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SOFT WEAK BAIRE SPACES

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Abstract

It has been noticed that among the different fields of theoretical mathematics where soft sets theory is applied extensively, the most of papers are related to topology. In this paper, the concept of soft weak baire spaces, i.e., the soft generalization of the concept of weak baire spaces as defined by Renukadevi and Muthulakshmi [1] has been presented. Finally, the basic properties of such spaces and accordingly the defined continuous soft functions between these spaces have been studied.

Keywords: soft set; soft weak structure, soft weak baire space.

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1 Introduction

Many real life problems have uncertainties and so many theories have emerged to address it like probability, fuzzy and rough sets and interval mathematics theory. But these are not enough to represent the continuously appeared uncertainties in many branches, such as medicine, economics, engineering, social sciences, etc.

The Russian mathematician Molodtsov [2] introduced the concept of soft set as an extension of crisp sets. He successfully applied it to many areas such as probability theory, operation researches, game theory, smoothness of functions, Perron integration, theory of measurement and so on. Since then, the theory of soft sets has been widely and intensively discussed.

The notion of soft topology was initiated by Shabir and Naz [3] as a generalization of general topology. Since then, it has been widely and deeply discussed [4–10]. Császár [11] defined the generalized topology μ on set X by neglecting X from the collection μ and dropping the finite intersection condition. The properties of generalized topology are investigated by Császár and many other authors (see [12–15]).

Császár continued to try to find a more general structure from general topology, generalized topology, and minimal structure. In 2010, Császár [11] introduced the notion of weak structures and proved that it can replace the already defined structures. A sub-collection $w \subset P(X)$ is said to be a weak structure on X if and only if it contains the empty set. Its properties have been investigated intensively (see [16–22]).

Recently, Zakari et al. [23] integrated the soft sets theory with weak structures to define the soft weak structures. Moreover, they discussed and verified the separation axioms and compactness of soft weak structures. Their results are an extension to the corresponding notions in weak structure, minimal structure, generalized topology, general topology and soft topology. However, Shi and Pang [24] demonstrated the redundancies of soft topologies that this can not be achieved on soft weak structures. Also, Zakari et al. [23] proved that many properties in weak structures can not be achieved in soft weak structures.

The applications of baire spaces are varied in complete metric spaces. To improve its applications, some spaces like hyperspace and Volterra space have been studied by some researchers [25,26]. Later on, m-baire spaces are introduced by Chakrabarti and Dasgupta [27]. The aim of this paper is to introduce and study soft weak baire spaces. Therefore, some characterizations and properties of soft weak baire spaces are investigated. Finally, the soft images and inverse soft images of such spaces are presented.

2 Preliminaries

In the sequel, X and E refers to a non-empty set and the parameters set, respectively. 2^X denotes the power set of X and A be a non-empty subset of E. The function $f_A : A \longrightarrow 2^X$ is said to be a soft set [2] over X. For each $e \in A$, $f_A(e)$ is the set of e-approximate elements of the soft set f_A. A soft set f_A is called a finite (resp. countable) soft set if $f_A(e)$ is finite (resp. countable) for any $e \in A$. In 2008, Majumdar and Samanta [28] extended the soft set by re-defining it as a function $f_A : E \longrightarrow 2^X$ such that $f_A(e) \neq \emptyset$ if $e \in A \subset E$ and $f_A(e) = \emptyset$ if $e \notin A$. From now on, $\tilde{X}_E$ refers to the collection of all soft sets over X with respect to the attributes set E. A null (resp. absolute) soft set $f_\emptyset \in \tilde{X}_E$ (resp. $f_E \in \tilde{X}_E$), denoted by Φ (resp. $\tilde{X}$), defined by $f_\emptyset(e) = \emptyset$ (resp. $f_E(e) = X$) for all $e \in E$. For any two soft sets $f_A, g_B \in \tilde{X}_E$, we have:

(1) $f_A \sqsubseteq g_B$ iff $f_A(e) \subseteq g_B(e)$ for any $e \in E$.

(2) $f_A = g_B$ iff $f_A \sqsubseteq g_B$ and $g_B \sqsubseteq f_A$.

(3) $f_A \sqcup g_B = h_C$, where $h_C(e) = f_A(e) \cup g_B(e)$ for any $e \in E$ and $C = A \cup B$.

(4) $f_A \sqcap g_B = k_D$, where $k_D(e) = f_A(e) \cap g_B(e)$ for any $e \in E$ and $D = A \cap B$.

(5) If $f_A \in \tilde{X}_E$, then the complement of f_A, given by f'_A, is given by $f'_A(e) = X - f_A(e)$. It is clear that $\Phi' = \tilde{X}$ and $\tilde{X}' = \Phi$.

(6) If $Y \subset X$ and $f_A \in \tilde{X}_E$, then the soft set ${}^Y f_A$ is given by ${}^Y f_A(e) = Y \cap f_A(e)$ for any $e \in A$, i.e., ${}^Y f_A = \tilde{Y} \sqcap f_A$.

Definition 1. *[29] The soft point $(x_0)_A$, where $A \subset E$, is a soft set defined by $(x_0)_A(e) = \{x_0\}$ for each $e \in A$ and some $x_0 \in X$, and $(x_0)_A(e) = \emptyset$ for each $e \in E - A$. If $A = E$, then $(x_0)_E$ is called absolute soft point. A soft point $(x_0)_A \tilde{\in} g_B$ if $x_0 \in g_B(e)$ for each $e \in A$.*

Aygünoğlu and Aygün [29] proved that $(x_0)_A \tilde{\in} g_B \sqcup h_C$ does not lead to $(x_0)_A \tilde{\in} g_B$ or $(x_0)_A \tilde{\in} h_C$, while this leads to the existence of two soft points $(x_0)_{A_1} \tilde{\in} g_B$ and $(x_0)_{A_2} \tilde{\in} h_C$ such that $(x_0)_A = (x_0)_{A_1} \sqcup (x_0)_{A_2}$.

Definition 2. *[30] Let $\tilde{X}_E$ and $\tilde{Y}_F$ be two collections of soft sets, and let $\alpha : X \longrightarrow Y$, $\beta : E \longrightarrow F$ be two functions. The image and preimage of a soft set under the soft function $\varphi_{\beta\alpha} : \tilde{X}_E \longrightarrow \tilde{Y}_F$ are given by*

$$\varphi_{\beta\alpha}(f_A)(s) = \begin{cases} \bigcup\limits_{e \in \beta^{-1}(s) \cap A}, & \text{if } \beta^{-1}(s) \cap A \neq A; \\ \emptyset, & \text{otherwise.} \end{cases}$$

and

$$\varphi^{-1}_{\beta\alpha}(g_B)(e) = \begin{cases} \alpha^{-1}\left(g_B(\beta(e))\right), & \text{if } \beta(e) \in B; \\ \emptyset, & \text{otherwise.} \end{cases}$$

for each $f_A \in \tilde{X}_E$ and $g_B \in \tilde{Y}_F$. A soft function $\varphi_{\beta\alpha}$ is said to be injective (resp. surjective) if both α and β are injective (resp. surjective).

For additional soft set properties and operations, we refer to [2–10,23,28–31].

Definition 3. *[23] Let $\tilde{X}_E$ be a collection of soft sets on X with respect to the set of attributes E. Then, the collection $\sigma \subset \tilde{X}_E$ is called a soft weak structure iff $\Phi \in \sigma$. The triplet (X, E, σ) is said to be a soft weak structure.*

A soft weak structure (X, E, σ) have the property **I** if σ is closed under finite soft intersection. A soft set f_A is said to be σ-open (resp. σ-closed) soft set iff $f_A \in \sigma$ (resp. $f'_A \in \sigma$). Moreover, σ is said to be a strong soft weak structure if $\tilde{X} \in \sigma$.

Proposition 1. *[23] For any soft weak structure (X, E, σ), $\sigma_e = \{f_A(e) | f_A \in \sigma\}$ is a weak structure on X for any $e \in E$.*

Definition 4. *[23] For any soft weak structure* (X, E, σ) *and* $Y \subset X$*, the relative soft weak structure or the soft weak substructure* (Y, E, σ_Y) *is defined by* $\sigma_Y = \{^Y f_A : f_A \in \sigma\}$.

For any soft weak structure (X, E, σ) and $f_A \in \tilde{X}_E$, Zakari et al. [23] defined $\widehat{f_A}$ as the soft intersection of all σ-closed soft supsets of f_A and $\widetilde{f_A}$ as the soft union of all σ-open soft subsets of f_A. The following theorem presents the main properties of the introduced operators:

Theorem 1. *[23] For any soft weak structure* (X, E, σ) *and soft sets* f_A, $g_B \in \tilde{X}_E$*, we have the following properties:*

(1) $\widetilde{f_A} \sqsubseteq f_A \sqsubseteq \widehat{f_A}$.

(2) If $f_A \sqsubseteq g_B$*, then* $\widetilde{f_A} \sqsubseteq \widetilde{g_B}$ *and* $\widehat{f_A} \sqsubseteq \widehat{g_B}$.

(3) $\widetilde{\widetilde{f_A}} = \widetilde{f_A}$ *and* $\widehat{\widehat{f_A}} = \widehat{f_A}$.

(4) $\widetilde{(f'_A)} = \left(\widehat{f_A}\right)'$ *and* $\widehat{(f'_A)} = \left(\widetilde{f_A}\right)'$.

For any soft weak structure (X, E, σ) and $f_A \in \tilde{X}_E$, if f_A is σ-closed (resp. σ-open) soft set, then $f_A = \widehat{f_A}$ (resp. $f_A = \widetilde{f_A}$). The converse need not be true in general as shown in [23]. A soft set f_A is called σ^*-open (resp. σ^*-closed) soft set if $\widetilde{f_A} = f_A$ (resp. $\widehat{f_A} = f_A$). By $\mathcal{G}$, we refer to the collection of all σ^*-open soft sets with respect to the soft weak structure (X, E, σ). It is easily to verify that $\mathcal{G}$ is a generalized soft topology.

Proposition 2. *[23] Let* (X, E, σ) *be a soft weak structure and* $g_B \in \tilde{X}_E$*. Then:*

(a) If there exists a soft set $h_C \in \sigma$ *such that* $(x_0)_A \tilde{\in} h_C \sqsubset g_B$*, then* $(x_0)_A \tilde{\in} \widetilde{g_B}$.

(b) $(x_0)_A \tilde{\in} \widehat{g_B}$ *iff* $h_C \sqcap g_B \neq \Phi$ *for all* $h_C \in \sigma$ *such that* $(x_0)_A \tilde{\in} h_C$.

Proposition 3. *[23] Let* σ_Y *be a soft weak substructure of* σ *and* $f_A \in \tilde{Y}_E$*. Then* f_A *is* σ_Y*-closed soft set if and only if* $f_A = \tilde{Y} \sqcap g_B$ *for some* σ*-closed soft set* g_B.

Proposition 4. *[23] Let* σ_Y *be a soft weak substructure of* σ *and* $f_A, g_B \in \tilde{X}_E$*. Then the following statements hold:*

(1) $\widehat{g_B}^Y = \widehat{f_A} \sqcap \tilde{Y}$.

(2) $\widetilde{f_A} \sqcap \tilde{Y} \sqsubseteq \widetilde{f_A}^Y$.

Where $\widehat{g_A}^Y$ *and* $\widetilde{f_A}^Y$ *denote the relative closure and relative interior of* g_B *and* f_A*, respectively.*

Theorem 2. *Let* (X, E, σ) *be a soft weak structure and* (Y, E, σ_Y) *be open soft weak substructure of* (X, E, σ) *having the* **I***-property. If* g_B *is a* σ^**-open soft set, then* $g_B \sqcap \tilde{Y}$ *is a* σ^*_Y*-open soft set.*

Proof. Let g_B be a σ^*-open soft set and $(x_0)_A \tilde{\in} g_B \sqcap \tilde{Y}$, then $\widetilde{g_B} = g_B$. Since $(x_0)_A \tilde{\in} g_B$ implies that $(x_0)_A \tilde{\in} \widetilde{g_B}$ which implies that $(x_0)_A \in \widetilde{g_B} \sqcap \tilde{Y}$ and so $(x_0)_A \tilde{\in} \widetilde{g_B}^Y$. Also, it follows that $(x_0)_A \tilde{\in} \widetilde{\tilde{Y}}^Y$. Therefore, $(x_0)_A \tilde{\in} \widetilde{g_B} \sqcap \widetilde{\tilde{Y}}$. Since σ has the property **I**, σ_Y has the same property and so $(x_0)_A \in \widetilde{g_B \sqcap \tilde{Y}}^Y$. Therefore, $g_B \sqcap \tilde{Y} \sqsubseteq \widetilde{g_B \sqcap \tilde{Y}}^Y$. Since $\widetilde{g_B \sqcap \tilde{Y}}^Y \sqsubseteq g_B \sqcap \tilde{Y}$. Hence $g_B \sqcap \tilde{Y}$ is a σ^*-open soft set in $\tilde{Y}_E$. □

Definition 5. *[23] Let* (X, E, σ) *be a soft weak structure and* $f_A \in \tilde{X}_E$*. Then* f_A *is called:*

(1) σ*-dense soft set if* $\widehat{f_A} = \tilde{X}$.

(2) σ*-nowhere dense or* σ*-rare soft set if* $\widetilde{\left(\widehat{f_A}\right)} = \Phi$.

Lemma 1. *Let* (X, E, σ) *be a soft weak structure and* $f_A \in \tilde{X}_E$*. Then the following statements hold:*

(1) If f_A *is a* σ*-rare soft set, then* f'_A *is a* σ*-dense soft set*

(2) If f_A is a σ-dense, σ^*-open soft set, then f'_A is a σ-rare soft set.

Proof. (1) Suppose f_A is σ-rare soft set. Then $\widetilde{\left(\widehat{f_A}\right)} = \Phi$. Now

$$\widehat{(f'_A)} = \left(\widetilde{f_A}\right)' \sqsupset \left(\widetilde{\left(\widehat{f_A}\right)}\right)' = \tilde{X},$$

and so f_A is σ-dense soft set.

(2) Suppose f_A is a σ-dense, σ^*-open soft set. Then $\widehat{f_A} = \tilde{X}$ and $\widetilde{f_A} = f_A$. Now

$$\widetilde{\left(\widehat{(f'_A)}\right)} = \widetilde{\left(\widetilde{(f_A)}'\right)} = \widetilde{(f'_A)} = \widehat{(f_A)}' = \tilde{X}' = \Phi,$$

and so f'_A is σ-rare soft set. □

The converse Lemma 1 (1) is not true in general. Moreover, we can not drop the condition of σ^*-openness in (2) as shown by following example:

Example 1. *Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$, and $\sigma = \{\Phi, f_E, g_E, h_E, k_E\}$ such that f_E, g_E, h_E, and k_E are soft sets given by*

$$\begin{array}{ll} f_E(e_1) = \{x_1\}, & f_E(e_2) = \{x_3\}, \\ g_E(e_1) = \{x_4\}, & g_E(e_2) = \{x_2\}, \\ h_E(e_1) = \{x_2, x_3\}, & h_E(e_2) = \{x_1, x_4\}, \\ k_E(e_1) = \{x_1, x_3\}, & k_E(e_2) = \{x_1, x_3\}. \end{array}$$

Then σ is a soft weak structure. If the soft set l_E is defined by

$$l_E(e_1) = \{x_1, x_2, x_4\}, \quad l_E(e_2) = \{x_2, x_3, x_4\},$$

then l_E is σ-dense soft set but l'_E is not σ-rare soft set. Moreover, l_E is not σ^*-open, where $\widetilde{l_E} = m_E \neq l_E$, and

$$m_E(e_1) = \{x_1, x_4\}, \quad m_E(e_2) = \{x_2, x_3\}.$$

Definition 6. *Let (X, E, σ) be a soft weak structure. The soft set f_A is called:*

*(1) σ-first category (briefly, σ-**FCat**) soft set if f_A can be written as a countable soft union of σ-rare soft sets.*

*(2) σ-second category (briefly, σ-**SCat**) soft set if it is not a σ-**FCat**.*

Theorem 3. *For a soft weak structure (X, E, σ), we have the following statements:*

*(1) (X, E, σ) is of the σ-**SCat**, if $\prod_{n=1}^{\infty} f_{A_n} \neq \Phi$ for each countable family $\{f_{A_n} : n \in \mathbb{N}\}$ of σ-dense soft sets.*

*(2) (X, E, σ) is of the σ-**SCat**, if $\prod_{n=1}^{\infty} f_{A_n} \neq \Phi$ for each countable family $\{f_{A_n} : n \in \mathbb{N}\}$ of σ-dense and σ^*-open soft sets.*

Proof. (1) Suppose that $\prod_{n=1}^{\infty} f_{A_n} = \Phi$ for each countable family $\{f_{A_n} : n \in \mathbb{N}\}$ of σ-dense soft sets, then $\bigsqcup_{n=1}^{\infty} f_{A_n} = \tilde{X}$ where f_{A_n} is a σ-rare soft set. Therefore,

$$\left(\bigsqcup_{n=1}^{\infty} f_{A_n}\right)' = \Phi = \prod_{n=1}^{\infty} f'_{A_n}$$

is a soft intersection of σ-dense soft sets, which contradicts with the hypothesis. Thus $\bigsqcup_{n=1}^{\infty} f_{A_n} = \tilde{X}$ for each collection of σ-rare soft sets. Hence (X, E, σ) is of the σ-**SCat**.

(2) If $\{f_{A_n} : n \in \mathbb{N}\}$ is a countable collection of σ-dense σ^*-open soft sets. Then $\{f'_{A_n} : n \in \mathbb{N}\}$ is a countable collection of σ-rare soft sets. Then $\bigsqcup_{n=1}^{\infty} f'_{A_n}$ is of the σ-**FCat**. But (X, E, σ) is of the σ-**SCat**, then $\bigsqcup_{n=1}^{\infty} f'_{A_n} \neq \tilde{X}$ and so $\left(\prod_{n=1}^{\infty} f_{A_n}\right)' \neq \tilde{X}$. Therefore $\prod_{n=1}^{\infty} f_{A_n} \neq \Phi$.

□

Corollary 1. *A soft weak structure (X, E, σ) is of σ-**Scat** in itself if $\prod_{n=1}^{\infty} f_{A_n} \neq \Phi$ for each countable collection $\{f_{A_n} : n \in \mathbb{N}\}$ of σ-dense and σ^*-open soft sets.*

3 Soft functions and soft weak structures

Definition 7. *Let (X, E, σ_1) and (Y, F, σ_2) be two soft weak structures and $\varphi_{\beta\alpha} : \tilde{X}_E \longrightarrow \tilde{Y}_F$ be a soft function. Then, $\varphi_{\beta\alpha}$ is said to be:*

(1) σ-open if $\varphi_{\beta\alpha}(f_A) \in \sigma_2$, for each $f_A \in \sigma_1$.

(2) almost σ-open if $\varphi_{\beta\alpha}(\widetilde{f_A}) \sqsubseteq \widetilde{\varphi_{\beta\alpha}(f_A)}$, for each $f_A \in \tilde{X}_E$.

(3) σ-continuous if for each $(x_0)_A \in \tilde{X}_E$ and $g_{\beta(A)} \in \sigma_2$ containing $\varphi_{\beta\alpha}((x_0)_A)$, there exists $h_C \in \sigma_1$ containing $(x_0)_A$ such that $\varphi_{\beta\alpha}(h_C) \sqsubseteq g_{\beta(A)}$.

Lemma 2. *Let (X, E, σ_1) and (Y, F, σ_2) be two soft weak structures and $\varphi_{\beta\alpha} : \tilde{X}_E \longrightarrow \tilde{Y}_F$ be a soft function:*

(1) $\varphi_{\beta\alpha}$ is σ-continuous soft function.

(2) $\varphi^{-1}_{\beta\alpha}(g_B) = \widetilde{\varphi_{\beta\alpha}(g_B)}$, for each $g_B \in \sigma_2$.

(3) $\varphi_{\beta\alpha}(\widehat{f_A}) \sqsubset \widehat{\varphi_{\beta\alpha}(f_A)}$, for each $f_A \in \tilde{X}_E$.

(4) $\widehat{\varphi^{-1}_{\beta\alpha}(g_B)} \sqsubset \varphi^{-1}_{\beta\alpha}(\widehat{g_B})$, for each $g_B \in \tilde{Y}_F$.

(5) $\varphi^{-1}_{\beta\alpha}(\widetilde{g_B}) \sqsubset \widetilde{\varphi^{-1}_{\beta\alpha}(g_B)}$, for each $g_B \in \tilde{Y}_F$.

(6) $\widehat{\varphi^{-1}_{\beta\alpha}(g_B)} = \varphi_{\beta\alpha}(g_B)$, for each $g_B \in \tilde{Y}_F$ such that $g'_B \in \sigma_2$.

Proof. (1) $\Rightarrow$ (2) : Let $g_B \in \sigma_2$ and $(x_0)_A \tilde{\in} \varphi^{-1}_{\beta\alpha}(g_B)$. Then $\varphi_{\beta\alpha}((x_0)_A) \tilde{\in} g_B$. Then there exists $h_C \in \sigma_1$ such that $(x_0)_A \tilde{\in} h_C$ and $\varphi_{\beta\alpha}(h_C) \sqsubset g_B$. Thus $(x_0)_A \tilde{\in} h_C \sqsubset \varphi^{-1}_{\beta\alpha}(g_B)$ and so $(x_0)_A \tilde{\in} \widetilde{\varphi^{-1}_{\beta\alpha}(g_B)}$. Hence $\varphi^{-1}_{\beta\alpha}(g_B) \sqsubset \widetilde{\varphi^{-1}_{\beta\alpha}(g_B)}$. By Theorem 1, we have $\widetilde{\varphi^{-1}_{\beta\alpha}(g_B)} \sqsubset \varphi^{-1}_{\beta\alpha}(g_B)$. Therefore, $\varphi_{\beta\alpha}(g_B) = \widetilde{\varphi_{\beta\alpha}(g_B)}$.

(2) $\Rightarrow$ (3) : Let $f_A \in \tilde{X}_E$, $(x_1)_B \tilde{\in} \widehat{f_A}$, and $h_C \in \sigma_2$ such that $\varphi_{\beta\alpha}((x_1)_B) \tilde{\in} h_C$. Then $(x_1)_B \tilde{\in} \varphi_{\beta\alpha}(h_C) = \widetilde{\varphi_{\beta\alpha}(h_C)}$. There exists $k_D \in \sigma_1$ such that $(x_1)_B \in k_D \sqsubset \varphi_{\beta\alpha}(h_C)$. Since

$$\varphi_{\beta\alpha}(k_D \sqcap f_A) \sqsubset \varphi_{\beta\alpha}(k_D) \sqcap \varphi_{\beta\alpha}(f_A) \sqsubset \varphi_{\beta\alpha}(k_D) \sqcap h_C$$

implies that $h_C \sqcap \varphi_{\beta\alpha}(f_A) \neq \Phi$. Since $h_C \in \sigma_2$ such that $\varphi_{\beta\alpha}((x_1)_B) \tilde{\in} h_C$, then $\varphi_{\beta\alpha}((x_1)_B) \tilde{\in} \widehat{\varphi_{\beta\alpha}(f_A)}$ and therefore $\varphi_{\beta\alpha}(\widehat{f_A}) \sqsubset \widehat{\varphi_{\beta\alpha}(f_A)}$.

(3) $\Rightarrow$ (4) : Let $g_B \in \tilde{Y}_F$, then

$$\varphi_{\beta\alpha}\left(\widehat{\varphi^{-1}_{\beta\alpha}(g_B)}\right) \sqsubset \left(\varphi_{\beta\alpha}\left(\widehat{\varphi^{-1}_{\beta\alpha}(g_B)}\right)\right) \sqsubset \widehat{g_B}.$$

Hence $\widehat{\varphi^{-1}_{\beta\alpha}(g_B)} \sqsubset \varphi^{-1}_{\beta\alpha}(\widehat{g_B})$.

$(4) \Rightarrow (5)$: Let $g_B \in \tilde{Y}_F$, then

$$\left(\widetilde{\varphi^{-1}_{\beta\alpha}(g_B)}\right)' = \widehat{\varphi^{-1}_{\beta\alpha}(g_B)'} = \widehat{\varphi^{-1}_{\beta\alpha}(g'_B)} \sqsubset \varphi^{-1}_{\beta\alpha}(\widehat{g'_B})$$
$$= \varphi^{-1}_{\beta\alpha}\left((\widetilde{g_B})'\right) = \left(\varphi^{-1}_{\beta\alpha}(\widetilde{g_B})\right)'.$$

Hence, $\varphi^{-1}_{\beta\alpha}(\widetilde{g_B}) \sqsubset \varphi^{-1}_{\beta\alpha}(\widehat{g_B})$.

$(5) \Rightarrow (6)$: Let $g_B \in \tilde{Y}_F$ such that $g'_B \in \sigma_2$. By using (5), we have

$$\varphi^{-1}_{\beta\alpha}(g_B)' = \varphi^{-1}_{\beta\alpha}\left(\widetilde{g'_B}\right) \sqsubset \widetilde{\varphi^{-1}_{\beta\alpha}(g'_B)} = \widetilde{\varphi^{-1}_{\beta\alpha}(g_B)'} = \widehat{\varphi^{-1}_{\beta\alpha}(g_B)}'.$$

Hence,

$$\widehat{\varphi^{-1}_{\beta\alpha}(g_B)} \sqsubset \varphi^{-1}_{\beta\alpha}(g_B) \sqsubset \widehat{\varphi^{-1}_{\beta\alpha}(g_B)}.$$

Therefore, $\widehat{\varphi^{-1}_{\beta\alpha}(g_B)} = \varphi^{-1}_{\beta\alpha}(g_B)$.

$(6) \Rightarrow (1)$: Let $(x_0)_A \tilde{\in} \tilde{X}_E$ and $g_B \in \sigma_2$ such that $\varphi_{\beta\alpha}((x_0)_A) \tilde{\in} g_B$. By using (6), we have

$$\varphi^{-1}_{\beta\alpha}(g_B)' = \varphi^{-1}_{\beta\alpha}(g'_B) = \widehat{\varphi^{-1}_{\beta\alpha}(g'_B)} = \left(\widehat{\varphi^{-1}_{\beta\alpha}(g_B)'}\right) = \left(\widetilde{\varphi^{-1}_{\beta\alpha}(g_B)}\right)'.$$

Therefore, $(x_0)_A \tilde{\in} \varphi^{-1}_{\beta\alpha}(g_B) = \widetilde{\varphi^{-1}_{\beta\alpha}(g_B)}$. Hence, there exists $h_C \in \sigma_1$ such that $(x_0)_A \tilde{\in} h_C \sqsubset \varphi^{-1}_{\beta\alpha}(g_B)$. Then, h_C is σ_1-open soft set such that $(x_0)_A \tilde{\in} h_C$ and $\varphi_{\beta\alpha}(h_C) \sqsubset g_B$. Therefore, $\varphi_{\beta\alpha}$ is σ-continuous soft function. □

Definition 8. *Let (X, E, σ_1) and (Y, F, σ_2) be two soft weak structures and $\varphi_{\beta\alpha} : \tilde{X}_E \longrightarrow \tilde{Y}_F$ be a soft function. Then, $\varphi_{\beta\alpha}$ is said to be:*

(1) feebly σ-continuous (briefly, $f\sigma c$) if $\widetilde{\varphi^{-1}_{\beta\alpha}(g_B)} \neq \Phi$ for each $g_B \in \tilde{Y}_F$ such that $\widetilde{g_B} \neq \Phi$.

(2) feebly σ-open (briefly, $f\sigma o$) if $\widetilde{\varphi_{\beta\alpha}(f_A)} \neq \Phi$ for each $f_A \in \tilde{X}_E$ such that $\widetilde{f_A} \neq \Phi$

Theorem 4. *Let (X, E, σ_1) and (Y, F, σ_2) be two soft weak structures. A soft function $\varphi_{\beta\alpha} : \tilde{X}_E \longrightarrow \tilde{Y}_F$ is:*

(1) $f\sigma c$ if and only if $\varphi_{\beta\alpha}(f_A)$ is σ_2-dense soft set for each σ_1-dense soft set $f_A \in \tilde{X}_E$.

(2) $f\sigma o$ if and only if $\varphi^{-1}_{\beta\alpha}(g_B)$ is σ_1-dense soft set for each σ_2-dense soft set $g_B \in \tilde{Y}$.

Proof. (1) Suppose that $\varphi_{\beta\alpha}(f_A)$ is σ_2-dense soft set for each σ_1-dense soft set $f_A \in \tilde{X}_E$ and let $\Phi \neq g_B \in \tilde{Y}_F$ such that $\widetilde{g_B} \neq \Phi$. Suppose that $\widetilde{\varphi^{-1}_{\beta\alpha}(g_B)} = \Phi$, then $\tilde{X} = \left(\widetilde{\varphi^{-1}_{\beta\alpha}(g_B)}\right)' = \widehat{\varphi^{-1}_{\beta\alpha}(g_B)'}$. Then $\varphi^{-1}_{\beta\alpha}(g_B)'$ is σ_1-dense soft set. By the hypothesis, $\varphi_{\beta\alpha}\left(\varphi^{-1}_{\beta\alpha}(g_B)'\right)$ is σ_2-dense soft set and so

$$\tilde{Y} = \left(\varphi_{\beta\alpha}\left(\widehat{\varphi^{-1}_{\beta\alpha}(g_B)'}\right)\right) = \varphi_{\beta\alpha}\widehat{\varphi^{-1}_{\beta\alpha}(g'_B)} \sqsubset \widehat{g'_B} = \widetilde{g_B}',$$

which implies $\widetilde{g_B} = \Phi$, a contradiction. Therefore, $\widetilde{\varphi^{-1}_{\beta\alpha}(g_B)} \neq \Phi$ and hence $\varphi_{\beta\alpha}$ is $f\sigma c$-soft function.

Now, let $\varphi_{\beta\alpha}$ be a $f\sigma c$-soft function and $f_A \in \tilde{X}_E$ be a σ_1-dense soft set. Then $\widehat{f_A}' = \Phi$ and hence $\widetilde{f'_A} = \Phi$ which implies that $\left(\widetilde{\varphi^{-1}_{\beta\alpha}\varphi_{\beta\alpha}(f_A)}\right)' = \Phi$. Thus $\varphi^{-1}_{\beta\alpha}\left(\widetilde{\varphi_{\beta\alpha}(f_A)'}\right) = \Phi$. By the hypothesis, $\widetilde{\varphi_{\beta\alpha}(f_A)}' = \Phi$. Hence $\widehat{\varphi_{\beta\alpha}(f_A)}' = \Phi$ and so $\varphi_{\beta\alpha}(f_A) \in \tilde{Y}_F$ is σ_2-dense soft set.

(2) Suppose that $g_B \in \tilde{Y}_F$ is σ_2-dense soft set implies $\varphi_{\beta\alpha}^{-1}(g_B) \in \tilde{X}_E$ is σ_1-dense soft set and let $f_A \in \tilde{X}_E$ such that $\widetilde{f_A} \neq \Phi$. If $\widetilde{\varphi_{\beta\alpha}(f_A)} = \Phi$, then $\widetilde{\varphi_{\beta\alpha}(f_A)}' = \tilde{Y}$, which implies that $\widehat{\varphi_{\beta\alpha}(f_A)'} = \tilde{Y}$. By the hypothesis, $\varphi_{\beta\alpha}^{-1}\widehat{(\varphi_{\beta\alpha}(f_A)')} = \tilde{X}$ which implies that $\widehat{f'_A} = \tilde{X}$ so that $\widetilde{f_A}' = \tilde{X}$ and hence $\widetilde{f_A} = \Phi$, which is a contradiction. Therefore, $\widetilde{\varphi_{\beta\alpha}(f_A)} \neq \Phi$.

Now, suppose that $\varphi_{\beta\alpha}$ is $f\sigma o$ and $g_B \in \tilde{Y}_F$ is σ_2-dense soft set. Then $\widehat{g_B} = \tilde{Y}$ and hence $\widetilde{g'_B} = \Phi$ and so $\varphi_{\beta\alpha}\left(\widetilde{\varphi_{\beta\alpha}^{-1}(g_B)}\right) = \Phi$. By the hypothesis, we have $\widetilde{\varphi_{\beta\alpha}^{-1}(g'_B)} = \Phi$. Since

$$\widetilde{\varphi_{\beta\alpha}^{-1}(g'_B)} = \widetilde{\varphi_{\beta\alpha}^{-1}(g_B)'} = \left(\widehat{\varphi_{\beta\alpha}^{-1}(g_B)}\right)'.$$

Then $\widehat{\varphi_{\beta\alpha}^{-1}(g_B)} = \tilde{X}$. Therefore $\varphi_{\beta\alpha}^{-1}(g_B)$ is σ_1-dense soft set.

□

Theorem 5. *Every σ-continuous soft function is $f\sigma c$.*

Proof. Let (X, E, σ_1) and (Y, F, σ_2) be two soft weak structures and $\varphi_{\beta\alpha} : \tilde{X}_E \longrightarrow \tilde{Y}_F$ be σ-continuous soft function. If $g_B \in \tilde{Y}$ such that $\widetilde{g_B} \neq \Phi$, by using Lemma 2, we have $\varphi_{\beta\alpha}^{-1}(\widetilde{g_B}) \sqsubset \widetilde{\varphi_{\beta\alpha}^{-1}(g_B)}$. Since $\varphi_{\beta\alpha}$ is onto, $\varphi_{\beta\alpha}^{-1}(\widetilde{g_B}) \neq \Phi$, and hence $\widetilde{\varphi_{\beta\alpha}^{-1}(g_B)} \neq \Phi$. Therefore, $\varphi_{\beta\alpha}$ is $f\sigma c$. □

Theorem 6. *Every almost σ-open soft function is $f\sigma o$.*

Proof. Let (X, E, σ_1) and (Y, F, σ_2) be two soft weak structures, and $\varphi_{\beta\alpha} : \tilde{X}_E \longrightarrow \tilde{Y}_F$ be almost σ-open soft function. For every $f_A \in \tilde{X}_E$ such that $\widetilde{f_A} \neq \Phi$, we have $\varphi_{\beta\alpha}(\widetilde{f_A}) \neq \Phi$ and hence $\widetilde{\varphi_{\beta\alpha}(f_A)} \neq \Phi$. Therefore, $\varphi_{\beta\alpha}$ is a $f\sigma o$. □

The converse of the above two theorems is not true in general as shown by the following example:

Example 2. *Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$, $\sigma_1 = \{\Phi, f_{1E}, f_{2E}, f_{3E}\}$ and $Y = \{y_1, y_2, y_3\}$, $F = \{k_1, k_2\}$, $\sigma_2 = \{\Phi, g_{1F}, g_{2F}\}$ such that*

$$\begin{aligned}
f_{1E}(e_1) &= \{x_1\}, & f_{1E}(e_2) &= \{x_2\},\\
f_{2E}(e_1) &= \{x_3\}, & f_{2E}(e_2) &= \{x_4\},\\
f_{3E}(e_1) &= \{x_3, x_4\}, & f_{3E}(e_2) &= \{x_1, x_4\},
\end{aligned}$$

$$\begin{aligned}
g_{1F}(k_1) &= \{y_1\}, & g_{1F}(e_2) &= \{y_2\},\\
g_{2F}(k_1) &= \{y_2\}, & g_{2F}(e_2) &= \{y_3\},
\end{aligned}$$

and let the soft function $\varphi_{\beta\alpha} : \tilde{X}_E \longrightarrow \tilde{Y}_F$ defined by:

$$\alpha(x_1) = \alpha(x_2) = y_1, \quad \alpha(x_3) = y_2, \quad \alpha(x_4) = y_3,$$

$$\beta(e_1) = k_1, \quad \beta(e_2) = k_2.$$

It is clear that $\varphi_{\beta\alpha}$ is $f\sigma c$ but not σ-continuous since $\varphi_{\beta\alpha}^{-1}(\widetilde{g_{1F}}) \not\sqsubset \widetilde{\varphi_{\beta\alpha}^{-1}(g_{1F})}$. Moreover, $\varphi_{\beta\alpha}$ is $f\sigma o$ soft function but not almost σ-open since $f_{3E} \in \sigma_1$ and $\varphi_{\beta\alpha}(f_{3E}) \notin \sigma_2$.

4 Soft weak baire space

In this section, we present the baire property of soft weak structures. We have called those soft weak structures that have baire property by soft weak baire spaces.

Definition 9. *A soft weak structure $(\tilde{X}_E, \sigma)$ is said to be a soft weak baire space if the soft intersection of each countable collection of σ-dense σ^*-open soft sets is σ-dense.*

Theorem 7. *Every soft weak baire space (X, E, σ) is of σ-**SCat**.*

Proof. Let $\{f_{A_n} : n \in \mathbb{N}\}$ be a countable family of σ-dense and σ^*-open soft sets, then $\widehat{\prod_{n=1}^{\infty} f_{A_n}} = \tilde{X}$ and so $\prod_{n=1}^{\infty} f_{A_n} \neq \Phi$. Therefore, (X, E, σ) is of σ-**SCat**. □

Theorem 8. *If (X, E, σ) is a soft weak structure, then (X, E, σ) is a soft weak baire space if and only if $\widetilde{\prod_{n=1}^{\infty} f_{A_n}} = \Phi$ implies $\widetilde{\coprod_{n=1}^{\infty} f_{A_n}} = \Phi$, for each countable family $\{f_{A_n} : n \in \mathbb{N}\}$ of σ^*-closed soft sets.*

Proof. Let $\{f_{A_n} : n \in \mathbb{N}\}$ be a countable family of σ^*-closed soft sets such that $\widetilde{\prod_{n=1}^{\infty} f_{A_n}} = \Phi$ and let $g_{B_n} = (f_{A_n})'$. Then g_{B_n} is σ^*-open soft set such that $\widehat{g_{B_n}} = \widehat{f'_{A_n}} = \left(\widetilde{f_{A_n}}\right)' = \tilde{X}$, i.e., g_{B_n} is σ-dense soft set for each $n \in \mathbb{N}$, i.e., $\{f_{A_n} : n \in \mathbb{N}\}$ is a countable collection of σ-dense σ^*-open soft set. Therefore, $\prod_{n=1}^{\infty} g_{B_n}$ is σ-dense soft set and so $\widehat{g_{B_n}} = \tilde{X}$ which implies that $(\widehat{g_{B_n}})' = \Phi$ and so $\left(\widetilde{\prod_{n=}^{\infty} g_{B_n}}\right) = \Phi$, i.e., $\widetilde{\coprod_{n=1}^{\infty} g'_{B_n}} = \Phi$ and so $\left(\widetilde{\coprod_{n=1}^{\infty} f_{A_n}}\right) = \Phi$.

Now, let $\{f_{A_n} : n \in \mathbb{N}\}$ be a countable collection of σ-dense and σ^*-open soft sets. Then $\{f'_{A_n} : n \in \mathbb{N}\}$ is a countable collection of σ^*-closed soft sets such that $\widetilde{f'_{A_n}} = \Phi$. Then by hypothesis, $\left(\widetilde{\coprod_{n=1}^{\infty} f'_{A_n}}\right) = \Phi$. Now, we have

$$\widehat{\prod_{n=1}^{\infty} f_{A_n}} = \widehat{\prod_{n=1}^{\infty} f''_{A_n}} = \left(\widehat{\coprod_{n=1}^{\infty} f'_{A_n}}\right)' = \left(\widetilde{\coprod_{n=1}^{\infty} f'_{A_n}}\right)'$$
$$= \tilde{X} - \Phi = \tilde{X}.$$

Hence $\prod_{n=1}^{\infty} f_{A_n}$ is σ-dense soft set and so (X, E, σ) is a soft weak baire space. □

Theorem 9. *For any soft weak structure (X, E, σ) with the property **I**, (X, E, σ) is a soft weak baire space if and only if every soft point has a σ-open soft set which is soft weak baire space.*

Proof. Let $\{g_{B_n} : n \in \mathbb{N}\}$ be a countable family of σ-dense and σ^*-open soft sets, and $(x_0)_A \tilde{\in} \tilde{Y}$ where $\tilde{Y} \in \sigma$. Then $\hat{\tilde{Y}} = \widehat{\tilde{Y} \sqcap g_{B_n}}$ and $\widehat{\tilde{Y} \sqcap g_{B_n}}^{Y} = \tilde{Y} \sqcap \widehat{\tilde{Y} \sqcap g_{B_n}}$ for each $n \in \mathbb{N}$. Therefore,

$$\widehat{\tilde{Y} \sqcap g_{B_n}}^{Y} = \tilde{Y} \sqcap \widehat{g_{B_n}} = \tilde{Y}$$

and $\tilde{Y} \sqcap g_{B_n}$ is σ-dense soft set with respect to $\tilde{Y}$ for each $n \in \mathbb{N}$. Now,

$$\widetilde{\tilde{Y} \sqcap g_{B_n}}^{Y} \sqsupset \tilde{Y} \sqcap \left(\widetilde{\tilde{Y} \sqcap g_{B_n}}\right) = \tilde{Y} \sqcap \tilde{\tilde{Y}} \sqcap \widetilde{g_{B_n}} = \tilde{Y} \sqcap g_{B_n}$$

and so $\widetilde{\tilde{Y} \sqcap g_{B_n}}^{Y} = \tilde{Y} \sqcap g_{B_n}$ so that $\tilde{Y} \sqcap g_{B_n}$ is σ^*-open soft set for each $n \in \mathbb{N}$. By the hypothesis, $\tilde{Y}$ is soft weak baire space and so it is of the σ-**SCat**. Hence $\prod_{n=1}^{\infty} \left(\tilde{Y} \sqcap g_{B_n}\right) \neq \Phi$ and so $\tilde{Y} \sqcap \left(\prod_{n=1}^{\infty} g_{B_n}\right) = \Phi$. Therefore, $(x_0)_A \tilde{\in} \left(\widehat{\prod_{n=1}^{\infty} g_{B_n}}\right)$, which implies that $\prod_{n=1}^{\infty} g_{B_n}$ is σ-dense soft set. Therefore (X, E, σ) is a soft weak baire space.

Conversely, if (X, E, σ) is soft weak baire space and $(x_0)_A \tilde{\in} \tilde{Y}$ where $\tilde{Y} \in \sigma$, then $\tilde{Y}$ is a soft weak baire space. Hence, each soft point has a σ-open soft set which is a soft weak baire space. □

Theorem 10. *For any soft weak structure (X, E, σ) has the* **I** *property, (X, E, σ) is a soft weak baire space if and only if every nonempty σ-open soft set is of σ-***SCat**.

Proof. Let $\{g_{B_n} : n \in \mathbb{N}\}$ be a countable family of σ-dense and σ^*-open soft set and let $(x_0)_A \tilde{\notin} \left(\widehat{\prod_{n=1}^{\infty} g_{B_n}}\right)$. Then, there exists a σ-open soft set $\tilde{Y}$ such that $(x_0)_A \tilde{\in} \tilde{Y}$ and $\tilde{Y} \sqcap \left(\prod_{n=1}^{\infty} g_{B_n}\right) = \Phi$ and so $\prod_{n=1}^{\infty} \left(\tilde{Y} \sqcap g_{B_n}\right) = \Phi$. Since $\tilde{Y} \in \sigma$ and g_{B_n} is σ-dense for each $n \in \mathbb{N}$, then $\tilde{Y} \sqcap g_{B_n}$ is σ-dense soft set with respect to $\tilde{Y}$. Moreover,

$$\left(\widehat{\tilde{Y} \sqcap g_{B_n}}\right) \sqsupset \tilde{Y} \sqcap \left(\widehat{\tilde{Y} \sqcap g_{B_n}}\right) = \tilde{Y} \sqcap \widehat{\tilde{Y}} \sqcap \widehat{g_{B_n}} = \tilde{Y} \sqcap g_{B_n}$$

and so $\tilde{Y} \sqcap g_{B_n}$ is σ^*-open soft set. By the hypothesis, $\tilde{Y}$ is of σ-**SCat** and so $\prod_{n=1}^{\infty} \left(\tilde{Y} \sqcap g_{B_n}\right) \neq \Phi$, which is a contradiction. Hence $(x_0)_A \tilde{\in} \left(\widehat{\prod_{n=1}^{\infty} g_{B_n}}\right)$ and so $\prod_{n=1}^{\infty} g_{B_n}$ is σ-dense. Therefore, (X, E, σ) is a soft weak baire space.

Conversely, let (X, E, σ) be a soft weak baire space and $\tilde{Y}$ is σ-open soft set. Then $\tilde{Y}$ is soft weak baire space and hence $\tilde{Y}$ is σ-**SCat**. □

Theorem 11. *Let (X, E, σ_1) and (Y, F, σ_2) be two soft weak structures, and $\varphi_{\beta\alpha} : \tilde{X}_E \longrightarrow \tilde{Y}_F$ be $f\sigma c$ soft function such that $\varphi_{\beta\alpha}^{-1}(g_B) \in \tilde{X}_E$ is σ_1-rare soft set, for each σ_2-rare soft set $g_B \in \tilde{Y}_F$. Then (Y, F, σ_2) is soft weak baire space if (X, E, σ_1) is soft weak baire space.*

Proof. Let (X, E, σ_1) be a soft weak baire space, $\{g_{B_n} : n \in \mathbb{N}\} \subseteq \tilde{Y}_F$ be a collection of σ_2-dense and σ_2^*-open soft sets, and let $h_{C_n} = \widetilde{\varphi_{\beta\alpha}^{-1}(g_{B_n})}$ for each $n \in \mathbb{N}$. Since g_{B_n} is σ_2-dense and σ_2^*-open soft set for each $n \in \mathbb{N}$, we have g'_{B_n} is σ_2-rare soft set and hence $\varphi_{\beta\alpha}^{-1}(g'_{B_n})$ is σ_1-rare soft set. Then

$$h_{C_n} = \left(\widehat{\varphi_{\beta\alpha}^{-1}(g_{B_n})'}\right)' = \left(\widehat{\varphi_{\beta\alpha}^{-1}(g'_{B_n})}\right)',$$

for each $n \in \mathbb{N}$ and $\varphi_{\beta\alpha}^{-1}(g'_{B_n})$ is σ_1-rare soft set, h_{C_n} is a σ_1-dense and σ_1^*-open soft set. Since (X, E, σ_1) is soft weak baire space, $\prod_{n=1}^{\infty} h_{C_n}$ is σ_1-dense soft set. By using Lemma 1, we have $\varphi_{\beta\alpha}\left(\prod_{n=1}^{\infty} h_{C_n}\right)$ is σ_2-dense soft set. By using Theorem 4, $\prod_{n=1}^{\infty} \widetilde{\varphi_{\beta\alpha}^{-1}(g_{B_n})} \sqsubset \varphi_{\beta\alpha}^{-1}(g_{B_n})$ for all $n \in \mathbb{N}$ which implies that $\varphi_{\beta\alpha}\left(\prod_{n=1}^{\infty} \widetilde{\varphi_{\beta\alpha}^{-1}(g_{B_n})}\right) \sqsubset \varphi_{\beta\alpha}\left(\varphi_{\beta\alpha}^{-1}(g_{B_n})\right)$ for each $n \in \mathbb{N}$ which in turn implies to $\varphi_{\beta\alpha}\left(\prod_{n=1}^{\infty} \widetilde{\varphi_{\beta\alpha}^{-1}(g_{B_n})}\right) \sqsubset g_{B_n}$ for each $n \in \mathbb{N}$ and so $\varphi_{\beta\alpha}\left(\prod_{n=1}^{\infty} \widetilde{\varphi_{\beta\alpha}^{-1}(g_{B_n})}\right) \sqsubset \prod_{n=1}^{\infty} g_{B_n}$. Hence $\varphi_{\beta\alpha}\left(\prod_{n=1}^{\infty} h_{C_n}\right) \sqsubset \prod_{n=1}^{\infty} g_{B_n}$. But $\varphi_{\beta\alpha}\left(\prod_{n=1}^{\infty} h_{C_n}\right)$ is σ_2-dense soft set, then so is $\prod_{n=1}^{\infty} g_{B_n}$. Therefore, (Y, F, σ_2) is soft weak baire space. □

5 Conclusion

In this paper, we studied soft weak baire spaces as an extension to weak baire spaces given in [1]. Further, we presented some soft functions between it. Also, the relation with the second category axiom is introduced and discussed. A soft weak baire space is a parameterized family of weak baire spaces. So, we think that our results will play, in fact, an important role in soft version of analysis, topology and mathematical logic. Therefore, we suppose that this is an extra justification for the results studied in this paper. In future, we intend to introduce the soft hereditary classes [32] and modify soft weak baire spaces based on it and the future research will be performed in the same direction.

6 Compliance with ethical standards

Conflict of interest: The authors declare that they have no conflict of interest.

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