}$ and $\overline{R}(X)=\{x\in U:[x]_R\cap X\neq\phi\}$, called the lower approximation and upper approximation of $X\subseteq U$ respectively. Also let $POS_R(X)=\underline{R}(X)$ denote the positive region of X , $NEG_R(X)=U-\overline{R}(X)$ denote the negative region of X and $BN_R(X)=\overline{R}(X)-\underline{R}(X)$ denote the borderline region of X .

The degree of completeness can also be characterized by the accuracy measure, in which $|X|$ represents the cardinality of set X as follows: $\alpha_R(X)=|\underline{R}(X)|/|\overline{R}(X)|, \overline{R}(X)\neq\phi$.

3. Topological approximation spaces

In this section, we introduce and investigate the concept of topological approximation space. Also, we introduce the concepts of topological lower approximation and topological upper approximation and study their properties.

Definition 3.1 Let $K=(U,R)$ be an approximation space with general relation R and τ_R is the topology associated with K . Then the triple $K_\tau=(U,R,\tau_R)$ is called a topological approximation space.

Definition 3.2 Let $K_\tau=(U,R,\tau_R)$ be a topological approximation space. If $X\subseteq U$, then the topological lower approximations of X are defined as follows:

(1) Semi-lower approximation of X , $\underline{R}_{semi}(X)=\cup\{G:G\in SEMI(U),G\subseteq X\}$ where $SEMI(U)$ is the set of all $semi-open$ sets in $K_\tau=(U,R,\tau_R)$.

(2) Pre-lower approximation of X , $\underline{R}_{pre}(X)=\cup\{G:G\in PRE(U),G\subseteq X\}$ where $PRE(U)$ is the set of all $pre-open$ sets in $K_\tau=(U,R,\tau_R)$.

(3) $\alpha-lower$ approximation of X , $\underline{R}_{\alpha}(X)=\cup\{G:G\in \alpha(U),G\subseteq X\}$ where $\alpha(U)$ is the set of all $\alpha-open$ sets in $K_\tau=(U,R,\tau_R)$.

(4) $\beta-lower$ approximation of X , $\underline{R}_{\beta}(X)=\cup\{G:G\in \beta(U),G\subseteq X\}$ where $\beta(U)$ is the set of all $\beta-open$ sets in $K_\tau=(U,R,\tau_R)$.

(5) Regular-lower approximation of X , $\underline{R}_{regular}(X)=\cup\{G:G\in REG(U),G\subseteq X\}$ where $REG(U)$ is the set of all $regular-open$ sets in $K_\tau=(U,R,\tau_R)$.

(6) Semi-regular lower approximation of X, $\underline{R}_{semi-regular}(X)=\cup\{G:G\in SEMI(U)\ or\ G\in CSEMI(U)$ where $SEMI(U)$ and $CSEMI(U)$ are the set of all $semi-open$ sets and $semi-closed$ sets in $K_\tau=(U,R,\tau_R)$ respectively.

(7) $\delta-lower$ approximation of X , $\underline{R}_{\delta}(X)=\cup\{G:G\in \delta(U),G\subseteq X\}$ where $\delta(U)$ is the set of all $\delta-closed$ sets in $K_\tau=(U,R,\tau_R)$.

(8) $g-lower$ approximation of X , $\underline{R}_{g}(X)=\cup\{G:G\in g(U),G\subseteq X\}$ where $g(U)$ is the set of all $g-closed$ sets in $K_\tau=(U,R,\tau_R)$.

(9) $sg-lower$ approximation of X $\underline{R}_{sg}(X)=\cup\{G:G\in sg(U),G\subseteq X\}$ where $sg(U)$ is the set of all $sg-open$ sets in $K_\tau=(U,R,\tau_R)$.

(10) $gs-lower$ approximation of X $\underline{R}_{gs}(X)=\cup\{G:G\in gs(U),G\subseteq X\}$ where $gs(U)$ is the set of all $gs-open$ sets in $K_\tau=(U,R,\tau_R)$.

(11) $\alpha g-lower$ approximation of X , $\underline{R}_{\alpha g}(X)=\cup\{G:G\in \alpha g(U),G\subseteq X\}$ where $\alpha g(U)$ is the set of all $\alpha g-closed$ sets in $K_\tau=(U,R,\tau_R)$.

(12) $g\alpha-lower$ approximation of X , $\underline{R}_{g\alpha}(X)=\cup\{G:G\in g\alpha(U),G\subseteq X\}$ where $g\alpha(U)$ is the set of all $g\alpha-closed$ sets in $K_\tau=(U,R,\tau_R)$.

(13) $g\alpha^{**}-lower$ approximation of X , $\underline{R}_{g\alpha^{**}}(X)=\cup\{G:G\in g\alpha^{**}(U),G\subseteq X\}$ where $g\alpha^{**}(U)$ is the set of all $g\alpha^{**}-closed$ sets in $K_\tau=(U,R,\tau_R)$.

(14) $gsp-lower$ approximation of X $\underline{R}_{gsp}(X)=\cup\{G:G\in gsp(U),G\subseteq X\}$ where $gsp(U)$ is the set of all $gsp-closed$ sets in $K_\tau=(U,R,\tau_R)$.

(15) $\delta g-lower$ approximation of X , $\underline{R}_{\delta g}(X)=\cup\{G:G\in \delta g(U),G\subseteq X\}$ where $\delta g(U)$ is the set of all $\delta g-closed$ sets in $K_\tau=(U,R,\tau_R)$.

(16) Q-lower approximation of X , $\underline{R}_{Q}(X)=\cup\{G:G\in Q(U),G\subseteq X\}$ where $Q(U)$ is the set of all $Q-sets$ in $K_\tau=(U,R,\tau_R)$.

Definition 3.3 Let $K_\tau=(U,R,\tau_R)$ be a topological approximation space. If $X\subseteq U$, then the topological upper approximations of X are defined as follows:

(1) Semi-upper approximation of X , $\overline{R}_{semi}(X)=\cup\{F:F\in CSEMI(U),F\cap X\neq\varphi\}$ where $CSEMI(U)$ is the set of all $semi-closed$ sets in $K_\tau=(U,R,\tau_R)$.

(2) Pre-upper approximation of X , $\overline{R}_{pre}(X)=\cup\{F:F\in CPRE(U),F\cap X\neq\varphi\}$

where $CPRE(U)$ is the set of all $pre-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(3) $\alpha-upper$ approximation of X, $\overline{R}_\alpha(X) = \cup\{F : F \in C\alpha(U), F \cap X \neq \varphi\}$ where $C\alpha(U)$ is the set of all $\alpha-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(4) $\beta-upper$ approximation of X, $\overline{R}_\beta(X) = \cup\{F : F \in C\beta(U), F \cap X \neq \varphi\}$ where $C\beta(U)$ is the set of all $\beta-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(5) Regular-upper approximation of X, $\overline{R}_{regular}(X) = \cup\{F : F \in CREG(U), F \cap X \neq \varphi\}$ where $CREG(U)$ is the set of all $regular-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(6) Semi-regular upper approximation of X, $\overline{R}_{semi-regular}(X) = \cup\{F : F \in CSEMI(U)\ , F \cap X \neq \varphi\}$ where $CSEMI(U)$ and $CSEMI(U)$ is the set of all $semi-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(7) $\delta-upper$ approximation of X, $\overline{R}_\delta(X) = \cup\{F : F \in \delta(U), F \cap X \neq \varphi\}$ where $\delta(U)$ is the set of all $\delta-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(8) $g-upper$ approximation of X, $\overline{R}_g(X) = \cup\{F : F \in g(U), F \cap X \neq \varphi\}$ where $g(U)$ is the set of all $g-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(9) $sg-upper$ approximation of X $\overline{R}_{sg}(X) = \cup\{F : F \in Csg(U), F \cap X \neq \varphi\}$ where $Csg(U)$ is the set of all $sg-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(10) $gs-upper$ approximation of X $\overline{R}_{gs}(X) = \cup\{F : F \in Cgs(U), F \cap X \neq \varphi\}$ where $Cgs(U)$ is the set of all $gs-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(11) $\alpha g-upper$ approximation of X, $\overline{R}_{\alpha g}(X) = \cup\{F : F \in \alpha g(U), F \cap X \neq \varphi\}$ where $\alpha g(U)$ is the set of all $\alpha g-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(12) $g\alpha-upper$ approximation of X, $\overline{R}_{g\alpha}(X) = \cup\{F : F \in g\alpha(U), F \cap X \neq \varphi\}$ where $g\alpha(U)$ is the set of all $g\alpha-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(13) $g\alpha^{**}-upper$ approximation of X, $\overline{R}_{g\alpha^{**}}(X) = \cup\{F : F \in g\alpha^{**}(U), F \cap X \neq \varphi\}$ where $g\alpha^{**}(U)$ is the set of all $g\alpha^{**}-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(14) $gsp-upper$ approximation of X $\overline{R}_{gsp}(X) = \cup\{F : F \in gsp(U), F \cap X \neq \varphi\}$ where $gsp(U)$ is the set of all $gsp-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(15) $\delta g-upper$ approximation of X, $\overline{R}_{\delta g}(X) = \cup\{F : F \in \delta g(U), F \cap X \neq \varphi\}$ where $\delta g(U)$ is the set of all $\delta g-closed$ sets in $K_\tau = (U,R,\tau_R)$.

(16) Q-upper approximation of X, $\overline{R}_Q(X) = \cup\{F : F \in Q(U), F \cap X \neq \varphi\}$ where $Q(U)$ is the set of all $Q-sets$ in $K_\tau = (U,R,\tau_R)$.

Motivation for topological rough set theory has come from the need to represent subsets of a universe in terms of topological classes of the topological base generated by the general binary relation defined on the universe. That base characterizes a topological space, called topological approximation space $K_\tau = (U,R,\tau_R)$. The topological classes of R are also known as the topological granules, topological elementary sets or topological blocks; we will use $G_{xm} \in \tau$, $m \in \{semi, pre, \alpha, \beta, regular, semi-regular, \delta, g, sg, gs, \alpha g, g\alpha, g\alpha^{**}, gsp, \delta g, Q\}$ to denote the topological class containing $x \in U$. In the topological approximation space, we consider two operators $\underline{R}_m(X) = \{x \in U : G_{xm} \subseteq X\}$ and $\overline{R}_m(X) = \{x \in U : G_{xm} \cap X \neq \phi\}$ called the topological lower approximation and topological upper approximation of $X \subseteq U$ respectively. Also let $POS_m(X) = \underline{R}_m(X)$ denote the topological positive region of X, $NEG_m(X) = U - \overline{R}_m(X)$ denote the topological negative region of X and $BND_m(X) = \overline{R}_m(X) - \underline{R}_m(X)$ denote the topological borderline region of X.

The degree of topological completeness can also be characterized by the topological accuracy measure, in which $|X|$ represents the cardinality of set X as follows:

$$\alpha_m(X) = \frac{|\underline{R}_m(X)|}{|\overline{R}_m(X)|}, \text{ where } X \neq \phi$$

Example 3. 1 Let $U = \{a,b,c,d\}$ be a universe and a relation R defined by $R = \{(a,a),(a,c),(a,d),(b,b),(b,d),(c,a),(c,b),(c,d),(d,a)\}$, thus $aR = \{a,c,d\}$, $bR = \{b,d\}$, $cR = \{a,b,d\}$ and $dR = \{a\}$. Then the topology associated with this relation R is $\tau_R = \{U,\varphi,\{a\},\{d\},\{a,d\},\{b,d\},\{a,b,d\},\{a,c,d\}\}$ and $\beta(U) = \{U,\varphi,\{a\},\{d\},\{a,c\},\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. So (U,R,τ_R) is a topological approximation space using $\beta-open$ sets.

Let (U,R,τ_R) be a topological approximation space. The Universe U can be divided into many regions with respect to any $X \subseteq U$ and with respect to any $m \in$ {semi, pre, α, β, regular, semi-regular, δ, g, sg, gs, αg, gα, gα^(**), gsp, δg, Q} as follows:

Let (U,R,τ_R) be a topological approximation space. For any $m \in$ {semi, pre, α, β, regular, semi-regular, δ, g, sg, gs, αg, gα, gα^(**), gsp, δg, Q}and for any subset $X \subseteq U$ we define the following memberships:

x belongs strong to X if $x \in \underline{R}(X)$,x belongs weak to X if $x \in \overline{R}(X)$,

x belongs m-strong to ***X*** if $x \in \underline{R}_m(X)$, x belongs m-weak to ***X*** if $x \in \overline{R}_m(X)$,

Example 3.2 Let U = {a, b, c, d} be a universe and a relation R defined by R = {(a, a),(d, c), (d, d), (c, a), (c, d), (c, c)}, thus aR = {a}, bR = φ, cR = {a, c, d} and dR = {c, d}. Then the topology associated with this relation is τR= {U, φ, {a}, {c, d}, {a, c, d}} . So $(U, R, \tau_R)$is a topological approximation space. Let X = {b, c, d}, we have b is m-strong belongs to X but b is not strong belongs to X . Also, let Y = {c} be another subset of U. Then we have d is weak belongs to Y but d is not m-weak belongs to Y.

The degree of topological completeness can also be characterized by the topological accuracy measure (m-accuracy), in which $|X|$ represents the cardinality of set X as follows:

$$\alpha_m(X) = \frac{|\underline{R}_m(X)|}{|\overline{R}_m(X)|}, \text{ where } X \neq \phi$$

According to Example 3.1, Table 1 showing the differences among the degree of Pawlak's accuracy measure α(X) and β-accuracy measure αβ (X) for some subsets of U if we take m=β.

X	$\alpha(X)$ (Pawlak)	$\alpha_\beta(X)$
{a}	50%	100%
{a,c}	50%	100%
{b,d}	33.3%	100%
{b,c,d}	66.6%	100%

Table 1:Comparision between accuracy measures

The β-accuracy measure of the class of all β-open sets is accurate than the others measures and the following example and its followed diagram illustrate this fact.

Example 3.3 Let $U = \{a, b, c, d\}$ be a universe and for some relations we have the topology $\tau_R = \{U, \phi, \{d\}, \{a,b\}, \{a,b,d\}\}$. So (U,R,τR) is a topological approximation space. The we have the following knowledge bases:

REG(U) = {U ,φ ,{d},{a,b}},
SEMI(U) = {U ,φ ,{d},{a,b},{c, d},{a,b,c},{a,b, d}},
PRE(U) = {U ,φ ,{a},{b},{d},{a,d},{a,b},{b, d},{a,c,d},{a,b, d},{b,c, d}},
α(U) = {U ,φ ,{d},{a,b},{a,b, d}},
β(U)= {U ,φ ,{a},{b},{d},{a,d},{a,b},{a,c},{b, d},{b,c}, {c,d},{a,b,c},{a,c,d},{a,b, d},{b,c, d}},

Proposition 3.2 For any topological approximation space $K_\tau = (U, R, \tau_R)$,the following are hold for any $X, Y \subseteq U$:

$\underline{R}_m(X \cup Y) \supseteq \underline{R}_m(X) \cup \underline{R}_m(Y)$,

$\overline{R}_m(X \cup Y) \supseteq \overline{R}_m(X) \cup \underline{R}_m(Y)$,

$\underline{R}_m(X \cap Y) \subseteq \underline{R}_m(X) \cap \underline{R}_m(Y)$,

$\overline{R}_m(X \cap Y) \subseteq \overline{R}_m(X) \cap \underline{R}_m(Y)$.

Proof: We prove Part 1 and other parts are similar to it. Since we have $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$. Then $\underline{R}_m(X) \subseteq \underline{R}_m(X \cup Y)$ and $\underline{R}_m(Y) \subseteq \underline{R}_m(X \cup Y)$ then $\underline{R}_m(X \cup Y) \supseteq \underline{R}_m(X) \cup \underline{R}_m(Y)$.

4. Medecal Applications Example

The basic concepts about information systems can be found in [1, 2, 3, 15, 20].

Table 2 is shown the patients information system and respective symptoms, and the data are of the discrete type.

Patients (U)	Conditional Attributes (C)			Decision (D)
	Temperature	Flu	Headache	Dengue
u1	Normal	No	No	No
u2	High	No	No	No
u3	Very High	No	No	Yes
u4	High	No	Yes	Yes
u5	Very High	No	Yes	Yes
u6	High	Yes	Yes	Yes
u7	Very High	Yes	Yes	Yes
u8	High	No	No	No
u9	Very High	Yes	No	Yes
u10	High	Yes	No	No
u11	Very High	Yes	No	No
u12	Normal	No	Yes	No
u13	High	No	Yes	Yes
u14	Normal	No	Yes	No
u15	Normal	Yes	Yes	No
u16	Normal	Yes	No	No
u17	High	Yes	No	No
u18	Very High	Yes	Yes	Yes
u19	Normal	Yes	No	No
u20	Normal	No	Yes	No

Table 2: Patients Information System

Where, U is the set of patients (objects) or registrations of the system, such that $U = \{u1, u2, u3, u4, u5, u6, u7, u8, u9, u10, u11, u12, u13, u14, u15, u16, u1$. The set of conditional attributes is represented by C={Temperature, Flu, Headache } and the set D represented the decision attribute, where D={Dengue}.

According to Pawlak's rough set approach Table 2 is reduced to Table 3 below.

Patients (U)	Conditional Attributes (C)			Decision (D)
	Temperature	Flu	Headache	Dengue
u1	Normal	No	No	No
u3	Very High	No	No	Yes
u4	High	No	Yes	Yes

Table 3: Reduced of Table 1 using Pawlak's rough approach

According to Table 2 above we have the following Decision rules:

$r1$: If patient $Temperature = Normal$ and $Flu = No$ and $Headache = No$ Then $Dengue = No$,

$r2$: If patient $Temperature = Very\ High$ and $Flu = No$ and $Headache = No$ Then $Dengue = Yes$,

$r3$: If patient $Temperature = High$ and $Flu = No$ and $Headache = Yes$ Then $Dengue = Yes$.

According to m-rough set approach Table 2 is reduced to Table 4 below.

Patients (U)	Conditional Attributes (C)		Decision (D)
	Temperature	Headache	Dengue
u1	Normal	No	No
u2	High	No	No
u3	Very High	No	Yes
u4	High	Yes	Yes
u5	Very High	Yes	Yes
u6	High	Yes	Yes
u7	Very High	Yes	Yes
u8	High	No	No
u10	High	No	No
u12	Normal	Yes	No
u15	Normal	Yes	No
u16	Normal	No	No
u19	Normal	No	No

Table 4: Reduced of Table 2 using m -rough set approach

According to Table 4 above we have the following elimination decision rules:

$r1$: If patient $Temperature = Normal$ and $Headache = No$ Then $Dengue = No$,

$r2$: If patient $Temperature = High$ and $Headache = No$ Then $Dengue = No$,

$r3$: If patient $Temperature = Very\,High$ and $Headache = No$ Then $Dengue = Yes$,

$r4$: If patient $Temperature = High$ and $Headache = Yes$ Then $Dengue = Yes$,

$r5$: If patient $Temperature = Very\,High$ and $Headache = Yes$ Then $Dengue = Yes$,

$r10$:If patient $Temperature = Normal$ and $Headache = Yes$ Then $Dengue = No$,

Using our approach we obtained rich decision rules that support many fields of applications in computer science such as artificial intelligence.

5. Conclusion

The objectives of this work are to study a new alternative methods of data mining. It is about the rough sets theory and its generalizations to topological notions used for the mining of decision rules. The advantage of these generalizations are a mathematic base of rough sets and the possibility of mathematic description of this problem.

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